Chapter 2

Beyond Standard Model scenarios

There are numerous beyond standard model (BSM) scenarios proposed to address the problems that are not answered in the standard model (SM), as well as to look for the possible new physics that could stand as a more fundamental theory. The SM would then correspond to the low energy effective theory of this fundamental theory. Super symmetry (SUSY), extra dimensions, techni-colour, unparticles, little Higgs and $Z'$ models are a few to name. Each of these scenarios are interesting in its own right when it comes to the understanding of how these models help address various issues and how rich and interesting will be the signals of these models at the collider experiments such as those at the Tevatron or the LHC. The gauge hierarchy problem is one among such issues. In SUSY, corresponding to each of the particles in the SM there is a super partner, called sparticle but with a different spin than that of the particle. For each of the fermions in the SM there is a corresponding boson and for each of the gauge boson in the SM there is a fermion. In this model the particle spectrum is rich and in number it contains almost the double of the particles of the SM. The existence of these super partners explains some of the problems in the SM and also gives rise to a much interesting particle phenomenology. But the fact that we do not see super particles in nature breaks the super symmetry and pushes the super particle spectrum to higher energy scales, may be of the order of a TeV scale.

The concept of extra dimensions is first introduced by Kaluza and Klein. These
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models are based on the idea that the space-time consists of $4 + n$ dimensions rather than the conventional 4 space-time dimensions. The space of $n$ extra dimensions is called bulk whereas the usual 4 dimensional space-time is called a domain wall (may be a brane, but need not be always). The compactification of the extra dimensions lead to a tower of modes. Thus a particle propagating the extra spatial dimensions will correspond to a tower of modes on the brane. In the case of universal extra dimensional model [1] (UED), all the SM particles are allowed to propagate the extra dimensions. As the SM has been tested down to $10^{-16}$ cm., and found no evidence for the new extra dimensions which the SM particles can traverse, the size of the extra dimensions is expected to be smaller than this length scale. Consequently, the modes will be very heavy and hence can be observed only in high energy collider experiments.

In the case of both large extra dimension model [2] (LED) and warped extra dimension model [3] (WED), only the gravity is allowed to propagate the extra dimensions while all the SM fields are confined to the brane. The concept of the localization of the SM fields to a domain wall comes from the string theory, according to which all the SM fields are harmonic excitations of the open strings whose one end lie on the brane and consequently they can not leave it and propagate into the bulk. Whereas the gravity and other anti-symmetric fields correspond to closed strings which can live in the bulk as well as on the brane. Hence only gravity is allowed to propagate the extra dimensions. Because the gravity has been tested down to sub millimeter range so far, the size of the extra dimensions in the LED could be as large as this length scale. In the case of warped extra dimension model unlike UED or LED case, the extra dimension is not flat but highly warped. Because of this large warp factor, the size of the extra dimension could be very small, may be of the order of Planck’s length. Both the LED and WED scenarios offer an explanation to the hierarchy problem. In what follows, we concentrate on both the large and warped extra dimension models and discuss them in detail. First, we begin with the Kaluza-Klein theories.
2.1 Extra Dimension models

2.1.1 Kaluza-Klein theories

One of the initial attempts to unify the long range forces seen in the nature, that of gravity and electromagnetism, were made by Kaluza and Klein (KK) with their theories of extra spatial dimensions [5]. The conventional electromagnetic $A_\mu$ and gravitational fields $h_{\mu\nu}$ in 4 dimensions are supposed to originate from a higher dimensional filed, a gravitational field $h_{AB}$ ($A,B = 0,1,2,3,5$) in $(4+1)$ dimensions. This $h_{AB}$ now stands as the unified field of gravity $h_{\mu\nu}$ ($\mu,\nu = 0,1,2,3$) and electromagnetic fields $A_\mu = h_{\mu5}$.

In a sense it is analogous to the way the electric and magnetic fields are unified in the electromagnetic field. In the Kaluza-Klein theories the extra dimensions are not similar to the usual spatial dimensions but are compactified on some manifold. In the case of one extra dimension, it could be compactified on a circle of radius $R$. In general, the space of $n$ extra dimensions could be a higher dimensional sphere or torus or some other manifold ($X^n$). For example, for an $n$-dimensional torus, this space is $X^n \sim L^n = (2\pi R)^n$. In the KK approach, the $(4+n)$ dimensional space is the direct product of 4-dimensional space-time $M^4$ and this compact space $X^n$ of extra dimensions. It is implicit that there is a particular dynamics of these $n$ extra dimensions, which leads to the suitable compactification of them leaving the Minkowski spacetime $M^4$ intact.

These extra dimensions have physical implications when the energy scales used can probe the length scales of the order $R$. A massless field in $4+n$ dimensions is equivalent to a tower of massive modes in 4 dimensions. For example, consider a massless scalar field in $4+1$ dimensions.

$$\mathcal{L} = -\frac{1}{2} \partial_A \Phi \partial^A \Phi, \quad A = 0, 1, 2, 3, 5$$

where $\Phi(x,y) \equiv \Phi(x_\mu, y)$ with periodic boundary condition $y = y + 2\pi R$. We expand the field in terms of the spherical harmonics. The coefficients are the conventional 4-
dimensional fields.

\[ \Phi(x, y) = \sum_{n=-\infty}^{+\infty} \phi_n(x)e^{iny/R}, \quad \phi_{-n}(x) = \phi^*(x) \]

After compactification, the effective action in 4-dimensions can be given as

\[ S = \int d^4x \left[ -\frac{1}{2} \partial_\mu \varphi_0 \partial^\mu \varphi_0 - \int d^4x \sum_{k=1}^{+\infty} \left[ \partial_\mu \varphi_k \partial^\mu \varphi_k^* + \frac{k^2}{L^2} \varphi_k \varphi_k^* \right] \right] \]

where \( \varphi_n \equiv \sqrt{2\pi R}\phi_n \). There are one massless mode and an infinite tower of massive modes \( \varphi_k \) with mass \( m_k^2 = k^2/R^2 \), together called Kaluza-Klein modes.

Similar is the case with electromagnetic field in \((4 + 1)\) dimensions. The electromagnetic lagrangian in \((4 + 1)\) dimensions is given by

\[ \mathcal{L} = -\frac{1}{4g_5^2} F_{AB} F^{AB}, \]

where the field strength \( F_{AB}^2 = F_{\mu\nu}^2 + 2(F_{\mu0}^2) \). As action is dimensionless, the coupling \( g_5 \) must be dimensionful here. After the KK decomposition, the fields \( A_\mu \) and \( A_5 \) can be expanded in terms of spherical harmonics as before

\[ A_\mu(x, y) = \sum_{n=-\infty}^{+\infty} A_\mu^{(n)}(x)e^{iny/R}, \quad A_5(x, y) = A_5^{(n)}(x)e^{iny/R} \]

After the compactification of the extra dimension on circle \( S^1 \), the effective lagrangin in 4 dimensions is given by

\[ \mathcal{L}_4 \sim F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + 2 \sum_{n=1}^{+\infty} \left[ F_{\mu\nu(k)} F^{*\nu(k)\mu} + \frac{n^2}{R^2} A_\mu^{(k)} A^{*\mu(k)} \right] + 2(\partial_\mu A_5^{(0)})^2 \]

The mass spectrum consists of a massless gauge field \( A_\mu^{(0)} \) with \( g_3^2 = g_5^2/2\pi R \), an infinite tower of massive gauge bosons with mass \( m_k^2 = k^2/R^2 \) and a massless scalar field \( A_5^{(0)} \). The massive scalar modes \( A_5^{(k)} \) are absorbed into the massive modes of \( A_\mu^{(k)} \), a phenomenon analogous to the Higg’s mechanism but driven by the geometry of the extra dimensions without requiring any Higg’s field. The n-dimensional gauge transformation will be reduced to an infinite number of 4-dimensional gauge transformations - one for each KK level.
Thus, a massless field propagating the extra dimensions is equivalent to that of a
tower of massive KK modes in 4 dimensions. However, as we did not see any such new
massive scalars or photons in the experiments so far, we can say that either the size
of the extra dimensions is sufficiently small that probing such modes was beyond the
experimental reach, or in a more intuitive way that these fields are confined to the 4-
dimensional spacetime (brane). This kind of localization of certain fields to the brane
and at the same time allowing fields like gravity to traverse the extra dimensions is
considered in a particular class of extra dimension models in order to explain some
of the issues that are not addressed in the SM, like the hierarchy problem. In what
follows, we will discuss such models that have gained lot of attention in the recent
past in both theoretical and experimental physics.

2.2. Large Extra Dimension model (ADD model)

There are two apparent physical scales in the particle physics, namely the electroweak
scale \( M_{\text{ew}} \sim \times 10^2 \text{ GeV} \) that governs the electroweak interactions and the Planck scale
\( M_{\text{Pl}} \sim 10^{19} \text{ GeV} \) that governs the gravitational interactions. If the SM is valid all the
way up to the Planck scale, then there is no mechanism or dynamics that can generate
a large gap of 16 orders of magnitude between these two scales, a problem known as
hierarchy in the particle physics. Or when put in the terminology of particle physics
that explains the fundamental interactions between the elementary particles, this is
to question as to why the gravity is very weak compared to the electroweak interac-
tions seen in the nature. This hierarchy problem is a long standing problem and has
been the main motivation for physics beyond the SM. The hierarchy between the elec-
tr o weak scale and the Planck scale has in the past been addressed by modifying the
particle content of the theory— supersymmetry and technicolor belong to this cate-
gory. A paradigm shift in this approach was proposed by Arkani-Hamed, Dimopoulos
and Dvali (ADD) [2], wherein they have explained the hierarchy with the help of ex-
tra spatial dimensions which gravity alone can ‘see’. In this model the spacetime is
supposed to have more than four dimensions. Similar to the Kaluza-Klein compactifications, the extra dimensions will form a compact manifold. The metric in higher dimensions is the direct product of four dimensional Minkowski spacetime with the compact manifold of the extra dimensions. The gravity can propagate all the dimensions whereas the SM fields live only on the $3 + 1$ dimensional domain wall, which does not extend in the compact directions. In string theory, D-branes are the natural candidates for such domain walls, and accordingly the domain wall is sometimes called a brane in the literature. It is worth noting that the SM domain wall, in this model, can correspond to a 3-brane, but need not necessarily be so. The KK theories of gravity in general are effective theories valid below a certain scale $\Lambda$ [6]. Because only the gravity is allowed to propagate the extra dimensions, it can undergo some modifications at length scales smaller than the size of these extra dimensions. In this context two cases are of much importance [6]. If $'E'$ is the energy scales that can be attained in the experiments, then (i) $E < 1/R << \Lambda$ corresponds to a case where a large number of $(\Lambda R)^n$ of KK modes are integrated out. Thus, though the coupling of each KK mode is Planck scale suppressed, they can contribute non-negligible higher dimension operators to the effective low energy theory. (ii) $1/R << E < \Lambda$ corresponds to the case where a large number $(ER)^n$ of KK modes are dynamically accessible and one can observe the effects of the extra dimensions in the experiments. Now, the question that needs to be addressed is what could be the size of the extra dimensions that gravity only is allowed to propagate. For this we will have to consider the gravitational force between two masses in the presence of extra dimensions.

Gravitational force in presence extra dimensions

According to the Newton’s inverse square law, the gravitational force of attraction between two point of masses, $M$ and $m$ is

$$\vec{F} = -\frac{G_NMm}{r^2} = m\vec{\nabla}\phi$$
where $G_N \sim 1/M_{Pl}^2$ is Newton’s constant and $\phi = -\frac{G_N M}{r}$ is the Newtonian potential. In $4 + d$ dimensions this Newtonian potential can be given as

$$\phi = -\frac{4\pi}{(n-2)V_{s^{4+n-2}}}G_{N}^{4+n} \frac{M}{r^{4+n-3}}$$

Here $G_{N}^{4+n}$ is the gravitational constant in $4 + n$ dimensions, and $V_{s^{4+n-2}}$ is the volume of $4 + d - 2$ dimensional sphere. Here $G_{N}^{4+n}$ can be obtained from the Gauss’ law which says that the total flux of the gravitational field over any closed surface area containing the gravitational charge (mass) be the same. For simplicity one can take infinite spherical surface area for the 4 dimensional space-time and the volume of $n$-dimensional torus for the extra dimensions, which is $(2\pi R)^n$. At infinity, in $(4 + n)$ dimensions, the total flux is $\Phi = 4\pi G_{N}^{4+n} M$ which is the same as $\Phi = 4\pi G_N (2\pi R)^n M$ obtained from the Newtonian potential in 4-dimensions and integrating over the extra $n$-dimensions. This gives us an important relation between $G_N$ and $G_{N}^{4+n}$ given by

$$G_{N}^{4+n} = G_N (2\pi R)^n \quad (2.1)$$

As the $G_N \sim 1/M_{Pl}^2$, the above relation can be re-written as:

$$M_{Pl}^2 = M_s^{n+2}(2\pi R)^n \quad (2.2)$$

where $M_s$ the scale of the extra dimensions, often called as the string scale. To solve the hierarchy problem, one expects $M_s$ to be of the order of a TeV.

$$L \sim 10^{-17+30/n}$$

For $2 \leq n \leq 6$, $10^{-1} \text{ cm.} \leq L \leq 10^{-6} \text{ cm.}$ Then the size of the extra dimension will be $L = 2\pi R \simeq 10^{13} \text{ m}$ for $n = 1$. Obviously, the possibility of $n = 1$ is thus ruled out. For $n = 2$, $L \simeq 1 \text{ mm}$ and is interesting scale which is being probed currently for the deviations from the inverse square law. For $n = 6$, the size of the extra dimensions will be as small as few $\mu m$. However the size of the extra dimensions in this model could be as large as few $\mu m$ and is clearly not ruled out by the current gravitational experiments that probe the inverse square law. (A brief study of the experiments on
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the inverse square law behaviour of gravity and the distance scales that have been
probed so far are taken up in the section(2.4). Hence, a value of \( n \geq 3 \) is acceptable
for the phenomenology of extra dimensions. Thus, there is a region in the parametric
space of \( n \) and \( L \) corresponding to \( M_s \sim \text{TeV} \) that is allowed by the present experiments
on Newtonian gravity. As a result the new physics can commence at the TeV scale
itself setting the UV of the SM at this scale instead of at \( M_{Pl} \). Thus the hierarchy
problem has been offered an explanation coming from the large extra dimensional
model.

Gravity in higher dimensions

The metric in this model is given by

\[
\text{d}s^2 = g_{\mu\nu} \text{d}x^\mu \text{d}x^\nu - r^2 d\Omega_n^2
\]

where \( g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) + \mathcal{O}(\kappa^2) \). In the weak gravitational field approximation
one can neglect the higher order terms in \( \kappa \) and work in the linearized gravity approx-
imation as given by \( g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \) where \( h_{\mu\nu} \) are the gravitational fluctuations
around the flat metric \( \eta_{\mu\nu} \). The action in higher dimensions now takes form

\[
S_{4+n} = -\frac{M_s^{n+2}}{2} \int d^4 x \int_0^{2\pi L} d^n y \sqrt{g^{4+n}} \mathcal{R}^{4+n}
\]

The linearized gravity lagrangian in \((4+n)\) dimensions or the Fierz-Pauli lagrangian
is given by

\[
\frac{1}{\kappa^{(d)2}} \sqrt{g^d} R^d = \frac{1}{4} \left( \partial^A h^{BC} \partial_A h_{BC} - \partial^A h^A h_{AB} - 2h^A h_A + 2h^A \partial_A h \right) + \mathcal{O}(\kappa^{(d)})
\]

where \( \kappa^{(d)2} = 16 \pi G^{(4+n)} \) with \( G^{(4+n)} \) the Newton’s constant in \((4+n)\) dimensions. This
lagrangian is invariant under general coordinate transformations: \( \partial h_{AB} = \partial_A \zeta_B + \partial_B \zeta_A \)
and yields the following d’Alembert equation of motion

\[
\Box_{(4+n)} (h_{AB} - \frac{1}{2} \eta_{AB} h) = 0,
\]
after imposing the de Donder gauge condition \( \partial^{A} (h_{AB} - \frac{1}{2} \eta_{AB} h) = 0 \). The KK reduction of this linearized gravity to 4-dimensions can be given as:

\[
h_{AB} = V_{n}^{-1/2} \begin{pmatrix}
 h_{\mu\nu} + \eta_{\mu\nu} \phi & A_{\mu i} \\
 A_{\nu j} & 2 \phi_{ij}
\end{pmatrix},
\]

where \( V_{n} \) is the volume of the compact space of \( n \) extra dimensions, \( \phi = \phi_{ii} \) and \( \mu, \nu = 0, 1, 2, 3 \) and \( i, j = 5, 6, \ldots, (4 + n) \). These together with the tracelessness condition \( h^{A}_{A} = 0 \) and the equation of the motion of gauge parameter will fix the physical degrees of freedom for a massless graviton in \((4 + n)\) dimensions. These fields can be compactified on an \( n \)-dimensional torus, and can have harmonic expansion as

\[
h_{\mu\nu}(x, y) = \sum_{\vec{k}} h_{\mu\nu}^{\vec{k}}(x) e^{i\vec{k}.\vec{y}/R} \quad \text{(2.3)}
\]

\[
A_{\mu i}(x, y) = \sum_{\vec{k}} A_{\mu i}^{\vec{k}}(x) e^{i\vec{k}.\vec{y}/R} \quad \text{(2.4)}
\]

\[
\phi_{ij}(x, y) = \sum_{\vec{k}} \phi_{ij}^{\vec{k}}(x) e^{i\vec{k}.\vec{y}/R}, \quad \text{(2.5)}
\]

where \( \vec{k} = \{k_{1}, k_{2}, \ldots, k_{n}\} \). For simplification the extra dimensions are all taken to be of the same size \( R \) and compactified on \( n \) dimensional torus. For, the extra dimensions of different size, the compactification on an asymmetric \( n \)-dimensional torus is straightforward by considering different \( R_{i} \) along the directions of different extra dimensions, denoted by \( k_{i} \). The compactification leads to a massless mode corresponding to \( \vec{k} = 0 \) and a tower of massive modes of spin-2, spin-1 and spin-0 particles, all are mass degenerate for any given KK level \( \vec{k} \). Each of these massive KK modes satisfy the following equation of motion:

\[
(\Box_{4} + m_{\vec{k}}^{2}) X^{\vec{k}} = 0 \quad \text{with} \quad m_{\vec{k}}^{2} = \frac{\vec{k}^{2}}{R^{2}}. \quad \text{(2.6)}
\]

where \( X^{\vec{k}} = (h_{\mu\nu}^{\vec{k}} - \frac{1}{2} \eta_{\mu\nu} h^{\vec{k}}), \quad A_{\mu i}^{\vec{k}}, \quad \phi_{ij}^{\vec{k}} \). To make these fields invariant under the gen-
eral coordinate transformations

\[
\delta h_{\mu\nu}^k = \partial_\mu \xi^k + \partial_\nu \phi^k_i + i\eta_{\mu\nu} \frac{k_i}{R} \xi^k ,
\]

\[
\delta A_{\mu i}^k = -i\frac{k_i}{R} \phi^k + \partial_\mu \phi^k_i ,
\]

\[
\delta \phi_{ij}^k = -i\frac{k_i}{2R} \phi^k_i - i\frac{k_j}{2R} \phi^k_j ,
\]

they will be redefined as [6]

\[
\tilde{h}_{\mu\nu}^k = h_{\mu\nu}^k - i\frac{k_i R}{k^2} (\partial_\mu A_{\nu i}^k + \partial_\nu A_{\mu i}^k) - (P_{ij} + 3\tilde{P}_{ij}) \left( \frac{2}{3} \partial_\mu \partial_\nu \frac{1}{3} \eta_{\mu\nu} \right) \phi_{ij}^k ,
\]

\[
\tilde{A}_{\mu i}^k = P_{ij} (A_{\mu j}^k - i\frac{2n_k R}{k^2} \partial_\mu \phi_{jk}^k) , \quad \tilde{\phi}_{ij}^k = \sqrt{3}(P_{ik} P_{jk} + aP_{ij} P_{kl}) \phi_{kl}^k ,
\]

(2.7)

where the projection operators are given by

\[
P_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2} , \quad \tilde{P}_{ij} = \frac{n_i n_j}{n^2}
\]

(2.8)

and \(a\) is the solution of the equation \(3(n-1)a^2 + 6a - 1 = 0\). As can be seen from the eqns.(2.7), that a massless spin-2 fields \(h_{\mu\nu}^k\) absorb the spin-1 and spin-0 fields at the same KK level and become massive. This is similar to Higg's mechanism but governed by the geometry of the extra dimensions without requiring to have any Higg's field. Hence the compactification of the extra dimensions lead to a tower of massive spin-2, spin-1 and spin-0 particles in 4-dimensions with masses \(m_k^2 = \frac{k^2}{R^2}\). As discussed before, the size of the extra dimension in this model could be as large as few tens of \(\mu m\) consistent with the present experiments on testing Newton’s gravity at smaller length scales. With this possible macroscopic size of the extra dimensions, there can be a large number of light KK modes that can play an important role in the phenomenology of particle physics.

Gravity couples to the SM fields via energy momentum tensor \(T^{\mu\nu}\). Hence all the above spin-0,spin-1 and spin-2 KK modes obtained from a gravity in higher dimensions will also couple to the SM via energy momentum tensor. From the relation

\[
P_{ij} \phi_{ij}^k = \frac{3\omega}{2} \phi^k
\]
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(where $\omega = \sqrt{\frac{2}{3(n+2)}}$), we see that the spin-0 modes couple through the dilation mode $\tilde{\phi}^k = \tilde{\phi}_{(i)}^k$, the trace of it, whereas the spin-1 KK modes decouple. Hence, the effective interaction lagrangian between the physical (redefined) KK modes and the SM fields is given by

$$L_{\text{int}} = -\frac{\kappa}{2} \sum_{k=0}^{\infty} \int d^4x \left[ \tilde{h}_{\mu\nu}^k(x) T^{\mu\nu}(x) + \omega \tilde{\phi}^k(x) T_{\mu}^\mu(x) \right]$$

where $\kappa = \sqrt{16\pi/M_{Pl}}$. The zero mode corresponds to the usual 4-dimensional massless graviton. Though, the coupling of each of the KK modes with the SM fields is $M_{Pl}$ suppressed, the effective coupling due to the many available KK modes need not be suppressed by again Planck scale but could be by the scale $M_s$ that governs this effective theory of large extra dimensions. This results in enhanced effective couplings that can lead to observable effects in the collider experiments. In any typical scattering cross section, one needs to perform a summation over all possible virtual states at the amplitude level (or in the propagator) itself. For incoherent processes, similar to that of the subprocess contribution involving different initial and final state particles but leading to the same physical observable, all possible contributions have to be added over at the cross section level. In the case where the KK modes participate in a scattering process at the intermediate level, thus, one has to perform the summation over these virtual KK modes at the amplitude level itself. In general, these virtual KK modes lead to the deviations from the expected SM background contribution. However for the processes involving the KK modes in the final state, KK modes with different mass correspond to different final states, and hence incoherent processes, that have to added at the cross section level. The real emission of the KK modes lead to the large missing $E_T$ signals at the collider experiments.

In a process involving a virtual exchange of KK modes between the SM particles, the sum of KK propagators with momentum $q$ is $D(Q^2)$ is given by ($q^2 = Q^2$)

$$\kappa^2 D(Q^2) = \kappa^2 \sum_k \frac{1}{Q^2 - m_k^2 + i\epsilon} \tag{2.9}$$
For large extra dimension case, the $m_k$ will be small and hence the spectrum of these light KK modes can be approximated to be a continuous one with the number of KK states lying in the interval $dm^2_k$ being

$$\Delta k^2 = \rho(m_k)dm^2_k$$

where $\rho(m_k)$ is the density of the KK states in the continuum limit. Hence the above summation over the KK modes can be approximated to the integral given by

$$\kappa^2 D(Q^2) = \frac{8\pi}{M^4_s} \left( \frac{Q}{M_s} \right)^{(n-2)} \left[ -i\pi + 2I(\Lambda/Q) \right], \tag{2.10}$$

The integral $I(\Lambda/Q)$ is a result of the summation over the non-resonant KK modes and the term proportional to $\pi$ is due to the resonant production of a single time-like KK mode [6]. Here the $\Lambda$ is the explicit cut-off on the KK sum which is identified with the scale of the extra dimension theory $M_S$ [6, 7]. The $\kappa^2$ suppression in a virtual exchange is compensated for by the high multiplicity, after the KK modes are summed over. For time-like KK modes, the integral can be read as

$$I(\omega) \sim \int_0^\omega dx \frac{x^{n-1}}{1-x^2}$$

$$= -\sum_{k=1}^{n/2-1} \frac{1}{2k} \omega^{2k} - \frac{1}{2} \log (\omega^2 - 1) \quad k = \text{even},$$

$$= -\sum_{k=1}^{(n-1)/2} \frac{1}{2k-1} \omega^{2k-1} + \frac{1}{2} \log \left( \frac{\omega+1}{\omega-1} \right) \quad k = \text{odd}. \tag{2.11}$$

Here it should be noted that for an exchange of space-like KK modes, there will be no factor of $\pi$ corresponding to the resonance in the above summation. The summation for space-like KK propagators is given by

$$\kappa^2 D(Q^2) = -\kappa^2 \sum_k \frac{1}{Q^2 + m^2_k} \tag{2.12}$$

where $Q^2 = |q^2|$ and the above integral for space-like KK modes takes the following
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form:

\[
I(\omega) = \int_0^\omega dx \frac{x^{n-1}}{1 + x^2}
\]

\[
= (-1)^{n/2+1} \sum_{i=k}^{n/2-1} (-1)^k \frac{1}{2k} \omega^{2k} + \frac{1}{2} \log (\omega^2 - 1) \quad k = \text{even, (2.13)}
\]

\[
= (-1)^{(n-1)/2} \sum_{k=1}^{(n-1)/2} (-1)^k \frac{1}{2k-1} \omega^{2k-1} + \tan^{-1} \omega \quad k = \text{odd}.
\]

As the coupling of the KK modes to the SM fields is via energy momentum tensor, the KK modes do not distinguish the color, flavor, electric charge or even the spin of the SM particles. Hence each KK mode couples with the same strength to the photons, leptons, weak bosons, quarks, gluons and scalars. However the vertex factors could be different. As per the phenomenology of these KK modes is considered, this particular feature of their coupling to all SM fields is very much interesting. For example, we know that the photons do not interact with each other and hence the production of a pair of photons from the hard scattering process at the photon colliders can be explained only with the terms that come at higher orders in perturbation theory in the SM, whereas in the extra dimension models this process can take place in the simplest $2 \rightarrow 2$ process itself because of the photon-graviton couplings. Another example is various pair production processes at the LHC. At the hadron colliders like LHC where the centre of mass energy is very high ($\sqrt{s} = 14$ TeV), the parton fluxes that can produce invariant mass of upto 1 TeV are very high. In particular, the gluon fluxes are very very high. This when combined with the fact that the KK modes couple to the gluons directly, makes various processes possible, like production of $l\bar{l}$, $\gamma\gamma$, di-jet, $ZZ$, $W^+W^-$, $hh$, that can be explained based on the simplest gluon-gluon initiated gravity mediated tree level $2 \rightarrow 2$ feynman diagrams [6–9]. For the scale $M_s \sim \mathcal{O}(\text{TeV})$, it can be likely that the cross sections in these processes could become large enough to dominate the SM background contribution. Thus it will be very much possible to look for the signals that these extra dimension models predict based on the virtual effects of the KK modes at the LHC. On the other hand, the phenomenology associated with
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the real production of KK modes is even interesting in the sense that it is associated with large missing transverse energies that are balanced by either a mono-jet or by a mono-photon ensuring the energy momentum conservation. These mono-photon and mono-jet productions do give a very clear signals. In the possible case of such single jet or single photon productions in the SM, the final states in the SM and in the extra dimension model are different and hence in such channels there will be no interreference effects (SM*ADD). Hence the study of such signals is easy to compare with the theoretical predictions and obtain the information about the unknown parameters of the model. The search for the large extra dimensions at the Tevatron experiments has already been carried out in the dielectron and diphoton channels using the 1.05 fb\(^{-1}\) data obtained from the \(p\bar{p}\) collisions at the center of mass energy of 1.96 TeV [10]. The data has been presented in the table 2.1.

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<td>Obs.</td>
<td>(M_s)</td>
<td>2.09</td>
<td>1.94</td>
<td>1.62</td>
<td>1.46</td>
<td>1.36</td>
</tr>
<tr>
<td>Exp.</td>
<td>(M_s)</td>
<td>2.16</td>
<td>2.01</td>
<td>1.66</td>
<td>1.49</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Table 2.1: Observed and expected lower limits at the 95% C.L. on the effective Planck scale, \(M_s\), in TeV [10].

The data has set a lower limit of 2.1 TeV to 1.3 TeV on the string scale \(M_s\) for 2 to 7 extra dimensions.

Summary and few remarks on the ADD model

1. The ADD model offers a possible explanation to the hierarchy between the electroweak scale and the Planck scale. The size ‘\(R\)’ of the extra dimensions in this model could be as large as few tens of micrometers, consistent with the present experimental results on the gravitational inverse square law. The compactification of these extra dimensions leads to a tower of spin-0, spin-1 and spin-2 Kaluza-Klein (KK) modes. The spin-2 and spin-0 (via its dilaton mode) couple to the SM fields while the spin-1 KK modes decouple.
2. At the LHC energies for $M_s \sim O(\text{TeV})$, a large number of KK modes are dynamically accessible and hence there is every possibility to look for these KK modes at the LHC experiments. The virtual KK modes will give raise to deviations from the SM predictions while the real emissions of these KK modes lead to a large missing transverse energy signals.

3. Above the scale $M_s$, the extra dimensions are dynamically accessible and the world looks $(4 + n)$ dimensional indeed and in that case the collider signals will be different from those of the compactified extra dimensions.

4. The signals of the ADD model are interesting for they could open up a window for the discovery of the gravitons in the laboratory experiments and eventually to probe the quantum structure of gravity.

2.3 Warped Extra Dimension model (RS model)

In the last chapter we have studied the ADD model, the first extra dimension model to address the hierarchy problem. The hierarchy between the electroweak scale and the Planck scale has been accounted for the large volume of the extra dimensions in that model. The string scale $M_s$, below which the effective theory of the extra dimensions is valid, can be as low as a few TeV and stands as a new UV cutoff for the SM. Because of the many available KK modes, both the virtual effects and the real emission of these can become significant and hence the collider phenomenology of this model is very rich. Although, the model is very interesting in various respects, there are few important points worth noting here, they are

- In explaining the hierarchy, a very large number of $N = (\Lambda R)^n$ of modes have been integrated over in this model, where $\Lambda$ is the scale below which the effective theory of the extra dimension model, as described by ADD, is valid. To address a large hierarchy, the model introduces a large number of KK modes.
• In explaining the hierarchy between $M_w$ and $M_{Pl}$, this model runs into another hierarchy between the scales $1/R$ and $M_w$. There is no apparent explanation for this new hierarchy.

In light of the above, a new approach to the hierarchy problem was taken up by Randall and Sundrum (RS) using a completely different geometry of the extra dimensions [3]. In this model also, only gravity is allowed to propagate the higher dimensional spacetime while the SM fields are confined to the 4-dimensional spacetime (brane). In the RS model there is only one extra spatial dimension and is compactified on a circle of circumference $2\pi r$ and further orbifolded by identifying points related by $y \rightarrow -y$. The higher dimensional metric in this model is not factorisable as given by

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$ (2.14)

where $y = r\phi$ is the coordinate of extra dimension, $x^\mu$ are the familiar 4-dimensional coordinates and $k$ is a scale of order the Planck scale $M_{Pl}$. This metric corresponds to anti de Sitter space $AdS_5$ for it is a maximally symmetric Lorentzian manifold with negative curvature. This metric is also a solution of the Einstein’s field equations.

The novel feature in this model is the exponential warp factor $e^{-2ky}$ which can be accounted for the hierarchy between the electroweak and the Planck scales, without requiring a large volume of the extra dimensional space. Thus, the size $r$ could be as small as possible that the compactification scale $\mu \sim 1/r$ need not be very different from the fundamental Planck scale. As the extra dimension has orbifold symmetry $S^1/Z_2$, the points corresponding to $0 \leq y \leq r\pi$ will specify the metric completely. Another significant thing in this model is that there are two 3-branes that are attached to the orbifold fixed points $y = 0$ and $y = r\pi$, as opposed to the familiar one 3-brane like in ADD model. The brane on which we live is the SM brane or the “visible” brane and the other one is called Planck brane or the “hidden” brane. The SM brane is placed at the point $y = r\pi$ in order to account for the apparent weakness of the gravity on it, consistent with the above metric. On the other hand the hidden brane is placed at $y = 0$ where the gravitational interactions are expected to become strong. The SM
brane is with negative tension while the Plank brane is with positive tension. Both of these branes can support the familiar 4-dimensional field theories but the excitations of the particle on the SM brane are at the TeV scale while those on the Planck brane are at the Planck scale. The classical action for this five dimensional system is given by

\[ S = S_{gr} + S_{sm} + S_{Pl} \]

(2.15)

where

\begin{align*}
S_{gr} &= \int d^4x \int_\pi^{\pi} d\phi \sqrt{-G} \left[ -\Lambda + 2M^3R \right] \\
S_{sm} &= \int d^4x \sqrt{-g_{sm}[\mathcal{L}_{sm} + 24M^3k]} \\
S_{Pl} &= \int d^4x \sqrt{-g_{Pl}[\mathcal{L}_{Pl} - 24M^3k]},
\end{align*}

(2.16)

and \( M \) is the \((4 + n)\) dimensional Planck scale. The action of gravity in 4-dimensions is

\[ S_{4gr} \sim M^2_{Pl} \int d^4x \sqrt{-g} R \]

(2.17)

By comparing the scales in this action and the effective action obtained from eqn.(2.16), one can easily show that

\[ M^2_{Pl} = \frac{M^3}{k} \left[ 1 - e^{-2kr\pi} \right] \]

(2.18)

From this we see that for \( M \sim M_{Pl} \) and large \( kr \) values, the 4-dimensional Planck scale does not get significant modifications because of the additional extra dimension. The metric on the SM brane will be \( g_{\mu\nu} \sim e^{-2kr\pi} \eta_{\mu\nu} \). This is an important outcome of the model for it can lead to a general conclusion that any fundamental mass parameter \( M_0 \) in higher dimensions corresponds to a physical mass \( m \) on the SM brane at \( \phi = \pi \) as given by

\[ m = M_0 e^{-kr\pi} \]

(2.19)
For $kr \sim 12$, one can generate a TeV scale from the Planck scale. This is remarkable in the sense that to explain a large hierarchy one really does not require a large volume of the extra dimensions but an exponential warp factor as considered above will be sufficient, with the size of the extra dimension not far away from the Planck length. All the fundamental parameters $k, M, \mu \sim 1/r$ (with $k < M$) are the order of the same scale and there is no new hierarchy among these scales. This model not only offers a possible explanation to the hierarchy between the electroweak scale and the Planck scale but also gives raise to the possibility that the gravitational interactions can become stronger on the brane at scales close to TeV. The gravitational interactions can have only TeV scale suppression as opposed to the usual Planck scale suppression. Further it has been showed that [11] the value of $kr\pi$ can be stabilized without fine tuning by minimizing the potential for the modulus field which describes the relative motion of the two branes. In the RS model graviton and the modulus field can propagate the full 5-dimensional space time while the SM is confined to the TeV brane.

### 2.3.1 KK modes in the RS model

The Kaluza-Klein compactification of the extra dimension in general involves the fields $h_{\mu\nu}(x, y), A_{\mu5}(x, y)$ and $\phi_{55}(x, y)$. In the particular set up of the branes at $\phi = 0$ and at $\phi = \pi$ as described here, there are no continuous isometries of the higher dimensions and hence all the off diagonal elements corresponding to these vector modes $A_{\mu y}^{(n)}$ are excluded in this effective theory. To perform the KK decomposition, the gravitational field $h_{\mu\nu}(x, y)$ can be expanded as

$$h_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)} \frac{f^{(n)}(\phi)}{\sqrt{r}}$$

The equation of the motion of $h_{\mu\nu}^{(n)}$ in the gauge $\partial^{\mu} h_{\mu\nu}^{(n)} = \eta^{\mu\nu} h_{\mu\nu}^{(n)} = 0$, is given by

$$(\Box - m_n^2) h_{\mu\nu}^{(n)} = 0$$

In addition the functions $f^{(n)}(\phi)$ are chosen to satisfy

$$\int_{-\pi}^{\pi} d\phi \ e^{-2k\phi} f^{(n)}(\phi) f^{(m)}(\phi) = \delta_{nm}$$
2.3. Warped Extra Dimension model (RS model)

From the above equation and the Einstein’s equation for the field $h_{\mu\nu}(x,\phi)$ sets the equation of the motion for the field $f^{(n)}(\phi)$ as

$$\frac{-1}{r^2} \frac{d}{d\phi} \left[ e^{4\phi} \frac{df^{(n)}}{d\phi} \right] = m_k^2 e^{-2ky} f^{(n)}$$

(2.23)

The solutions for the above differential equation are given by [11, 12]

$$f^{(n)}(\phi) = \frac{e^{2ky}}{C_n} \left[ J_2(\alpha_n) + a_n Y_2(\alpha_n) \right].$$

(2.24)

where $J_2(\alpha_n)$ and $Y_2(\alpha_n)$ are second order Bessel functions of first and second kind respectively, $\alpha_n = m_n e^{ky}/k$, $C_n$ are normalization and $a_n$ are constants. On the SM brane, if we define $x_n = \alpha_n(\pi)$ and in the limit $m_n << k$ (with $e^{kr} >> 1$), the requirement that $f^{(n)}(\phi)$ has to be continuous across the orbifold fixed points gives us $a_n \sim x_n^2 e^{-2kr\pi}$ and $J_1(x_n) = 0$. Here, the $x_n$ are simply the zeros the first order Bessel function. Hence the masses of the RS modes are given by

$$m_n = k \ x_n \ e^{-kr\pi}$$

(2.25)

where $x_1 = 3.8317$ and $x_n = 7.0156 + (n-2)\pi$ for $n > 2$. From the value of $a_n$, we can ignore $Y_2(x_n)$ in eqn.(2.24) in comparison to $J_2(x_n)$. The normalization constants hence can be approximated to

$$C_n = \frac{e^{kr\pi}}{\sqrt{kr}} J_2(x_n)$$

(2.26)

for $n > 0$ and for zero mode, it is $C_0 = 1/\sqrt{kr}$. As the masses of the RS modes depend on the zeros of the Bessel’s functions, the mass spectrum is not uniform, and the separation is decided by that between the zeros of the Bessel function. The coupling of the gravity to the matter fields is given by the interaction of the higher dimensional gravitational field $h_{\mu\nu}(x,\phi)$ given in eqn.(2.20) at $\phi = \pi$, with the energy momentum tensor $T^{\mu\nu}$ of the SM fields on the brane at $\phi = \pi$

$$\mathcal{L}_{\text{int}} \sim -\frac{1}{M_{\text{pl}}^2} h_{\mu\nu}(x,\phi) T^{\mu\nu}(x)$$

(2.27)
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Figure 2.1: Bounds on the RS model parameters obtained using 1 fb$^{-1}$ of data from $p\bar{p}$ collisions at the Tevatron, collected by the D0 detector [13]. The shaded region is excluded at 95% confidence level. The solid thin line corresponds to the previously published exclusion contour. The area below the dashed line is excluded by precision electro weak measurements ($M_{Pl} = M_{Pl}/\sqrt{8\pi}$).

Using eqn. (2.18), eqn.(2.20) and eqn.(2.26), the above interaction Lagrangian takes the following form

$$\mathcal{L}_{int} \sim -\frac{1}{M_{Pl}} T^{\mu\nu} h_{(0)}^{\mu\nu} - \frac{1}{M_{Pl}} e^{-kr\pi} \sum_{n=1}^{\infty} T^{\mu\nu} h_{(n)}^{\mu\nu}. \quad (2.28)$$

We see here that the coupling of the zero mode is $M_{Pl}$ suppressed while the those of the higher graviton modes are suppressed by only TeV scale ($\sim M_{Pl} e^{-kr\pi}$) with $kr\pi \sim 12$. In this model there are two parameters which are $c_0 = k/M_P$, the effective coupling and $m_1$ the mass of the first KK mode. Except for an overall warp factor the Feynman rules of RS are the same as those of the ADD model.

The RS modes, when produced in high energy collider experiments, being heavy
can decay into the SM fields with a decay width $\Gamma_n$. The graviton propagator with momentum $Q$ and mass $m_n$ is $P_G(Q) = i B_{\mu\nu\rho\sigma} D(Q^2)$ (see eqn. B.0.1) with

$$D(Q^2) = \sum_n \frac{1}{Q^2 - m_n^2 + \Gamma_n},$$

$$= \frac{1}{m_0^2} \sum_n \frac{x^2 - x_n^2 - i \Gamma_n x_n}{(x^2 - x_n^2)^2 + \frac{\Gamma_n^2}{m_0^2} x_n^2},$$ (2.29)

where $x = Q/m_0$ and $x_n = m_n/m_0$. The summation over $n$ is kinematically bounded. Further the RS KK mode of mass $m_n$ if decays only to SM particles, it can be computed and hence the the decay width $\Gamma_n$ is not unknown. Hence, these heavy RS modes will show up themselves as a resonance pattern in the invariant mass distribution of the pair production processes like Drell-Yan, di-photon production processes, at the high energy collider experiments. This resonance pattern is very much distinct from the signals of the contact type interactions as in the case of ADD model which have a smooth deviations from the SM predictions, say in the tail of the invariant mass distribution.

For example, at the LHC ($\sqrt{S} = 14$ TeV) it is very much possible to probe the TeV scale physics, like the RS model with KK modes that are of the order of a few TeV. As the gravity couples to the energy momentum tensor of the SM fields, the KK modes couple to quarks and gluons with the same coupling of $c_0 = k/M_{Pl}$. At the LHC where the gluon fluxes are very high, higher than the quark anti-quark fluxes, the RS modes can be produced in the gluon fusion subprocesses which subsequently decay into all the possible SM particles, like a pair of leptons $ll$, photons $\gamma\gamma$, jets $jj$, weak bosons $ZZ$ and Higgs bosons $hh$. In addition to the gluon gluon fusions, the quark anti-quark annihilations also can produce these RS modes. These RS modes will appear, then, as resonance pattern in the invariant mass of the above mentioned channels. It should be noted here that the coupling of the each of the RS modes, except the zero mode, is very large as given by $c_0$ and hence each RS mode can prominently decay into the SM particles, whereas in the ADD model, the coupling of each of the KK modes is Planck scale suppressed but due to the many available light KK modes the effective coupling
is $M_s$ scale suppressed. The decay width of ADD gravitons is very small owing to its negligible coupling to the SM particles and hence is neglected in the propagator. The typical value chosen for this effective coupling is $0.01 < c_0 < 0.1$. Smaller couplings $c_0 << \mu$ corresponds to $k << M_{Pl}$ and gives raise to additional hierarchy between the fundamental scales $k$ and $M_{Pl}$. Therefore to avoid new hierarchy in the model, smaller couplings are not allowed.

The Tevatron experiments ($\sqrt{S} = 1.96$ TeV) for the first time have searched for the RS model signatures using the di-lepton and di-photon final states. At the Tevatron for its relatively low center of mass energy, the gluon fluxes corresponding to TeV scale invariant mass are smaller than the quark anti-quark fluxes. So the quark anti-quark annihilations subprocesses have dominant contributions over those of the gluon fusion subprocesses. The latest bounds obtained from the Tevatron using the 1 fb$^{-1}$ of data collected by the $D_0$ detector on the parameters of the RS model are shown in the fig.(2.1). This data has set a lower bound at 95% confidence limit on the first RS mode to be $m_1 > 850$ GeV corresponding to a coupling $c_0 \sim 0.01$.

Summary and a few remarks on the RS model

1. The RS model provides an alternate explanation to the large hierarchy between the electroweak scale and the Planck scale with a new geometry of the extra dimensions that is completely different from the one in the ADD model. In this model there is only one extra dimension which is a slice of anti de Sitter space-time $AdS_5$, with orbifold symmetry. Two branes, namely the Plank and the SM brane, are attached at the orbifold fixed points of the extra dimension. The $(4+1)$ dimensional metric in this model has an exponential warp factor $e^{-2kr\phi}$ which is a function of the extra dimension. The large hierarchy has been accounted for this exponential warp factor with a particular choice of $kr \sim 12$.

2. All the higher dimensional parameters $M, k, \mu \sim 1/r$ are of the order the same scale, Planck scale, and hence there is no additional hierarchy among these
2.3. **Warped Extra Dimension model (RS model)**

scales, unlike the ADD model case where in explaining the existing hierarchy, the model runs into another hierarchy between the weak scale and the compactification scale of the extra dimensions.

3. Any fundamental mass parameter $M_0$ in higher dimensions corresponds to a physical mass $m = e^{-kr\pi}M_0$ on the SM brane. Thus the fundamental scales could be of the order Planck scale but the physical scales on the brane are only of the order of a few TeV.

4. Although, the model successfully addresses the hierarchy problem, the explanation is based on the particular choice $kr \sim 12$, which plays a key role in addressing the hierarchy problem. This choice does not come from the theory and there is no explanation for it so far.

5. The bulk moduli field accounts for the stability of the two branes against their mutual attraction towards each other. However, this moduli field has a back reaction on the branes, which has been ignored in this model.

6. The compactification of the extra dimension leads to a tower of graviton modes on the SM brane, the masses of whose are determined by the zeros of the Bessel function. The modes could be of order a TeV scale and the spectrum of these modes is non-uniform.

7. The zero mode has a Planck scale suppressed coupling while all other higher modes have TeV scale suppressed couplings to the SM fields. Thus the zero mode decouples in the RS mode spectrum.

8. These graviton modes being heavy when produced at the high energy colliders like LHC, will decay into SM particles and appear as resonance enhancements over the SM predictions.

9. The RS model has a very distinct collider signatures such as a series of resonances whereas those of the ADD model give a tail raising pattern in the invar-
Beyond Standard Model scenarios

ant mass distributions. Hence distinguishing these two models is very easy at the collider experiments.

10. The concept of localization of the fields to the branes has gained lot of interest following the work in this model, and hence the attempts have been made to localize the gravity in a model known as RS(II) model by considering the infinite size extra dimension and taking the second brane to infinity. A significant research on the concept of localization of the fields in the non-factorisable metric of higher dimensions is on progress.

2.4 Tests of the gravitational inverse square law

According to the Newton’s Inverse Square Law (ISL), the gravitational force of attraction between two point masses $m_1$ and $m_2$ separated by a distance $r$ is given by

$$ F = -G_N \frac{m_1 m_2}{r^2} $$

(2.30)

where $G_N \simeq 6.673 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ is the gravitational constant. Until few years ago, this inverse square law is assumed to be valid for length scales ranging from infinity to Planck length ($\sim 10^{-35} \text{m}$). However, motivated by many new physics scenarios which predict that the gravitational inverse square law could be altered at sub-millimeter range, this ISL has been put to experimental verification by various collaborations at shorter length scales. The modifications to the inverse square law can be accounted for the Yukawa type of corrections to the familiar Newtonian potential that can be written as [14].

$$ V(r) = -G_N \frac{m_1 m_2}{r} \left[1 + \alpha e^{-r/\lambda}\right] $$

(2.31)

where $\alpha$ is the dimensionless strength parameter and $\lambda$ is a length scale. This Yukawa potential describes the interaction from the exchange of a scalar of mass $m \sim 1/\lambda$ between the two masses $m_1$ and $m_2$. The deviations from the ISL can then be obtained from the experimental bounds on these $\alpha$ and $\lambda$ parameters.
2.4. Tests of the gravitational inverse square law

2.4.1 Some theoretical speculations

In this section we outline some of the possible new physics scenarios that are relevant in the search of deviations from the Newtonian ISL [14].

1. *Large extra dimensions:*

   The observed gauge hierarchy between the Planck scale and the weak scale has been offered an explanation in an interesting theoretical approach that deals with the existence of large extra spatial dimensions. Only gravity ‘sees’ the extra dimensions and the Planck scale in higher dimensions can assume as low value as a few TeV in this scenario. Consequently the gravity is expected to be stronger at the length scales that are of the order of the size of these extra dimensions. The modification to the Newtonian potential can then assume the form given in eqn. (2.31) with the strength \( \alpha \) and the range \( \lambda \) given by

   \[
   \lambda = R \quad \text{and} \quad \alpha = \frac{8n}{3} \quad (2.32)
   \]

   where \( R \) is the size and \( n \) the number of extra dimensions. As is seen in the ADD model, for Planck scale of the order of a TeV scale, the possibility of \( n = 1 \) corresponds to a length scale of the order of \( 10^{12} \) m which has been ruled out. For \( n = 2 \), the radius of the extra dimension is \( R \sim 10^{-1} \) mm, a length scale which is of the current interest for the experiments that probe the ISL of gravity.

2. *Exchange of radions:*

   In string theories, the radion is a hypothetical scalar particle that appears as a component of the metric tensor or gravitational field in higher dimensions. It is also called as *graviscalar*. Consequently the spacetime is somewhat dynamical with the radii of the new dimensions whose volume must be stabilized with radions. If there is more than one extra dimension, then there could exist several such particles. This radion exchange is expected to produce a force that
corresponds to a strength and the range given as
\[ \alpha = \frac{n}{n + 2}; \quad \lambda \sim \sqrt{\frac{1}{G M^4}} \approx 2.4 \left(\frac{1\text{TeV}}{M_*}\right)^2 \text{mm}. \]

In general the radion mediated force is the long range effect of new dimensions and, unlike the large extra dimension case, does not fall with the number of extra dimensions.

3. *Exchange of moduli:*

In quantum field theories, the moduli fields refer to the scalar fields whose potential energy has a continuous global minima. The vacuum expectation values of these scalar fields are used to represent the respective vacua, consequently these expectation values are used as the parameters of the effective theory. This type of potential energy functions often occur in supersymmetric theories. The moduli couple weakly to the supersymmetric-breaking sector and these couplings are computable in any given vacuum, so that ISL tests are the best way to search for these particles. The best known particles of this type are *dilatons*.

4. *Exchange of axions:*

Axion is an elementary particle postulated to explain the strong CP problem in QCD. The force resulting from the exchange of boson with parity either \(0^-\) or \(1^+\) is purely spin-dependent and vanishes between unpolarized objects. In the case where the CP violating term \(\Theta_{\text{strong}}\) will not go down to zero, the axions can be accounted for as to acquire a small CP-violating scalar admixture. This results in a spin-dependent Yukawa potential between Nucleons with
\[ \alpha = \left[ \left(\frac{\Theta_{\text{strong}}}{10^{-10}}\right) \left(\frac{m_a}{1\text{meV}}\right) \right]^2 \times 1.3 \times 10^{-6}; \quad \lambda = \frac{1}{m_a} \approx \left(\frac{1\text{meV}}{m_a}\right) 0.2 \text{ mm}. \]

2.4.2 Experimental results

In what follows we outline two types of experiments carried out to test the inverse square law at length scales smaller than 100 \(\mu m\) and present the bounds obtained in these experiments on the parameters \(\alpha\) and \(\lambda\).
2.4. Tests of the gravitational inverse square law

1. Torsion pendulum experiments

The torsion pendulum based experiments are in general used to measure the weak forces like gravitational force. In a recent experimental set-up [14, 15] for testing the ISL, a new type of torsion-balance has been developed where the test masses are the "missing masses" of holes bored into cylindrically symmetrical plates. The torsion pendulum is a thin ring containing 10 equally spaced cylindrical holes and is suspended above a uniformly rotating circular attractor disk containing 10 similar holes. Due to the gravitational field $V(\phi)$ of the disk the pendulum experiences a torque $\tau(\phi) = \partial V(\phi)/\partial \phi$ for a displacement $\phi$ of the disk, and undergoes oscillations. This torque, however, will be zero in the absence of these holes. Attaching to the first disk another thicker circular disk containing the holes, leads to almost null measurements of the torque experienced by the pendulum. The dimensions of the holes in the second disk and the orientation of this disk with respect to the first one are so chosen that the torque produced in the pendulum because of the first disk is canceled by that of the second one. The cancellation of the forces due to the Newtonian gravity is very much dependent on the distance of the pendulum from the circular disk. On the other hand, the Yukawa type of potential monotonically decreases with this distance. As a result, the location of the exact cancellation will be very sensitive to the deviations from the ISL and any deviation from the characteristic of Newtonian gravity can be used to keep the bounds on the parameters $\alpha$ and $\lambda$.

2. Cantilever based experiments

Another recent attempt to probe the Newtonian ISL is made by the Stanford group [16], using their cantilever based experiments. In these experiments, a single-crystal silicon bar for the cantilever and the test masses, made up of gold and of the size comparable to the distance between them, are used. A drive mass which has an alternating pattern of gold and silicon is used to create a varying gravitational field and is mounted on a piezoelectric bimorph for its horizontal movement. The test
mass is mounted on the free end of the cantilever and is subjected to experience the varying gravitational force due to the drive mass placed below it at a distance. The test mass and the drive mass are separated by a shield made up of metalized silicon nitride to minimize the non-gravitational backgrounds. Then, the drive mass is put to oscillations at the subharmonic of the resonance frequency of the cantilever. Any coupling between the test mass and the drive mass would then create a force on the cantilever at harmonics of this drive frequency, including the cantilever’s resonant frequency. The motion of the cantilever can be measured using a fiber interferometer and from that the force between the masses can be deduced. The forces are measured as a function of the horizontal distance from the equilibrium position of the drive mass, maintaining a fixed vertical separation between the test mass and the drive mass. By comparing these measurements with the predictions of the finite element analysis (FEA), the bounds on the Yukawa type corrections can be obtained.

The results obtained from these ISL testing experiments conducted by various collaborations are summarized in fig.(2.2). They are presented as bounds at 95\% confidence level on the Yukawa coupling strength $\alpha$ as a function of the range $\lambda$. Implications of these results on some of the theoretical speculations discussed above are very much informative and are useful in constraining the parameters of different models. In specific we will concentrate on the large extra dimension case. The bounds on the Yukawa range $\lambda$ will give an upper limit on the size of the extra dimensions. The possibility of only one extra dimension is ruled out as no deviations from the ISL are observed at the Solar length scales. Then the other possibility that will be of interesting is the case with two extra dimensions corresponding to a scale $M_s = 1$ TeV. According to eqn.(2.32), if one of the extra dimensions is very large, which can see deviations from the ISL, then it would correspond to $\alpha = \frac{8}{3}$. Corresponding to this $\alpha$ value, the 95\% confidence limit on the size of the extra dimension will be $R \leq 160 \, \mu m$. For two extra dimensions of equal size the possibility of $M_s = 1$ TeV corresponds to $R \sim 380 \, \mu m$ and $\alpha = \frac{16}{3}$ (see eqn.(2.32) and is ruled out from the above bounds. Instead, the 95\% confidence limit on the size i.e. $R \leq 130 \, \mu m$ for
2.4. Tests of the gravitational inverse square law

Figure 2.2: Constraints on Yukawa violations of the gravitational inverse square law [15]. The shaded region is excluded at the 95% confidence level. Heavy lines labeled Eot-Wash 2006, Eot-Wash 2004, Irvine, Colorado and Stanford show experimental constraints from the works in Refs. [14, 15], [17], [18], and [19], [16], respectively. Lighter lines show various theoretical expectations summarized in Ref. [20].

two equal extra dimensions case is used to obtain a bound on the corresponding scale of the new physics, which turned out to be $M_s \geq 1.7$ TeV. Implications on other theoretical scenarios are discussed in [14].

2.4.3 RS type corrections to ISL

Recently, there has been a proposal for an experiment to search for the Randall-Sundrum type of corrections to the Newton’s ISL [21]. The RS type of correction to the ISL is given by

$$V(r) = -\frac{G_N m}{r^2} \left(1 + \frac{l_s^2}{r^2}\right)$$  \hfill (2.33)
where the RS parameter $l_s^2 = \frac{2}{3} l^2$, $l$ is the curvature scale of 5-dimensional anti-deSitter space-time, $G_N$ is the Newton’s gravitational constant, $m$ is the mass and $r$ is the distance in the three dimensional space. The experimental set up is based on the torsion pendulum and the expected sensitivity of the experiment is to probe the RS parameter up to 10 microns.

### 2.5 Unparticle scenario

Banks and Zaks (BZ) [22] studied non-abelian gauge theories by considering the number of fermions to be a continuous parameter such that the two loop beta function vanishes at an infra-red fixed point. Below this fixed point the theory becomes scale invariant. In any scale invariant sector the fields correspond to either massles particles or particles with a continuous mass distribution. Consequently in such scale invarinat sector with a non-integer number of fermions, the conventional particle interpretation is lost.

Following this idea, Georgi [23] has proposed that the theory at high energies can have both the SM fields as well as the BZ fields. These two sectors interact through the exchange of particles with a very large mass scale $M_U$. Below this scale $M_U$ the couplings of the interaction have a generic form

$$\frac{1}{M^{d_{SM}+d_{BZ}-4}} O_{SM} O_{BZ},$$

where $O_{SM}$ and $O_{BZ}$ are operators constructed out of SM and BZ fields with mass dimension $d_{SM}$ and $d_{BZ}$ respectively. This hidden $BZ$ sector can have an infra-red (IR) fixed point $\Lambda_U$ below which the scale invariance emerges in the theory. As the couplings in the $BZ$ sector need not be zero (trivial) at the fixed point, it corresponds to a non-trivial IR fixed point. The scale invariant sector below this fixed point $\Lambda_U$ can not be described in terms of well defined particles but with “unparticles”. In this effective theory valid below $\Lambda_U$, the above interaction of the SM fields with the $BZ$
2.5. Unparticle scenario

sector matches onto the form

\[
C_U \frac{\Lambda^{d_{SU}}_{U} \tilde{d}_U}{M^{d_{SM}+d_{SU}-4}_{SM} \tilde{O}_{SM} O_U},
\]

(2.35)

where \(d_U\) is the scaling dimension of unparticle operator \(O_U\). In principle, unparticle operators can have different tensor structures, such as scalar, vector or tensor. The effective interaction for scalar unparticle consistent with the SM gauge symmetries are:

\[
\mathcal{L}_{\text{int}} \supset \frac{\lambda_s}{4 \Lambda_u^{d_s}} F_{\mu \nu} F^{\mu \nu} O_u + \frac{\lambda_s}{\Lambda_u^{d_s-1}} \bar{\psi} \psi O_u.
\]

(2.36)

The coupling of the vector unparticles can be given as

\[
\mathcal{L}_{\text{int}} \supset \frac{\lambda_v}{\Lambda_u^{d_v}} \bar{\psi} \gamma_\mu \psi O_u^\mu,
\]

(2.37)

For the tensor unparticle, we assume that the SM fields couple to the unparticle operator \(O_u^{\mu \nu}\) via SM energy momentum tensor \(T_{\mu \nu}\):

\[
\mathcal{L}_{\text{int}} \supset \frac{\lambda_t}{\Lambda_u^{d_t}} T_{\mu \nu} O_u^{\mu \nu},
\]

(2.38)

where \(d_s, d_v t\) and \(d_t\) are the scaling dimensions of scalar, vector and tensor unparticle operators respectively, and \(\lambda_s, \lambda_v\) and \(\lambda_t\) are dimensionless coupling constants given by

\[
\lambda_u = C_U \frac{\Lambda^{d_{SU}}_{U} \tilde{d}_U}{M^{d_{SM}+d_{SU}-4}_{SM} \tilde{O}_{SM} O_U}.
\]

Unitarity imposes constraint \(d_s > 1\) on the scaling dimension of scalar unparticle [25] and scale invariance restricts [26] \((d_v, d_t) \geq 3\). The scale invariance fixes the two-point functions of unparticle operators, apart from an overall normalization, without requiring any detailed knowledge of the theory at high energies. The propagators for

\[\text{[Footnote]}\]

There exist no known examples of scale invariant local field theories that are not conformally invariant [24]
the scalar, vector and tensor unparticles are respectively given as [26–28]

\[
\int d^4x \ e^{-ik \cdot x} \langle 0 | TO_u(x) O_u(0) | 0 \rangle = -i \ C_S \frac{\Gamma(2 - d_s)}{4^{d_s-1}} \Gamma(d_s) (-k^2)^{d_s-2} \]

where \(C_S\) are overall normalization constants, in practice are set equal to unity. The terms given by ellipses in general do not contribute owing to the conservation of the vertex factors in the case of tensor unparticles.

The phase space for the real emission of unparticles is the same as that of \(d_u\) (scaling dimension) number of massless particles as given by [23, 28]

\[
dP_{d_u} = \frac{16\pi^2}{(2\pi)^{2d_u}} \frac{\Gamma(d_u + 2)}{\Gamma(d_u - 1) \Gamma(2d_u)} q^{d_u-2} \]

where \(q = (p_1 + p_2 + ... p_{d_u})\)

Recently unparticle phenomenology, in the context of present and future colliders have been explored in great detail [29].