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Journal Publications

1. Generation of wakefields and terahertz radiation in laser-magnetized plasma interaction
   Pallavi Jha, Akanksha Saroch and Rohit Kumar Mishra

2. Laser wakefield acceleration in magnetized plasma
   Pallavi Jha, Akanksha Saroch, Rohit Kumar Mishra and Ajay Kumar Upadhyay

3. Wakefield generation via two color laser pulses
   Pallavi Jha, Akanksha Saroch and Nirmal Kumar Verma

4. Simulation study of wakefield generation by two color laser pulses propagating in homogeneous plasma
   Rohit Kumar Mishra, Akanksha Saroch and Pallavi Jha

5. Wakefield generation and electron acceleration by intense super-Gaussian laser pulses propagating in plasma
   Pallavi Jha, Akanksha Saroch and Rohit Kumar Mishra
Conference Proceedings

1. Generation of wakefields via laser-magnetized plasma interaction
   **Akanksha Saroch**, Pallavi Jha and Rohit Kumar Mishra
   26th National Symposium on Plasma Science and Technology-2011 (Patna).

2. THz radiation generation via laser-magnetized plasma interaction
   **Akanksha Saroch**, Pallavi Jha and Rohit Kumar Mishra
   27th National Symposium on Plasma Science and Technology-2012 (Puducherry).

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   28th National Symposium on Plasma Science and Technology-2013 (Bhubaneswar).
Summary

With the advent of short pulsed, high intensity lasers, the study of interaction of intense radiation with plasma became a field of great importance. The interaction of high intensity laser radiation with preformed plasma can be applied to the study of a variety of novel, nonlinear phenomena which include high harmonic as well as Terahertz (THz) radiation generation, X-ray sources, inertial confinement fusion and wakefield generation for particle acceleration. Conventional particle accelerators cannot sustain electric field gradients greater than 100 MV/m due to cavity breakdown. However, plasma being pre-ionized does not have a breakdown limit and therefore plasma based accelerators can support much larger (10-100 GV/m) electric fields. Therefore, plasma based accelerators are an attractive alternative to conventional accelerators.

The acceleration of charged particles by plasma based accelerators utilizes the concept of generation of plasma waves or wakefields, driven by an electron beam as in plasma wakefield accelerator (PWFA) or by intense laser pulses. The laser-plasma based accelerators include (i) the plasma beat wave accelerator (PBWA), which uses two conventional long (~100 ps) laser pulses of modest intensity (~$10^{14}$-$10^{16}$ W/cm$^2$); (ii) the self-modulated laser wakefield accelerator (SMLWFA) in which enhanced acceleration of the charged particles is achieved via resonant self-modulation of the laser pulse and (iii) the laser wakefield accelerators
(LWFA) which uses the concept of acceleration by wakefields generated by compact, short (< 1ps), high intensity ($\geq 10^{18}$W/cm$^2$) laser pulses propagating in plasma. The ponderomotive force arising due to a pulsed laser beam, propagating through underdense plasma, pushes the electrons away from the region of high laser intensity. Since the ionic mass is much greater than the electronic mass, the ions are assumed to form a stationary background in the plasma system. The displacement of electrons creates a local charge separation. The electrostatic restoring force generated by the space charge tends to restore the perturbed plasma electron density distribution, thereby creating alternating regions of positive and negative charges, inside and behind the laser pulse. These oscillations follow the laser pulse in the form of a wakefield of wavelength $\lambda_p = \frac{2\pi}{\omega_p}$. 

The present thesis deals with a detailed analytical as well as simulation study of wakefield generation due to the interaction of short (~fs) laser pulses with preformed plasma. The plasma is assumed to be cold so that prior to the passage of the laser pulse, the plasma electrons are at rest. The study includes (i) terahertz radiation generation due to the interaction of linearly polarized laser pulses with transversely magnetized plasma (ii) enhancement of wakefields and electron acceleration by circularly polarized laser pulses propagating in axially magnetized plasma as well as super-Gaussian pulses propagating in homogeneous plasma and (iii) generation of wakefields by two-colour laser pulses interacting with homogeneous plasma.
Chapter 1 includes the basic properties of plasma and condition required for the propagation of electromagnetic waves (laser) inside the plasma. A brief summary of the development of plasma based accelerators is presented. Nonlinear phenomena of wakefield generation by the propagation of the short laser pulses through uniform plasma, in the mildly as well as highly relativistic regimes of laser intensity, has been discussed briefly. Further, the description of trapping and acceleration of externally injected test electron by the generated axial wakefields has also been presented. A concise description of the numerical simulations is also given.

Chapter 2 deals with the generation of axial and transverse wakefields due to the propagation of a linearly polarized laser pulse through plasma which is uniformly magnetized, perpendicular to the propagation as well as polarization directions of the laser pulse, in the mildly relativistic regime of the laser intensity. In this regime, a perturbative technique involving orders of the laser strength parameter has been used. The electric and magnetic fields are derived with the help of time dependent Maxwell’s equations. The source driving these equations is obtained using the Lorentz force and continuity equations. Further, quasistatic approximation is used to study the evolution of the slow plasma electron velocities and wakefields. For weak applied magnetic fields, a perturbative approach is used to obtain the superposition of wakefields for unmagnetized (unperturbed) and magnetized (perturbed) plasma to give the total axial and transverse electric &
magnetic wakefields generated within & behind the laser pulse. It is seen that the axial wakefields generated in unmagnetized and magnetized plasma is nearly the same in both the cases, while the transverse wakefields are significantly enhanced by the presence of external magnetic field. Further, the mutually perpendicular transverse electric and magnetic wakefields are seen to have the same amplitude and oscillate with the plasma frequency both within & behind the laser pulse, thus generating THz radiation. This field is generated due to the coupling of the slow, longitudinal plasma electron velocities with the externally applied transverse magnetic field. This study will be significant for the generation of THz radiation through laser-magnetized plasma interaction.

In Chapter 3, a one-dimensional (1-D) numerical model has been set up to study the generation of longitudinal electrostatic wakefields via propagation of a circularly polarized Gaussian laser pulse through axially magnetized plasma. The direction of the external magnetic field is considered to be along as well as opposite to the direction of propagation of the laser pulse. The nonlinear fluid equations, describing the interaction of the laser pulse with uniformly magnetized plasma are transformed to the laser pulse frame. These equations are reduced to a time independent form using the quasistatic approximation and solved numerically to study the generation of longitudinal, electrostatic wakefields in the highly as well as mildly relativistic regimes. It is seen that the slow transverse velocities and hence transverse wakefields do not arise for such a configuration of the laser and
external magnetic field. However, the longitudinal wakefields are affected by the presence of externally applied field. It is observed that the potential and field of the longitudinal electrostatic wakefields for reversed (forward) magnetic field increases (decreases) as compared to the unmagnetized case. The numerically predicted results when compared with 2-D PIC simulation results, obtained using XOOPIC code, show the same trend of the wakefield amplitudes as predicted via numerical study. Trapping and energy gain of a test electron, externally injected into the electrostatic longitudinal wakefields, are studied by plotting the separatrix curves. It is important to note that in the mildly relativistic regime, the external magnetic field does not cause any enhancement in the effective energy gain by the test electron. However, for the highly relativistic regime, a significant enhancement in effective gain in energy by the test electron is obtained when a reversed magnetic field is applied.

The 1-D numerical model is further used to analyze the longitudinal electrostatic wakefields generated by the interaction of a super-Gaussian laser pulse with homogeneous plasma. The generated wakefields are compared with the wakes driven by a Gaussian laser pulse. It is observed that the generated wake amplitude for super-Gaussian laser pulse shows a remarkable increase as compared to that generated by a Gaussian laser pulse. The 2-D PIC simulation results are found to be in close agreement with the numerically obtained results. Further, the trapping and acceleration of an externally injected test electron by the generated longitudinal
wakes show that the energy gain of the test electron accelerated by the wakes generated by a super-Gaussian laser pulse is appreciably enhanced as compared to the energy gain for Gaussian pulse case.

Chapter 4 focusses on the study of the generation of longitudinal as well as transverse electric wakefields, via passage of two-colour, sinusoidal laser pulses in uniform plasma. A perturbative approach is employed to study the generation of electrostatic wakefields in the mildly relativistic regime. The frequency difference between the two laser pulses is considered to be equal to the plasma frequency. The two laser pulses are linearly polarized along arbitrary directions. The relative angle between the two directions of polarization is varied and the amplitudes of the generated wakefields are compared. It is seen that the net ponderomotive force driving the longitudinal wakefields arises due to two contributions. The first is the sum of the ponderomotive force due to the envelopes of the two individual laser pulses, while the second contribution arises due to the combined effect of both the pulses. The former ponderomotive force is out of (in) phase with respect to the latter, when the two laser pulses are polarized in the same (opposite) directions, thereby reducing (enhancing) the net ponderomotive force. The electrostatic wakefields driven by this ponderomotive force also show the same trend of results, when plotted for same and oppositely polarized laser pulses. It is seen that the wakefield amplitude generated by two colour, oppositely polarized laser pulses is the same as that generated by a single laser pulse having intensity greater than the
sum of the intensities of the constituent pulses, in a two colour pulse system. Further, VORPAL code has been used to perform 2-D PIC simulations, in order to validate the results reported via analytical study. It is seen that the longitudinal electric wakefields show the same trend of results as reported analytically. This study may be looked upon as a combination of the laser wakefield as well as plasma beat wave mechanisms.

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4.84×10^{16} \text{W/cm}^2, \quad \lambda_1 = 1.064 \mu \text{m}, \quad \lambda_2 = 1.1006 \mu \text{m}, \quad L_1 = 30 \mu \text{m}, \quad L_2 = 31 \mu \text{m} \text{ and } \lambda_p = 32 \mu \text{m}.

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\(4.84 \times 10^{16} \text{W/cm}^2\) (corresponding to \(a_{01} = 0.2\), \(a_{02} = 0.207\))

\[\lambda_1 = 1.064 \mu m, \quad \lambda_2 = 1.1006 \mu m, \quad L_1 = 30 \mu m, \quad L_2 = 31 \mu m, \quad r_{01} = 40 \mu m, \]

\[x = y = 6 \mu m\] and \(\lambda_p = 32 \mu m\). Dotted curve shows the wakes generated by a single laser pulse with \(\lambda_0 = 1.064 \mu m\), \(a_{01} = 0.41\) \((I_{01} = 2.1 \times 10^{17} \text{W/cm}^2)\) and \(L = 30 \mu m\).

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**Fig. 4.7:** Surface plot of the longitudinal electric wakefield \((E_x)\) generated by two laser pulses with same polarization direction, using 2-D VORPAL simulation code.
Fig. 4.8: Surface plot of the longitudinal electric wakefield \( (E_z) \) generated by two laser pulses with opposite polarization direction, using 2-D VORPAL simulation code.
Plasma is a quasi-neutral gas of charged and neutral particles that exhibits collective behaviour. Irving Langmuir, the Nobel laureate who pioneered the scientific study of a glowing ionized gas, produced by electric discharge in a tube, gave the name \textit{plasma} to this fourth state of matter. However any ionized gas cannot be called plasma. A more rigorous definition requires three criteria to be satisfied. Firstly, the Debye length \( \lambda_D \) (\( = \frac{k_B T}{4 \pi n_0 e^2} \) where \( k_B \), \( T \), \( n_0 \) and \( e \) represent the Boltzmann constant, absolute temperature, plasma electron density and electronic charge respectively), must be smaller than the plasma system size. Debye length is the measure of the distance to which the electric field of an individual charged particle extends, before it is effectively shielded by the oppositely charged particles. Secondly, the number of particles within a sphere of radius equal to the Debye length must be much greater than unity \((\frac{4 \pi \lambda_D^3 n_0}{3}) >> 1\). According to the third criterion, a condition of electrical neutrality or quasi-neutrality must be maintained by the plasma. Microscopic variation of quasi-neutrality leads to plasma electron oscillations with a frequency \( \omega_p = (4 \pi n_0 e^2 / m)^{1/2} \) where \( m \) is the electron mass, known as the plasma frequency. This characterizes plasma as an elastic medium [1-3]. The ionic mass is much greater than the electronic mass. Therefore, the ion oscillation frequency is
very small as compared to that of the electrons. The ions can therefore be regarded as a stationary immobile background in the plasma system.

It is well known that 99% of the matter of our universe is in the plasma state. The plasmas in nature as well as man-made plasmas on earth cover an extremely wide range of particle densities. The most obvious application of plasma science in the field of space science and astrophysics include the study of aurora borealis, solar wind, magnetospheres of the earth and Jupiter, solar corona, sunspots and black holes. Laboratory plasmas find applications in industry and extractive metallurgy. Surface treatments such as plasma spraying, etching in microelectronics, metal cutting, welding, reduced permeation for nitriding surfaces against corrosion and abrasion are some of the industrial applications of plasma [4]. Study of plasma physics achieved great impetus with the startup of research on controlled nuclear fusion for the generation of energy by the production of thermonuclear reactions on earth, similar to that used by stars. With the advent of short pulsed, high intensity lasers [5] the study of interaction of intense radiation with plasma became a field of great importance. The interaction of high intensity laser radiation with preformed plasma can be applied to the study of a variety of novel, nonlinear phenomena which include higher-order harmonic [6-8] as well as Terahertz (THz) [9-11] generation, X-ray sources [12-14], inertial confinement fusion [15,16] and wakefield generation for particle acceleration [17-19].
Plasma based particle accelerators are being developed for studying high-energy physics. Accelerator based experiments have produced many breakthroughs in the field of particle physics during the past 50 years. Conventional particle accelerators like synchrotrons and radiofrequency linear accelerators (RF-LINACs) are capable of generating highly energetic bunches of charged particles which are of scientific interest for the investigation of the fundamental structure of matter. In addition, particle acceleration is a key to provide an advanced research tool in many fields like material science, nanotechnology, nuclear medicine, fusion research, food sterilization, transmutation of nuclear waste and cancer therapy. These particle accelerators are able to reach higher energy regimes, but as the energy increases so does the size and cost of accelerators.

Construction of high energy conventional particle accelerators is an expensive and demanding task. Large facilities such as Large Hadron Collider (LHC) at CERN and Stanford Linear Accelerator (SLAC) have particle accelerators that are kilometers long. In general, the particle accelerators consist of a metal cavity wherein particles are accelerated using an alternating electric field. However, electric field gradients greater than 100 MV/m cannot be sustained by these accelerators due to cavity breakdown resulting in the destruction of the accelerator [20]. Therefore particle energies have to be enhanced by increasing the length over which the particles are accelerated. However, increasing the length of particle accelerators to obtain higher energies is not economically viable. With a
circumference of about 27 km LHC at CERN is already the largest accelerator in
the world and is one of the most expensive scientific instruments ever built [21].
Plasma based accelerators offer a solution to this problem. Plasma being pre-
ionized does not have a breakdown limit and can support much larger (10-100
GV/m) electric fields. For these reasons, plasma based accelerators are an attractive
alternative to conventional accelerators.

The acceleration of charged particles by plasma based accelerators utilizes
the concept of generation of plasma waves or wakefields, driven by an electron
beam as in plasma wakefield accelerator (PWFA) or by an intense laser pulse. The
concept of using electron beam driven plasma waves to accelerate charged particles
was first proposed by Fainberg in 1956 [22]. First experiments on the PWFA
process were carried out by Berezin and co-workers [23]. The basic concept of the
PWFA involves the passage of an ultrarelativistic charged particle bunch through
stationary plasma. The Coulomb force of the beam’s space charge expels the
plasma electrons which again rush back and overshoot when the driving beam
passes, thus setting up a plasma oscillation [24]. In single electron-bunch
experiments, the head of the bunch creates the plasma and drives the wake [25].
The wake generates a high-gradient longitudinal field, which in turn accelerates
particles at the back of the bunch. In this case, the length of the electron bunch is
greater than the plasma wavelength ($\lambda_p$). Alternatively, one can also use two
electron bunches injected into plasma in which the length of each bunch is shorter
than $\lambda_p$. In this the transverse Coulomb forces of the first bunch drives the wakefield that subsequently accelerates the second [26]. PWFA scheme has been popular since the early days of plasma based acceleration techniques till date [27-29]. However, a major disadvantage of PWFA is that the velocity and hence the energy of the leading bunch decreases as it drives the wakefield. This leads to the depletion in energy of the wakefield itself. As a result, the maximum energy of the trailing bunch cannot exceed twice the value of the energy of the leading bunch [30].

To overcome this limitation, a novel concept for particle acceleration using intense laser pulses was proposed. Since then a number of methods are being pursued vigorously to achieve ultrahigh acceleration gradients by laser-plasma based accelerators. These include (i) the plasma beat wave accelerator (PBWA) mechanism, which uses two, conventional, long (~100 ps) laser pulses of modest intensity; (ii) the self-modulated laser wakefield accelerator (SMLWFA) in which enhanced acceleration of the charged particles is achieved via resonant self-modulation of the laser pulse and (iii) the laser wakefield accelerator (LWFA) which uses the concept of acceleration by wakefields generated by compact, short (< 1ps), high intensity ($\geq 10^{18}$W/cm$^2$) lasers propagating in plasma.

The concept of plasma beat wave mechanism was first proposed by Rosenbluth and Liu in 1972 for plasma heating operations [31]. However, Tajima and Dawson were the first to utilize this beat wave concept as an acceleration
technique [32]. In PBWA scheme, two long laser pulses of frequencies $\omega_1$ and $\omega_2$ are used to resonantly excite a plasma wave. This is done by appropriately adjusting the laser and plasma frequencies such that the resonance condition $\omega_1 - \omega_2 = \omega_p$ is satisfied. Since the beat wave excitation is a resonant process, relatively modest laser intensities ($\sim 10^{14}$-$10^{16}$ W/cm$^2$) and fairly long ($\sim$ 100ps) pulses were required to generate large amplitude plasma oscillations [33].

The first two-dimensional (2-D) particle-in-cell simulation of the PBWA scheme was performed by Forslund et al [34]. Subsequently, PBWA mechanism was studied by various researchers theoretically [35, 36], experimentally [37, 38] as well as via simulation [39]. Kitagawa et al have reported the acceleration of background plasma electrons using beat wave mechanism [40]. Tochitsky et al have studied experimentally the enhancement in energy of externally injected electrons in the laser beat-wave induced plasma channel [41]. Parametric excitation of plasma waves by counterpropagating lasers has also been reported analytically by Shvets et al [42].

The invention of chirped pulse amplification (CPA) technique [43], was a major step towards the development of high power lasers. In the early 1990’s Nd:glass lasers were used to generate subpicosecond pulses having few Terawatts (TW) power [44]. These pulses were intense enough to excite a relativistic plasma wave via a parametric instability known as Forward Raman Scattering (FRS) [45]. In FRS, the laser beam decays into a forward propagating Stokes wave, anti-Stokes
wave and a relativistic plasma wave. Once the Stokes and anti-Stokes waves become sufficiently intense, they beat with the pump wave to produce an amplitude modulated envelope of the electric field. Under appropriate conditions the plasma wave traps and accelerates the plasma electrons. Such a laser plasma based acceleration technique is called Self-Modulated Laser Wakefield Accelerator (SMLWFA) [30].

Several studies for the acceleration of electrons by relativistic plasma waves generated by a modulated laser pulse have been reported [46-48]. Joshi et al were the first to point out experimentally the role of FRS in electron acceleration [49]. The self modulation of relativistically guided laser pulses was first observed via fluid simulations by Andreev et al [50]. Krall et al [51] simulated the self-modulated LWFA, including the acceleration of an injected electron bunch. The most impressive experimental result on SMLWFA reported the acceleration of background plasma electrons upto 100 MeV by Modena et al in 1995 [52]. An interesting consequence of SMLWFA experiments is the observation of relativistic self-focussing and filamentation of the laser beam in plasma [53]. Acceleration of electron bunches using SMLWFA technique in a self-guided channel has been studied experimentally by Kamperidis et al [54]. Recently, Rao et al [55] have shown the effect of chirp on SMLWFA in the regime of quasi-monoenergetic electron beam generation. However, a major drawback of SMLWFA technique is
that the modulated pulse structure eventually diffracts, thereby leading to reduced energy gain (~100 MeV) by accelerated electrons [30].

The third type of laser-plasma based accelerator is the Laser Wakefield Accelerator. In LWFA, a single, short (< ps), high intensity (≥10^{18} W/cm^2) laser pulse drives a plasma wave. This wave is driven most efficiently when the laser pulse length is of the order of the plasma wavelength [20]. At sufficiently high laser intensity, the ponderomotive force associated with the intensity gradient of the laser pulse, expels a significant amount of plasma electrons away from the path of the pulse, whereas the ions remain stationary due to their large mass. As a result, the laser pulse provides a charge separation. The Coulomb force generated by the charge separation tends to restore the neutrality of the plasma and thus pushes the electrons back towards their equilibrium position. In this way, a plasma wave or wakefield is generated behind and within the laser pulse. When an electron bunch is properly injected into the accelerating region of the wakefield, the electrons co-propagate with the laser wakefield and are accelerated to ultra-relativistic energies [56].

After Tajima and Dawson in 1979 [32], the LWFA was reinvented independently by Gorbunov and Kirsanov [57]. 1-D and 2-D nonlinear theories of LWFA were developed respectively by Bulanov et al [58] and Sprangle et al [59]. The first experimental evidence for plasma wave generation by LWFA mechanism was demonstrated by Hamster et al [60]. Dewa et al have reported electron
acceleration upto 100 MeV through LWFA experiments, using a 2 TW laser system [61]. These experiments successfully proved the basic working of laser wakefield acceleration concept. However, the energy spread of the externally injected accelerated electrons remained extremely broad (~100%). Various researchers have reported the generation of quasi-monoenergetic beams with narrow energy spread [62-64]. Using a 40 TW laser system, Leemans et al have experimentally demonstrated the generation of monoenergetic electron beams with 1 GeV bunch energy and a reasonably low energy spread of 2.5%, using a plasma channel as a guiding structure [65]. Wang et al have also proved experimentally as well as through simulation studies, the generation of accelerated electron bunches with 2 GeV energy and 10% energy spread [66]. Recently, Kim et al have shown electron energy enhancement upto 3 GeV by a dual stage laser-wakefield accelerator, pumped by 1 petawatt (PW) laser system [67].

Another laser-plasma based accelerator system has been demonstrated for very high intensities (~10^{19} W/cm^2). In this case, all the plasma electrons are expelled from the vicinity of the propagation axis of the laser. The heavy immobile ions create an ion column or cavity surrounded by a thin layer of expelled electrons. This regime is referred to as the ‘blow-out’, ‘bubble’ or ‘cavitation’ regime. In addition to electron cavitation, a fraction of the plasma electrons are self-trapped in the ion cavity and are accelerated to high energies [68-70].
The blow-out wake has led to an energy gain of more than 40 GeV for a fraction of electrons in the tail of the bunch [71]. Laser blowout with short ($L \approx \lambda_p$), ultraintense pulses was studied using PIC simulations and theoretical modelling by Pukhov and Meyer-ter-Vehn [72] and Lu et al [73]. Simulation studies conducted by Kostyukov et al [74] revealed that the background plasma electrons are self-injected near the back of the cavity and accelerated to high energies.

The interaction of intense laser pulses with plasma also finds application in the generation of radiation. The generated radiation can either be a higher order harmonic of the frequency of the laser [75-80] or Terahertz (THz) radiation [81-84]. Terahertz radiation is of great current interest owing to its ability to nondestructively analyze a wide range of materials in detail. It also finds applications in the field of medical imaging, material characterization, explosive detection, outer space communication and homeland security [85]. Many physical mechanisms have been reported for THz generation in plasma on interaction with a high power laser pulse [86-90]. A number of simulations, experimental and theoretical analyses have been done to study the generation of THz radiation. Sheng et al [91] have reported THz radiation from the vacuum-plasma interface driven by ultrashort laser pulses, by 2D PIC simulation. Electromagnetic radiation from Cherenkov wakes excited by short laser pulses in magnetized plasma has been shown through 1D PIC simulations [92]. Wu et al [93] have reported emission of
THz radiation from laser wakefields in inhomogeneous magnetized plasma via simulation. A theoretical model for THz generation using four wave mixing of two colour laser pulses in collisional plasma has also been reported [94, 95]. Generation of THz fields via relativistic optical rectification in unmagnetized homogeneous plasma [96] and rippled density magnetized plasma [97] has been theoretically examined. Recently, generation of THz radiation by focusing bichromatic laser pulses in preformed plasma has been shown experimentally by Chizhov et al [98].

The propagation dynamics of laser pulses propagating in plasma is significantly affected by the presence of self-generated [99-103] or externally applied [104-107] magnetic fields. The magnetic field significantly alters the plasma electron trajectory. The presence of external magnetic field affects the physical processes such as harmonic generation [108], self-focussing [109] and wakefield generation [110-112]. Hosakai et al have shown the effect of external static magnetic field on the emittance [113] and energy [114] of electron beams generated by laser wakefield acceleration. Trigger and control of self-injection in the presence of a static transverse magnetic field in the bubble regime has been studied by Vieira et al [115].

1.1 Nonlinear interaction of laser pulses with plasma

Since all applications based on laser-plasma interactions utilize intense laser pulses, it is important to study nonlinear behaviour of the interaction process.
Consider an intense linearly polarized laser field, represented by the vector potential

\[ \vec{A} = \hat{\mathbf{A}}_0(r, z, t) \sin(k_0 z - \omega_0 t) \quad (1.1) \]

(where \( k_0 \) and \( \omega_0 \) are respectively the wavenumber and frequency of the laser pulse), to be copropagating through preformed plasma of ambient electron density \( n_0 \, cm^{-3} \). The ions are assumed to form an immobile background owing to the large mass difference between electrons and ions. The propagation of the laser field through homogeneous plasma is governed by the wave equation

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \vec{A} = -\frac{4\pi}{c} \vec{J} = \frac{4\pi}{c} n_e e \vec{v} \quad (1.2)
\]

where \( \vec{J} = -n_e e \vec{v} \) is the current density of the plasma electrons arising due to the influence of the laser field. For intense laser fields, the plasma electron velocity becomes relativistic. This causes relativistic nonlinearity to set in and drive the laser field. In order to evaluate the nonlinear current density, the motion of the plasma electrons under the influence of the laser field can be described by the Lorentz force equation
\[
\frac{d\vec{p}}{dt} = e \frac{\partial \vec{A}}{\partial t} - e \frac{1}{c} \{\vec{v} \times (\vec{\nabla} \times \vec{A})\}
\] (1.3)

where \(\vec{p} = \gamma mv\) and \(\gamma = (1 - v^2 / c^2)^{-1/2}\) is the relativistic factor and \(\vec{v}\) are respectively the relativistic momentum and velocity of the plasma electrons. Since \(\vec{v} \times (\vec{\nabla} \times \vec{A}) = (\vec{\nabla} \cdot \vec{A})\vec{v} - (\vec{v} \cdot \vec{A})\vec{A}\) and the Coulomb gauge condition implies \(\vec{\nabla} \cdot \vec{A} = 0\), Eq. (1.3) may be represented as

\[
\frac{d\vec{p}}{dt} = e \frac{d\vec{A}}{dt},
\] (1.4)

While writing the right hand side of Eq. (1.4), the convective derivative \(\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \nabla)\) has been used. The equation (1.4) of motion can be exactly integrated to give respectively the relativistic transverse quiver momentum and hence the quiver velocity of the plasma electrons as

\[
\vec{p} = \frac{e\vec{A}}{c}
\] (1.5a)

and

\[
\vec{v} = \frac{e\vec{A}}{mc\gamma} = \frac{e\gamma}{c} \sin(k_0z - \omega_0t)
\] (1.5b)
where \( a_0 = (eA_0 / mc^2) \) is the normalized amplitude of the vector potential and the plasma electrons have been assumed to be at rest, prior to the passage of the laser pulse. \( a_0 \) is related to the peak laser intensity as \( a_0 = 8.544 \times 10^{-10} \times \lambda_0 \mu m \times \sqrt{I_0 (W/cm^2)} \) (where \( \lambda_0 \) is the laser wavelength). For a typical Ti:Sapphire laser having \( \lambda_0 = 800 \) nm, \( a_0 = 1 \) corresponds to \( I_0 \approx 10^{18} W/cm^2 \). When \( a_0 << 1 \) (\( a_0 \geq 1 \)), the laser-plasma interaction lies in the mildly (highly) relativistic regime.

Substituting the quiver velocity (Eq.1.5b) into the wave equation (1.2) gives

\[
\omega_0^2 - \frac{\omega_p^2}{\gamma} = c^2 k_0^2. \tag{1.6}
\]

Eq. (1.6) describes the nonlinear dispersion of an intense laser field as it propagates through homogeneous plasma. In the limit \( \gamma \rightarrow 1 \), Eq. (1.6) reduces to the standard linear dispersion relation for an electromagnetic wave (of frequency \( \omega_0 \)) propagating in homogeneous plasma. The group (\( v_g \)) and phase (\( v_p \)) velocities of the laser pulse propagating in homogeneous plasma are respectively given by

\[
v_g = c(1 - \frac{\omega_p^2}{\gamma \omega_0^2})^{1/2} \tag{1.7}
\]

and
$$v_p = c(1 - \frac{\omega_p^2}{\gamma \omega_0^2})^{1/2}. \quad (1.8)$$

From Eqs. (1.7) and (1.8) it may be concluded that the laser pulse cannot propagate with a group velocity greater than the velocity of light; however, $v_p$ is greater than $c$. Further, the dispersion relation (Eq. 1.6) can be used to write the refractive index

$$\eta = \frac{c k_o}{\omega_0}$$

of the plasma as

$$\eta = (1 - \frac{\omega_p^2}{\gamma \omega_0^2})^{1/2}. \quad (1.9)$$

If $\omega_p < \sqrt{\gamma} \omega_0$, the propagation constant and hence the index of refraction is real and the plasma having density $n < m \gamma \omega_0^2 / 4 \pi e^2$ is said to be underdense. If $\omega_p$ is increased to a value equal to $\sqrt{\gamma} \omega_0$, the plasma density attains a critical value $n_c = m \gamma \omega_0^2 / 4 \pi e^2$ and the plasma is said to be critically dense. With further increase in plasma frequency $\omega_p > \sqrt{\gamma} \omega_0$, the plasma becomes overdense $n > n_c$. Under this condition the refractive index (propagation constant) becomes imaginary and the laser cannot propagate. In the relativistic regime when $\gamma > 1$, the factor $\sqrt{\gamma} \omega_0$ may become greater than $\omega_p$ even though $\omega_0 < \omega_p$, thereby making the plasma
transparent to the laser light. This phenomenon is called Electromagnetically Induced Transparency (EIT) [116, 117].

The generation of intense laser radiation is in the form of short pulses [30]. The propagation of intense, spatially varying laser pulses through plasma sets up the ponderomotive nonlinearity which leads to the generation of (slow) plasma waves [118]. In order to study the generation of plasma waves (or wakefields), the plasma electron velocities are considered to be a superposition of slow \( \vec{v}_s \) and the fast \( \vec{v}_r \) components oscillating respectively at the plasma \( \omega_p \) and laser \( \omega_0 \) frequencies. The Lorentz force equation (1.3) can be rewritten for the total plasma electron momentum as,

\[
\frac{\partial \tilde{p}_T}{\partial t} + (\vec{v}_T, \vec{v}_r) \tilde{p}_T = e \frac{\partial \vec{A}}{\partial t} - \frac{e}{c} \left\{ \frac{1}{m \gamma} \tilde{p}_T \times (\vec{V} \times \tilde{p}_T) \right\} + e \vec{V} \phi
\]  

(1.10)

where \( \tilde{p}_T (= \tilde{p} + \tilde{p}_r) = \gamma m (\vec{v} + \vec{v}_r) \) and \( \phi \) is the scalar potential of the space charge fields. While writing Eq. (1.10), the fast quiver momentum (Eq. 1.5a) has been used. Substituting the vector identity

\[
\frac{1}{m \gamma} \tilde{p}_T \times (\vec{V} \times \tilde{p}_T) = \frac{1}{2m \gamma |p_T|^2} - \frac{1}{m \gamma} (\tilde{p}_T, \vec{V}) \tilde{p}_T
\]  

(1.11)
into Eq. (1.10) and separating the slow and fast terms gives the evolution equation for the slow plasma electron velocity as

$$\frac{\partial (\mathbf{v}_e)}{\partial t} = \frac{e}{m} \nabla \phi - \frac{e^2}{2\gamma} \mathbf{v}(a^2).$$

(1.12)

The first and the second terms on the right hand side of Eq. (1.12) represent respectively the space charge and the ponderomotive force, responsible for driving the plasma wakefields. The ponderomotive force arising due to a pulsed laser beam, propagating through underdense plasma, pushes the electrons away from the region of high laser intensity. Since the ions are stationary, the displacement of electrons creates a local charge separation. The electrostatic restoring force generated by the space charge tends to restore the perturbed plasma electron density distribution, thereby creating alternating regions of positive and negative charges, inside and behind the laser pulse. These oscillations follow the laser pulse in the form of a wakefield of wavelength $\lambda_p = \frac{2\pi c}{\nu_p}$ [32]. The ponderomotive nonlinearity thus modifies the local charge density and hence the refractive index, causing the characteristics of the propagating laser to be modified.

1.2 Wakefield generation in the mildly-relativistic regime

In order to study the generation of wakefields in the mildly relativistic regime
a perturbative technique is used to expand the Lorentz force equation (1.12) along with the continuity and Poisson’s equations given respectively by

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0
\]  
\[
(1.13)
\]

and

\[
\nabla^2 \phi = 4\pi n_0 (n - 1),
\]  
\[
(1.14)
\]

(where \( n = n_e / n_0 \)) in orders of the normalized laser field amplitude (a). The zeroth order describes the equilibrium plasma state, prior to the passage of the laser pulse, such that \( \vec{v}^{(0)} = \phi^{(0)} = 0 \) and \( n^{(0)} = \gamma^{(0)} = 1 \). The first order quiver velocity of the plasma electrons in the mildly relativistic regime can be obtained from Eq. (1.5b) as \( \vec{v}^{(1)} = \hat{z} \omega_0 a \sin(k_0 z - \omega_0 t) \). Also, \( n^{(1)} = \gamma^{(1)} = \phi^{(1)} = 0 \). The second order expansion of momentum, continuity and Poisson’s equations yield,

\[
\frac{\partial \vec{v}_x^{(2)}}{\partial t} = \frac{e}{m} \nabla \phi^{(2)} - \frac{c^2}{4} \nabla |a|^2,
\]  
\[
(1.15)
\]

\[
\frac{\partial n^{(2)}}{\partial t} + n^{(0)} \nabla \cdot (\vec{v} \cdot \vec{v}_x^{(2)}) = 0
\]  
\[
(1.16)
\]
and \[ \nabla^2 (\phi^{(2)}) = 4\pi en_0 n^{(2)}. \] (1.17)

Differentiating Eq. (1.16) with respect to time, taking the divergence of Eq. (1.15), using Eq. (1.17) and eliminating \( \tilde{\gamma}^{(2)} \), gives the equation governing the evolution of the slow density perturbation as,

\[ \left( \frac{\partial^2}{\partial t^2} + \omega_p^2 \right) n^{(2)} = \tilde{V}^2 \frac{c^2 |a|^2}{4}. \] (1.18)

Similarly, eliminating \( n^{(2)} \) and substituting \( (\tilde{E}^{(2)} = -\tilde{V} \phi^{(2)}) \), yields the equations describing the evolution of the electric wakefield as,

\[ \left( \frac{\partial^2}{\partial t^2} + \omega_p^2 \right) \tilde{E}^{(2)} = -\frac{mc^2 \omega_p^2}{e} \tilde{V} \frac{|a|^2}{4}. \] (1.19)

From Eq. (1.19), it may be noted that the axial wakefields are generated only for pulsed laser systems whereas continuous lasers do not give rise to such fields since the axial gradient of the laser field amplitude would not exist in this case.

In order to solve Eqs. (1.18) and (1.19), the quasi-static approximation (QSA) [119, 120] is used. Under QSA, it is assumed that the electron transit time through the laser pulse is small as compared to the characteristic laser pulse evolution time. This implies that, QSA is valid if \( \tau_L \ll \tau_E \) where \( \tau_L (= L/c, L \) is
the laser pulse length) is the laser pulse duration and $\tau_E$ is the laser pulse evolution time. Transforming the plasma fluid equations (1.18) and (1.19) to the laser pulse frame ($\xi = z - ct$ and $\tau = t$) and neglecting derivatives with respect to $\tau$, under QSA, gives

$$\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) n^{(2)} = \left( \nabla_{\perp}^2 + \frac{\partial^2}{\partial \xi^2} \right) \frac{|a|^2}{4}$$

and

$$\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \tilde{E}^{(2)} = \left( \nabla_{\perp}^2 + \frac{\partial}{\partial \xi} \right) \frac{|a|^2}{4}.$$  \hspace{1cm} (1.20)

From Eq. (1.21), it may be noted that apart from the axial electric wakefields, transverse wakes are also generated due to the Gaussian radial profile of the laser beam. For broad ($k_p r_0 \gg 1$, where $r_0$ is the laser spot size) laser beams, the transverse ponderomotive effect is negligible whereas narrow laser beams give rise to significant transverse wakefields. From Eq. (1.21), it may be concluded that the generated axial electric wakefields depend on the pulse profile of the laser.

1.2(a) Evaluation of axial electric wakefields

The longitudinal component of wakefield evolution equation (1.21) given by
\[
\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \hat{E}_z^{(2)} = -\frac{mc^2k_p^2}{4e} \frac{\partial |u|}{\partial \xi},
\]

(1.22)

can be solved to study the generation of axial electrostatic wakefields. Eq. (1.22) is an inhomogeneous differential equation for which the corresponding homogeneous differential equation

\[
\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) E_z^{(2)} = 0
\]

(1.23)

has a solution of the form

\[
E_z^{(2)} = C_1(\xi) \cos(k_p\xi) + C_2(\xi) \sin(k_p\xi).
\]

(1.24)

Differentiating Eq. (1.24) with respect to \( \xi \) gives

\[
\frac{\partial E_z^{(2)}}{\partial \xi} = C_1'(\xi) \cos(k_p\xi) - k_p C_1(\xi) \sin(k_p\xi) + C_2'(\xi) \sin(k_p\xi) + C_2(\xi) k_p \cos(k_p\xi)
\]

(1.25)

where \( C_1' = \frac{\partial C_1}{\partial \xi} \) and \( C_2' = \frac{\partial C_2}{\partial \xi} \). In order to obtain \( C_1 \) and \( C_2 \), it is assumed that
\[ C_1 \cos(k_p \xi) + C_2 \sin(k_p \xi) = 0. \quad (1.26) \]

Eq. (1.25) is further differentiated with respect to \( \xi \) to give

\[
\frac{\partial^2 E_z^{(2)}}{\partial \xi^2} = -C_1 k_p^2 \cos(k_p \xi) - k_p C_1 \sin(k_p \xi) + C_2 k_p \cos(k_p \xi) - C_2 k_p^2 \sin(k_p \xi).
\]  

(1.27)

Substituting Eqs. (1.24) and (1.27) into Eq. (1.22) gives

\[
-k_p C_1 \sin(k_p \xi) + k_p C_2 \cos(k_p \xi) = -\frac{mc^2 k_p^2}{4e} \frac{\partial |a|^2}{\partial \xi}. \quad (1.28)
\]

Multiplying Eq. (1.26) by \( k_p \sin(k_p \xi) \) and Eq. (1.28) by \( \cos k_p \xi \) and adding gives \( C_2 \) which can be integrated to give

\[
C_2 = -\frac{mc^2 k_p^2}{4e} \int (\frac{\partial |a|^2}{\partial \xi} \cos k_p \xi) d\xi. \quad (1.29)
\]

Similarly, eliminating \( C_2 \) gives

\[
C_1 = -\frac{mc^2 k_p^2}{4e} \int (\frac{\partial |a|^2}{\partial \xi} \sin k_p \xi) d\xi. \quad (1.30)
\]
Substituting the value of $C_1$ and $C_2$ respectively from Eqs. (1.30) and (1.29) in Eq. (1.24) gives

$$E_{z}^{(2)} = -\frac{mc^2k_p^2}{4e} \left\{ \left( \int \frac{\partial |A|^2}{\partial \xi} \sin k_p \xi d\xi \right) \cos k_p \xi + \left( \int \frac{\partial |A|^2}{\partial \xi} \cos k_p \xi d\xi \right) \sin k_p \xi \right\}.$$

(1.31)

Eq. (1.31) is the general solution for the axial wakefield evolution equation (1.22). Substituting the Gaussian $(a^2 = a_0^2 \exp(-2r^2/r_0^2) \exp(-\xi^2/L^2))$ pulse profile into Eq. (1.31) and integrating between $-\infty$ and $+\infty$ gives the axial wakefields as

$$E_{z}^{(2)} = \frac{mc\omega_p}{e} \left( \frac{\sqrt{\pi}a_0^2}{4} \right) k_p \exp \left( -\frac{k_p^2 L^2}{4} \right) \cos k_p \xi$$

(1.32)

where $a_0^2 = a_0^2 \exp(-2r^2/r_0^2)$. Eq. (1.32) explicitly shows the dependence of the wake amplitude on the pulse length $L$. In particular, the wakefields generated by a Gaussian laser pulse attains a maximum amplitude when $L = \lambda_p / \pi \sqrt{2}$ [30].

Fig. (1.1) depicts the variation of the normalized, longitudinal, on-axis electric field amplitude $E_{nc} (= eE_z / mc\omega_p)$ generated by a Gaussian laser pulse with respect to $k_p \xi$, for $a_o = 0.3$ (corresponding to $I = 1.926 \times 10^{17} W/cm^2$),
Fig. 1.1 Variation of the normalized, longitudinal, on-axis electric wakefield 

\[ E_{wz}(= eE_{c} / m c \omega_{p}) \] 
generated by a Gaussian laser pulse with respect to 

\[ k_{p} \xi \] 
for \( a_{0} = 0.3 \) \( (I = 1.926 \times 10^{17} W/cm^2) \), \( \lambda_{0} = 0.8 \mu m \), 

\( \lambda_{p} = 15.0 \mu m, L = 3.8 \mu m \) and \( n_{0} = 4.958 \times 10^{18} cm^{-3} \).
\[ \lambda_0 = 0.8\mu m, \quad \lambda_p = 15.0\mu m, \quad L = 4.0\mu m \quad (= \lambda_p / \pi \sqrt{2}) \quad \text{and} \quad n_0 = 4.958 \times 10^{18} \, \text{cm}^{-3}. \]

These axial wakefields can be utilized for accelerating electrons to high energies.

1.2(b) Evaluation of transverse wakefields

In addition to the axial wakefields, the transverse (Gaussian) profile of the laser beam will lead to the generation of transverse wakefields \( E_r \). These fields can be obtained by using Panofsky-Wenzel theorem [121, 122]

\[ \frac{\partial E_z}{\partial r} = \frac{\partial E_r}{\partial \xi}. \]  \hspace{1cm} (1.33)

Substituting the axial electrostatic wakefields from Eq. (1.32) into Eq. (1.33) yield the transverse wakes generated by a Gaussian laser pulse as,

\[ E_r^{(2)} = \frac{mcow}{e} \frac{\sqrt{\pi} a^2 r}{r_0^2} \exp \left( -\frac{k_p^2 L^2}{4} \right) \sin k_p \xi. \]  \hspace{1cm} (1.34)

These transverse wakefields provide a focusing force to the electrons undergoing acceleration by the axial wakefields.
1.3 Wakefield generation in the relativistic regime

For laser intensities beyond \(10^{18} W/cm^2 (a_0 \geq 1)\), relativistic factor \(\gamma\) plays a dominant role in determining the propagation characteristics of a laser pulse propagating in plasma. Therefore, a perturbative approach cannot be used in this regime. Applying the transformation \(\xi = z - v_g t\) and \(\tau = t\), to the plasma fluid equations (1.12) - (1.14) and using QSA gives

\[
\frac{\partial (\gamma u_z)}{\partial \xi} = \frac{1}{2\beta_g} \frac{\partial a^2}{\partial \xi} - \frac{1}{\beta_g} \frac{\partial \Phi}{\partial \xi},
\]

(1.35)

\[
\frac{\partial^2 \Phi}{\partial \xi^2} = k_p^2 (n - 1)
\]

(1.36)

and

\[
\frac{\partial}{\partial \xi} [n(\beta_g - u_z)] = 0
\]

(1.37)

where \(\beta_g = v_g / c\), \(u_z = v_z / c\) and \(\Phi = e\Phi / mc^2\) are the normalized laser-plasma parameters. Further, combining Eqs. (1.35) - (1.37) yields,

\[
\frac{\partial^2 \Phi}{\partial \xi^2} = k_p^2 \gamma_g^2 \left[ \frac{\beta_g(1 + \Phi)}{(1 + \Phi)^2 - \gamma_g^2 (1 + a^2)^{1/2} - 1} \right]
\]

(1.38)
where $\gamma_g^2 = (1 - \beta_g^2)^{-1/2}$. The nonlinear ordinary differential equation (1.38), governing the evolution of the wake potential, can be integrated numerically, for a given pulse profile, to obtain the potential and hence the axial electric field $(E_z = -\partial \Phi / \partial \xi)$ of the generated wakes [123].

Curves $a$ and $b$ in Fig. (1.2) show respectively the variation of the wake potential (normalized by $e/mc^2$) and the axial wakefield (normalized by $e/mc\omega_e$) with respect to $k_p\xi$. The wakefield is generated by a Gaussian laser pulse (curve-$c$) of intensity $I = 2.14 \times 10^{18} W/cm^2$ (corresponding to $a_0 = 1.0$), $\lambda_0 = 0.8\mu m$ and pulse length $L = 5.0\mu m$ propagating in preformed plasma of density $n_0 = 4.958 \times 10^{18} cm^{-3}$ ($\lambda_p = 15.0\mu m$). Comparing the axial wakes generated in the mildly-relativistic (Fig. 1.1) and the highly relativistic (Fig. 1.2) regimes of the laser intensity, it may be concluded that the wakefield curve steepens as the intensity of the propagating laser pulse increases.

1.4 Acceleration of externally injected test electrons

The acceleration mechanism of a laser wakefield can be studied by injecting a test electron behind the laser pulse. The motion of the test electron, injected into the plasma wave, is described by the Hamiltonian [124]
Fig. 1.2 Variation of the normalized, longitudinal, on-axis electric potential \( \Phi \) (curve-a) and wakefield \( E_w \) (curve-b) generated by a Gaussian laser pulse (curve-c) with respect to \( k_p \xi \), for \( a_0 = 1.0 \) 

\( I = 2.14 \times 10^{18} \text{ W/cm}^2 \), \( \lambda_0 = 0.8 \mu\text{m} \), \( \lambda_p = 15.0 \mu\text{m} \), \( L = 5.0 \mu\text{m} \) and \( n_o = 4.958 \times 10^{18} \text{ cm}^{-3} \).
\[ H(\gamma, \psi) = \gamma(1 - \beta \beta'_p) - \phi(\psi) \]  
\( (1.39) \)

where \( \beta(v/c) \) and \( \beta'_p(v'_p/c) \) are respectively the test electron velocity and phase velocity of the generated plasma wave and \( \psi = k_p \xi \). Since the generated plasma wave follows the laser pulse (propagating with velocity \( v_g \)), the phase velocity of the plasma wave is assumed to be equal to the group velocity of the laser pulse \( v'_p = v_g \). Assuming the system energy to be conserved, the conservation equation

\[ H(\gamma_e, \psi) = H(\gamma, \psi) \text{ or } H(\gamma_e, \psi_{\text{max}}) = H(\gamma'_p, \psi_{\text{min}}) \]  
\( (1.40) \)

must be satisfied. Assuming that the initial velocity of the test electron is equal to the phase velocity of the plasma wave, the relativistic factor \( \gamma_i \) representing the initial energy of the test electron, is equal to \( \gamma'_p = (1 - v'^2_p/c^2)^{-1/2} \). \( \gamma_e \) represents the relativistic factor corresponding to the maximum energy attained by the test electron. \( \psi_{\text{max}}(\psi_{\text{min}}) \) in Eq. (1.40) represents the phase where the wake potential is maximum (minimum). Combining Eqs. (1.39) and (1.40) yields

\[ \gamma_e(1 - \beta_e \beta'_p) - \phi(\psi) = \gamma_p(1 - \beta'_p^2) - \phi_{\text{min}}(\psi_{\text{min}}), \]  
\( (1.41) \)
where $\beta_e$ is the normalized final velocity associated with the accelerated test electron. The closed orbits plotted in the phase space ($\gamma_e, \psi$) represent the separatrix which separates the closed orbits (trapped electrons) from the open orbits (untrapped electrons). In order to plot the separatrices characterizing the just trapped test electron in the phase space, the test electron is injected at $\psi_{\min}$ where the wake potential is minimum.

Fig. (1.3) is the phase space plot depicting the separatrix (solid curve) which separates the untrapped electrons (dotted curve) from the trapped electrons (dashed curve). $\gamma_e(\text{max})$ and $\gamma_e(\text{min})$ represent respectively the relativistic factor corresponding to the maximum and minimum energies attained by the test electron. The peak energy of the test electron, as obtained from the separatrix plots, is further used to determine the energy gain of the accelerated electrons.

1.5 Numerical simulations

Traditionally the behaviour of complex physical systems has been investigated through theoretical and experimental approach. However, there are a large number of physical problems for which precise experiments and diagnostics are difficult and the simultaneous nonlinear interaction of complex processes make theoretical analysis difficult, even though the fundamental laws that govern the system are known. With the advent of high speed computers, simulations based on
Fig. 1.3 Phase space plots depicting the separatrix (solid curve) separating untrapped electrons (dotted curve) from the trapped electrons (dashed curve).
numerical physical models have been used for numerical experiments which can
give as much information about the details of the system as one desires.

Plasmas interacting with lasers span a large range of densities and
temperatures. Based on typical time and length scales of problems, numerical
plasma models are generally classified into two categories: fluid and kinetic. Fluid
codes are widely used to simulate huge complex experiments, such as tokamak and
astrophysical processes like gamma ray bursts and supernovae [125], while kinetic
simulations have been a major tool for research of laser-plasma interaction physics
[126]. In the last few decades, particle-in-cell (PIC) methods, initially proposed by
Dawson [127] and developed by Birdsall [128], proved to be a very reliable method
for kinetic simulations.

PIC codes model plasma as particles which interact self-consistently with
the electromagnetic field, obtained by solving Maxwell’s equations calculated on a
grid. Such codes work at the most fundamental microscopic level and are one of the
most computer-intensive models in plasma physics. The PIC algorithm is valid for
both, the electromagnetic as well as electrostatic limits. PIC codes generally follow
three important steps in the main iteration loop. The first is the deposit process i.e.
some particle quantity, such as a charge, is accumulated on a grid via interpolation
to produce a source density. Various other quantities like current density can also
be deposited on a grid, depending on the model. The second important process is
the field solver, which solves the field equations to obtain the electric and magnetic
wakefields from the source densities. In the electromagnetic limit, a charge conserving current deposition algorithm enables the integration of Maxwell’s equations, while in the electrostatic limit, it solves Poisson’s equation at every time step. Finally, once the fields are obtained, the particle sources are found by interpolation from the grid. PIC codes are used in almost all areas of plasma physics, such as fusion energy research, space physics, ion propulsion and plasma accelerators. XOOPIC [129,130], VORPAL [131], OSIRIS [132] and VLPL [133] are some of the electromagnetic PIC simulation codes that have significantly influenced the development of nonlinear laser-plasma physics.

1.6 Aim

Acceleration of charged particles to ultrarelativistic energies (~GeV) is an important requirement for the study of high energy physics as well as medical research. In this context, laser-plasma based accelerator system promises to be viable and economical option. For this purpose analytical and simulation study of generation of wakefields via laser-plasma interaction is an important area of research. With a view to enhance the wakefield amplitude for the purpose of electron acceleration, the present thesis envisages an extensive study of wakefield generation for different laser and plasma configurations in the mildly \( a_0 \ll 1 \) and highly \( a_0 \geq 1 \) relativistic regimes of the laser intensity. The enhanced axial
wakefields generated due to the propagation of a circularly polarized laser pulse through preformed plasma, uniformly magnetized along the propagation direction of the pulse, can be utilized for the purpose of acceleration of charged particles. Likewise, the transverse wakes obtained by the interaction of a linearly polarized laser pulse with magnetized (perpendicular to the directions of polarization as well as propagation of the laser field) plasma, will prove to be a useful tool for terahertz radiation generation. The concept of using two-colour laser pulses or pulses of different profiles, for the purpose of wakefield amplitude enhancement, has also been investigated analytically and via simulations.

The present thesis will prove to be useful for applications in electron acceleration by wakefields generated during laser-plasma interaction process as well as emission of electromagnetic radiation in the THz frequency range.

1.7 Approach

The present thesis deals with a detailed analytical as well as simulation study of wakefield generation due to the interaction of short (~fs) laser pulses with preformed plasma. The plasma is assumed to be cold so that prior to the passage of the laser pulse, the plasma electrons are at rest. Since the ionic mass is much greater than the electronic mass, the ions are assumed to form a stationary background in the plasma system. In order to enhance the amplitude of the
generated wakes, the contributions of (i) magnetized plasma (ii) two-colour laser pulse system and (iii) super-Gaussian pulse profile of the propagating laser pulse, have been taken into account.

The axial and transverse wakefields generated due to the propagation of a linearly polarized laser pulse through plasma which is uniformly magnetized, perpendicular to the propagation as well as polarization directions of the laser pulse, have been evaluated in the mildly relativistic regime of the laser intensity. In this regime, a perturbative technique involving orders of the laser strength parameter has been used. The electric and magnetic fields are derived with the help of time dependent Maxwell’s equations. The source driving these equations is obtained using the Lorentz force and continuity equations. Further, quasistatic approximation is used to study the evolution of the slow plasma electron velocities and wakefields. For weak applied magnetic fields, a perturbative approach is used to obtain the superposition of wakefields for unmagnetized (unperturbed) and magnetized (perturbed) plasma to give the total axial and transverse electric & magnetic wakefields generated within & behind the laser pulse. Further, the analysis of transverse wakefields leading to the generation of THz radiation in magnetized plasma has also been presented. A perturbative procedure is also used to evaluate the fields generated by two-colour laser pulses propagating in homogeneous plasma. The amplitude of the wakes thus generated is compared with those generated by a single laser pulse.
In the relativistic regime when $a_0 \geq 1$, the propagation dynamics of laser pulse through homogeneous plasma is influenced significantly by relativistic factor $\gamma$. Therefore, a perturbative approach is not valid in this regime. The plasma fluid equations viz. continuity and Poisson’s equations along with the Lorentz force equation are solved numerically using the fourth order Runge-Kutta algorithm to study the generation of wakefields. The enhancement in axial wakefields generated due to the propagation of a circularly polarized laser pulse through plasma which is uniformly magnetized along the propagation direction of the laser pulse, can be used for accelerating the charged particles. The same numerical technique is utilized to study the enhancement of amplitude of the generated wakes as well as electron acceleration by super-Gaussian laser pulse. Further, using XOOPIC and VORPAL particle-in-cell simulation codes, the analytical studies have been validated.
Chapter 2

Generation of wakefields and terahertz radiation via laser-magnetized plasma interaction

In this chapter, analytical study of terahertz (Thz) radiation generation due to transverse wakefields produced by the propagation of short, linearly polarized laser pulses through homogeneous, magnetized plasma has been presented [134]. The external, uniform magnetic field is considered to be applied along a direction perpendicular to the electric vector as well as the propagation direction of the laser pulse. A perturbative technique is used to obtain the electric and magnetic wakefields, generated within and behind the laser pulse, in the mildly relativistic regime. Under appropriate conditions, these wakefields lead to the generation of terahertz radiation.

2.1 Formulation

Consider a linearly polarized laser pulse propagating along the z-direction, through homogeneous plasma, embedded in a uniform magnetic field \((\hat{e}, b)\). The electric vector of the laser, polarized along the x-direction, is given by
\[ \vec{E}_L = e, E_0(r,z,t) \cos(k_0 z - \omega_0 t) \] (2.1)

where \( E_0(r,z,t), k_0 \) and \( \omega_0 \) are respectively, the slowly varying amplitude, wavenumber and frequency of the laser beam. The propagation of the laser pulse through plasma is governed by the fluid equations

\[
\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m} \left\{ \vec{E} + \frac{\vec{v} \times (\vec{B} + \vec{b})}{c} \right\} + \vec{v} \times (\vec{\nabla} \times \vec{v}) - \frac{1}{2} \vec{v} (\vec{v} \cdot \vec{v}) \] (2.2)

and

\[
\frac{\partial n_e}{\partial t} + \vec{v} (n_e \vec{v}) = 0; \] (2.3)

as well as Maxwell’s time dependent equations

\[
\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \] (2.4a)

and

\[
\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}. \] (2.4b)
where $\vec{v}$, $-e$ and $m$ are respectively, the velocity, electric charge and rest mass of the plasma electrons. In Eq. (2.2), $\vec{B}$ represents the magnetic field of the laser while in Eq. (2.4b) $\vec{J}(-n_ee\vec{v})$ is the current density. It may be noted that in Eq. (2.2), the nonlinear terms $\vec{v} \times \vec{B}$ and $\vec{v} \times (\vec{V} \times \vec{v})$ representing vortex motion, cancel each other.

Using perturbative technique all parameters ($P$) are expanded in orders of the radiation field as

$$ P = P^{(0)} + P^{(1)} + P^{(2)} . \tag{2.5} $$

Thus, the first order expansion of Eqs. (2.2) and (2.3) gives,

$$ \frac{\partial \vec{u}^{(1)}}{\partial t} = -\frac{e}{m} \vec{E}^{(1)} + \frac{\vec{v}^{(1)} \times \vec{B}}{c} \tag{2.6a} $$

and

$$ \frac{\partial n_e^{(1)}}{\partial t} + \nabla (n_e^{(0)} \vec{v}^{(1)}) = 0 \tag{2.6b} $$

Here, $n_e^{(0)}(= n_0)$ is the ambient (unperturbed) plasma electron density prior to the passage of the laser pulse. The equations governing the evolution of the first order perturbed plasma electron velocity components are given by,
\[
\frac{\partial \vec{v}_z^{(1)}}{\partial t} = -\frac{e}{m} \vec{E}_z^{(1)} + \omega_c v_{z}^{(1)}, \quad (2.7a)
\]

\[
\frac{\partial \omega_z^{(1)}}{\partial t} = 0, \quad (2.7b)
\]

and

\[
\frac{\partial \vec{v}_x^{(1)}}{\partial t} = -\omega_c v_{x}^{(1)}, \quad (2.7c)
\]

where \( \omega_c (= eb/mc) \) is the electron cyclotron frequency. Differentiating Eq. (2.7a) with respect to \( t \) and substituting Eq. (2.7c) gives

\[
\frac{\partial^2 \vec{v}_z^{(1)}}{\partial t^2} + \omega_c^2 v_{z}^{(1)} = -ca_o \omega_0^2 \sin(k_0 z - \omega_0 t), \quad (2.8)
\]

where \( a_o (= eE_0/mc\omega_0 << 1) \) is the laser strength parameter. Eq. (2.8) represents the forced oscillation of the first order perturbed transverse plasma electron velocity, driven by a force having frequency \( \omega_0 \) and amplitude \( ca_o \omega_0^2 \). The solution of Eq. (2.8) is given by

\[
v_{z}^{(1)} = -\frac{ca_o \omega_0^2}{\omega_c^2 - \omega_0^2} \sin(k_0 z - \omega_0 t). \quad (2.9a)
\]
Eq. (2.9a) represents the transverse quiver velocity of the plasma electrons (along the direction of polarization of the laser pulse) in magnetized plasma. Substituting Eq. (2.9a) into Eq. (2.7c) gives the velocity of the plasma electrons along the direction of propagation of the laser pulse as

$$v_z^{(1)} = \frac{ca_p \omega_p \omega_m}{\omega_z^2 - \omega_0^2} \cos(k_z z - \omega_m t). \quad (2.9b)$$

It may be noted that in the absence of the external magnetic field, Eq. (2.9a) reduces to the standard quiver velocity of the plasma electrons in unmagnetized plasma while Eq. (2.9b) gives $v_z^{(1)} = 0$. Substituting the transverse and longitudinal velocities (Eqs. 2.9) into the first order continuity equation (2.6b) gives the first order perturbed plasma electron density as

$$n_e^{(1)} = -\frac{n_0 a_p \omega_p \omega_m}{\omega_z^2 - \omega_0^2} \cos(k_z z - \omega_m t). \quad (2.10)$$

If the external magnetic field is switched off, $n_e^{(0)} = 0$.

The second order perturbative expansion of Eq. (2.2) gives

$$\frac{\partial v_z^{(2)}}{\partial t} = -\frac{e}{m} E_z^{(2)} + \omega_z v_z^{(2)} - \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( v_x^{(1)} v_x^{(1)} + v_x^{(1)} v_x^{(1)} \right) \right] \quad (2.11a)$$
\[
\frac{\partial v_y^{(2)}}{\partial t} = -\frac{e}{m} E_y^{(2)} - \frac{1}{2} \left[ \frac{\partial}{\partial y} \{v_x^{(1)} v_y^{(1)} + v_y^{(1)} v_z^{(1)} \} \right]
\]

(2.11b)

and

\[
\frac{\partial v_z^{(2)}}{\partial t} = -\frac{e}{m} E_z^{(2)} - \omega_x v_x^{(2)} - \frac{1}{2} \left[ \frac{\partial}{\partial z} \{v_x^{(1)} v_y^{(1)} + v_y^{(1)} v_z^{(1)} \} \right]
\]

(2.11c)

where \( E_x^{(2)} \), \( E_y^{(2)} \) and \( E_z^{(2)} \) represent the electric field components generated by the interaction process. The terms on the right hand side of Eqs. (2.11) contain fast terms oscillating at the second harmonic of the laser frequency as well as the constant terms contributing to the generation of slow velocities. These slow velocities will drive the electric and magnetic wakefields generated by laser-magnetized plasma interaction.

In order to study the generation of electric and magnetic wakefields, the Maxwell's equations (2.4) are written in terms of independent variables \( \xi = z - ct \) and \( \tau = t \). Quasistatic approximation [Chapter 1, Section 1.2] is applied to obtain the evolution of electric and magnetic fields as

\[
\frac{\partial E_y}{\partial y} - \frac{\partial E_z}{\partial z} = \frac{\partial B_x}{\partial \tau}
\]

(2.12a)

\[
\frac{\partial E_z}{\partial \xi} - \frac{\partial E_x}{\partial x} = \frac{\partial B_y}{\partial \tau}
\]

(2.12b)
\[
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \frac{\partial B_z}{\partial \xi} \quad \text{(2.12c)}
\]

and

\[
\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial \xi} = \frac{4\pi}{c} J_x = \frac{\partial E_x}{\partial \xi} \quad \text{(2.13a)}
\]

\[
\frac{\partial B_x}{\partial \xi} - \frac{\partial B_z}{\partial x} = \frac{4\pi}{c} J_y = \frac{\partial E_y}{\partial \xi} \quad \text{(2.13b)}
\]

\[
\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \frac{4\pi}{c} J_z = \frac{\partial E_z}{\partial \xi} \quad \text{(2.13c)}
\]

where \( E_{x,y,z} \) and \( B_{x,y,z} \) are respectively the components of the electric and magnetic wakefield and \( J_{x,y,z} \) are respectively the transverse and longitudinal current densities driving the slow components of plasma electron motion. Considering a weak applied magnetic field, the second order parameters can further be expanded perturbatively as

\[
F_{x,y,z}^{(2)} = F_{x,y,z}^{(2,0)} + F_{x,y,z}^{(2,1)} \quad \text{(2.14)}
\]

where \( F \) represents the electric and magnetic wakefields, plasma electron velocities and current density. The first superscript on the right side of Eq. (2.14)
represents the order of the radiation field while the second superscript shows the perturbation due to the presence of the magnetic field. Following this scheme, the perturbative expansion (in orders of the magnetic field) of the second order slow current densities is given by,

\[ J_{k,z}^{(2)} = J_{k,z}^{(2,0)} + J_{k,z}^{(2,1)} = \{-e(n_e^{(1,0)} v_{k,z}^{(1,0)} + n_0 v_{k,z}^{(2,0)})\} + \{-e(n_e^{(1,1)} v_{k,z}^{(1,1)} + n_e^{(1,0)} v_{k,z}^{(1,0)} + n_0 v_{k,z}^{(2,1)})\} \] (2.15a)

and

\[ J_{y}^{(2)} = J_{y}^{(2,0)} + J_{y}^{(2,1)} = \{-e(n_0 v_{y}^{(2,0)})\} + \{-e(n_0 v_{y}^{(2,1)})\}. \] (2.15b)

In Eq. (2.15a), \( n_e^{(1,0)}(v_{k,z}^{(1,0)}) \) and \( n_e^{(1,1)}(v_{k,z}^{(1,1)}) \) are respectively the first order plasma electron density (velocity components) in the absence and presence of the external magnetic field. Considering wakefield generation for the lowest (second) order of the radiation field, the first superscript pertaining to the order of the electromagnetic field is dropped for the sake of convenience, in the analysis that follows.

In the present study, the laser with spot size \( r_0 \) and pulse length \( L \) is assumed to have a sinusoidal pulse profile of the form, \( a^2 = a_r^2 \sin^2 \pi \xi / L \) where \( a_r^2 = a_0^2 \exp(-2r^2 / r_0^2) \) and \( r^2 = x^2 + y^2 \), represents a Gaussian beam. The slow
current densities driving the longitudinal and transverse electric and magnetic
wakefields are obtained by applying the QSA to Eqs. (2.11) as

\[
\frac{\partial (v_x^{(0)} + v_x^{(1)})}{\partial \xi} = \frac{e}{mc} (E_x^{(0)} + E_x^{(1)}) - \frac{\omega_e}{c} (v_z^{(0)}) - \frac{yca^2 (\omega_e^2 + \omega_0^2) \omega_0^2}{r_0^2 (\omega_e^2 - \omega_0^2)^2}, \quad (2.16a)
\]

\[
\frac{\partial (v_y^{(0)} + v_y^{(1)})}{\partial \xi} = \frac{e}{mc} (E_y^{(0)} + E_y^{(1)}) - \frac{yca^2 (\omega_e^2 + \omega_0^2) \omega_0^2}{r_0^2 (\omega_e^2 - \omega_0^2)^2}
\]

and

\[
\frac{\partial (v_x^{(0)} + v_x^{(1)})}{\partial \zeta} = \frac{e}{mc} (E_z^{(0)} + E_z^{(1)}) + \frac{\omega_e}{c} (v_z^{(0)}) + \frac{cc_0^2 (\omega_e^2 + \omega_0^2)}{4(\omega_e^2 - \omega_0^2)^2} \frac{\partial a^2}{\partial \xi}. \quad (2.16c)
\]

While deriving Eqs. (2.16), the first order velocity components (Eqs. 2.9) have
been used.

In order to determine the electric and magnetic wakefields generated in
unmagnetized plasma, the zeroth order (in the absence of external magnetic field)
of Eqs. (2.12), (2.13) and (2.16) are solved simultaneously. Considering a broad
laser beam \( r_0 >> L \), the wakefields are determined by first neglecting the
transverse gradients of the transversely generated fields [in Eqs.(2.12c) & (2.13c)].
Further, the fields arising due to the transverse gradients are evaluated. A
superposition of the fields thus obtained leads to the resultant axial and transverse
electric & magnetic wakefields generated by a laser pulse propagating in
unmagnetized plasma. Following the procedure given in Sec. 1.2 (a) of Chapter I, the electric wakefields generated within \(0 \leq \xi \leq L\) and behind \(\xi < 0\) the laser pulse, in unmagnetized plasma, are given by,

\[
E_{x,y}^{(0)} = -\frac{(x, y)\varepsilon}{2r_0^2} \left[ f \cos k_p (L - \xi) + (1 - f) \cos \frac{2\pi \xi}{L} - 1 \right], \quad \text{(2.17a)}
\]

\[
E_{x}^{(0)} = \frac{\varepsilon k_p f}{8} \left[ \sin k_p (L - \xi) + \frac{k_p L}{2\pi} \sin \frac{2\pi \xi}{L} \right], \quad \text{(2.17b)}
\]

\[0 \leq \xi \leq L\]

&

\[
E_{x,y}^{(0)} = -\left[ \frac{(x, y)\varepsilon}{2r_0^2} + \frac{8(x, y)\varepsilon}{k_p r_0^4} \left( 1 - \frac{x^2 + y^2}{r_0^2} \right) \right] \left( \cos k_p (L - \xi) - \cos k_p \xi \right)
\]

\[-\frac{(x, y)\varepsilon}{2r_0^2} \left( \cos \frac{2\pi \xi}{L} - 1 \right), \quad \text{(2.18a)}
\]

\[
E_{x}^{(0)} = \frac{\varepsilon k_p f}{8} \left[ \frac{\varepsilon f}{k_p r_0^4} \left( 1 - \frac{2x^2}{r_0^2} \right) \right] \left( \sin k_p (L - \xi) + \sin k_p \xi \right), \quad \text{(2.18b)}
\]

\[\xi < 0\]

where \(\varepsilon = mc^2 a_r^2 / e\) and \(f = (1 - k_p L^2 / 4\pi^2)^{-1}\). In both cases, no magnetic wakefields are generated. Now considering the zeroth order (absence of magnetic
field) of Eqs. (2.16) and substituting the electric wakefields (within and behind),
the transverse and axial plasma electron velocities are given by,

\[ v_{x,y}^{(0)} = \frac{e\mathcal{E}_f(x,y)}{2mc k_p r_0^2} \left[ \sin k_p (L - \xi) + \frac{k_p L}{2\pi} \sin \frac{2\pi \xi}{L} \right] \]  
\[ (x,y) \leq \xi \leq 0 \]  
\[ (2.19a) \]

\[ v_{z}^{(0)} = \frac{e\mathcal{E}_f}{8mc} \left[ \cos k_p (L - \xi) - \cos \frac{2\pi \xi}{L} \right] \]  
\[ 0 \leq \xi \leq L \]  
\[ (2.19b) \]

\[ v_{x,y}^{(0)} = \frac{e}{m c k_p} \left[ \frac{(x,y)\mathcal{E}_f}{2r_0^2} + \frac{8(x,y)\mathcal{E}_f}{k_p^2 r_0^4} \left( 1 - \frac{x^2 + y^2}{r_0^2} \right) \right] \left[ \sin k_p (L - \xi) + \sin k_p \xi \right] \]  
\[ (2.20a) \]

\[ v_{z}^{(0)} = \frac{e}{mc} \left[ \frac{\mathcal{E}_f}{8} + \frac{\mathcal{E}_f}{k_p^2 r_0} \left( 1 - \frac{2x^2}{r_0^2} \right) \right] \left[ \cos k_p (L - \xi) - \cos k_p \xi \right] - \frac{ca^2}{8} \cos \frac{2\pi \xi}{L} . \]  
\[ (\xi < 0) \]  
\[ (2.20b) \]

It may be noted that the on-axis transverse wakefields as well as the plasma
electron velocities generated within & behind the laser pulse, are zero [Eqs. (2.17a),
(2.18a), (2.19a) & (2.20a)]

2.2 Wakefield generation in magnetized plasma

The electric and magnetic wakefields, generated via propagation of a laser
pulse in magnetized plasma, are determined by solving Maxwell's equations [(2.12) & (2.13)]. Using Lorentz force equation (2.16), plasma electron velocities, perturbed by the presence of the magnetic field, are given by,

\[ \frac{\partial v_{x}^{(1)}}{\partial \xi} = \frac{e}{mc} E_{x}^{(1)} - \frac{\omega_{c}}{c} v_{z}^{(0)} \]  
\[ \frac{\partial v_{y}^{(1)}}{\partial \xi} = \frac{e}{mc} E_{y}^{(1)} \]  
and
\[ \frac{\partial v_{z}^{(1)}}{\partial \xi} = \frac{e}{mc} E_{z}^{(1)} + \frac{\omega_{c}}{c} v_{x}^{(0)}. \]

The second term on the right side of Eqs. (2.21a) & (2.21c), respectively, represent the force acting on the plasma electron due to coupling of the slow axial and transverse velocities with the applied magnetic field.

In order to evaluate the electric and magnetic wakefields, in presence of the external magnetic field, the transverse gradients of the transverse fields are neglected (in the broad beam limit). Under this approximation, the first order of Eqs. (2.12c) and (2.13c) gives

\[ B_{z}^{(1)} = 0 \]  

and
\[
\frac{\partial E_{z}^{(1)}}{\partial \xi} = \frac{4\pi}{c} J_{z}^{(1)} \quad \text{(2.22b)}
\]

The current density \( J_{z}^{(1)} \) may be evaluated with the help of Eqs. (2.9b), (2.10) and (2.15a). Differentiating Eq. (2.22b) with respect to \( \xi \) and combining with Eqs. (2.15a) and (2.21c) gives

\[
\left( \frac{\partial^2}{\partial \xi^2} + k^2_p \right) E_{z}^{(1)} = -\frac{k^2_p m_o}{\varepsilon} v_{p0} E_{z0}^{(0)} \quad \text{(2.23)}
\]

Eq. (2.23) shows that the lowest-order (unmagnetized) transverse velocity coupled with the magnetic field drives the longitudinal electric wakefield generated on account of the external magnetic field. The perturbed electric wakefield is obtained by solving the second order inhomogeneous differential equation (2.23). The corresponding homogeneous differential equation written as

\[
\left( \frac{\partial^2}{\partial \xi^2} + k^2_p \right) E_{z}^{(1)} = 0 \quad \text{(2.24)}
\]

has a solution of the form

\[
E_{z}^{(1)} = C_1(\xi) \cos(k_p \xi) + C_2(\xi) \sin(k_p \xi) \quad \text{(2.25)}
\]
where $C_1$ and $C_2$ are functions of $\xi$.

Differentiating Eq. (2.25) with respect to $\xi$ gives

$$\frac{\partial E^{(1)}_z}{\partial \xi} = C'_1(\xi)\cos(k_p\xi) - k_p C_1(\xi)\sin(k_p\xi) + C'_2(\xi)\sin(k_p\xi) + C_2(\xi)k_p\cos(k_p\xi)$$

(2.26)

where $C'_1 = \partial C_1 / \partial \xi$ and $C'_2 = \partial C_2 / \partial \xi$. In order to obtain $C_1$ and $C_2$, it is assumed that

$$C'_1 \cos(k_p\xi) + C'_2 \sin(k_p\xi) = 0.$$  

(2.27)

Eq. (2.26) is further differentiated with respect to $\xi$ to give

$$\frac{\partial^2 E^{(1)}_z}{\partial \xi^2} = -C'_1 k_p^2 \cos(k_p\xi) - k_p C'_1 \sin(k_p\xi) + C'_2 k_p \cos(k_p\xi) - C_2 k_p^2 \sin(k_p\xi).$$

(2.28)

Substituting Eqs. (2.28) and (2.25) into Eq. (2.23) gives

$$-k_p C'_1 \sin(k_p\xi) + k_p C'_2 \cos(k_p\xi) = -\frac{m\omega_j k_p^2}{e} \nu_{v_x}^{(0)}.$$  

(2.29)
Further, substituting the zeroth order velocities from Eq. (2.19a) (within the laser pulse) and Eq. (2.20a) (behind the laser pulse) respectively, reduces Eq. (2.29) to

\[-k_p C_1 \sin k_p \xi + k_p C_2 \cos k_p \xi = -\frac{x e \omega \kappa_p}{2c \tau_0^2} \left\{ \sin k_p (L - \xi) + \frac{L \kappa_p}{2\pi} \sin \frac{2\pi \xi}{L} \right\} \]

\[(0 \leq \xi \leq L) \quad (2.30a)\]

and

\[-k_p C_1 \sin k_p \xi + k_p C_2 \cos k_p \xi = -\frac{\omega k_p}{c} \left\{ \frac{x e \tau}{2r_0^2} + \frac{8x e \tau}{k_p^2 r_0^4} (1 - \frac{x^2}{r_0^2}) \right\} \times \]

\[\sin k_p (L - \xi) + \sin k_p \xi \right\} \]

\[(\xi < 0) \quad (2.30b)\]

Multiplying Eq. (2.27) by \(k_p \sin k_p \xi\) and Eq. (2.30) by \(\cos k_p \xi\) and adding gives

\[C_2 = -\frac{x e \omega k_p}{2c \tau_0^2} \left\{ \sin k_p (L - \xi) + \frac{L \kappa_p}{2\pi} \sin \frac{2\pi \xi}{L} \right\} \cos k_p \xi \]

\[(0 \leq \xi \leq L) \quad (2.31a)\]

and

\[C_2 = -\frac{\omega k_p}{c} \left\{ \frac{x e \tau}{2r_0^2} + \frac{8x e \tau}{k_p^2 r_0^4} (1 - \frac{x^2}{r_0^2}) \right\} \left( \sin k_p (L - \xi) + \sin k_p \xi \right) \cos k_p \xi \]

\[(\xi < 0) \quad (2.31b)\]
Similarly, Eqs. (2.27) and Eqs. (2.30) lead to

\[
C_1 = \frac{x_{\text{ef}} \omega_p k_p}{2cr_0^2} \left\{ \sin k_p (L - \zeta) + \frac{Lk_p}{2\pi} \sin \frac{2\pi \zeta}{L} \right\} \sin k_p \xi \quad (0 \leq \zeta \leq L)
\] (2.32a)

and

\[
C_1 = \frac{\omega_p k_p}{c} \left\{ \frac{x_{\text{ef}}}{2r_0^2} + \frac{8x_{\text{ef}}}{k_p^2 r_0^4} (1 - \frac{x^2}{r_0^2}) \right\} (\sin k_p (L - \zeta) + \sin k_p \zeta) \sin k_p \xi \quad (\zeta < 0)
\] (2.32b)

\[C_1^1\] and \[C_2^1\] obtained above are integrated further to obtain \[C_1\] and \[C_2\] within and behind the laser pulse. Substituting \[C_1\] and \[C_2\] in Eq. (2.25) lead to

\[
E_{\zeta}^{(1)} = -\frac{\omega_p x_{\text{ef}}}{2ck_p r_0^2} \left[ f \sin k_p (L - \zeta) + (f - 1) \frac{k_p L}{2\pi} \sin \frac{2\pi \zeta}{L} \right] \quad (2.33a)
\]

and

\[
E_{\zeta}^{(1)} = \frac{\omega_p Lx_{\text{ef}}}{4cr_0^2} \left[ 1 + \frac{16}{k_p^2 r_0^2} (1 - \frac{x^2}{r_0^2}) \right] (\cos k_p (L - \zeta) - \cos k_p \xi). \quad (2.33b)
\]

Eqs. (2.33a) and (2.33b) represent respectively, the axial electric wakefields generated within and behind the laser pulse due to magnetization of plasma. The sum of Eqs. (2.33a) and (2.17b) (Eqs. (2.33b) and (2.18b)) represent the net
longitudinal electric wakefield generated in magnetized plasma, within (behind) the laser pulse.

In Fig. 2.1, dotted and solid curves represent respectively the variation of normalized, longitudinal electric wakefields $E_{\text{ne}} (= eE_z / mc\omega_p)$ with respect to $\xi / L$, generated within ($\xi / L = 0$ to $1$) & behind ($\xi / L = 0$ to $-3$) the laser pulse, in unmagnetized and magnetized plasma. The laser and plasma parameters used are $a_0^2 = 0.1$ (which corresponds to laser intensity $I = 2.13 \times 10^{17} \text{W/cm}^2$), $r_0 = 20.0 \mu m$, $x = y = 6 \mu m$, $L = \lambda_p = 15.0 \mu m$, $n_0 = 4.958 \times 10^{19} \text{cm}^{-3}$ and $\omega_c / \omega_p = 0.1$ ($b = 713.78 kG = 71.378 Tesla$). It is seen that the peak amplitude of the net electric wakefield generated within (behind) the laser pulse in magnetized plasma is enhanced by 10.7% (2.85%) in comparison to that generated in unmagnetized plasma.

In order to obtain perturbation in the transverse electric wakefield due to the external magnetic field, the first order components of Maxwell’s Equations [(2.12b) & (2.13a) and (2.12a) & (2.13b)] are simultaneously solved to give the $x$ and $y$ components of the current density, respectively, as

$$J^{(1)}_{(x,y)} = -n_0 e v^{(1)}_{(x,y)} = \frac{c}{4\pi} \frac{\partial E_z^{(1)}}{\partial(x,y)}. \quad (2.34)$$
Fig. 2.1 Variation of the normalized longitudinal electric wakefield

$$E_{wz} = \frac{eE_z}{m c \omega_p}$$

generated in unmagnetized (dotted curve) and magnetized (solid curve) plasma, within ($\zeta/L = 0$ to 1) & behind ($\zeta/L = 0$ to -3) the laser pulse with respect to $\zeta/L$ for $a_0^2 = 0.1$

$I = 2.13 \times 10^{17} \text{W/cm}^2$, $\lambda_0 = 1 \mu m$, $r_0 = 20.0 \mu m$, $x = y = 6 \mu m$, $\lambda_p = L = 15.0 \mu m$, $n_0 = 4.958 \times 10^{18} \text{cm}^{-3}$ and $\omega_c/\omega_p = 0.1$

($b = 71.378 \text{Tesla}$).
While deriving Eq. (2.34), Eq. (2.22a) has been used. Differentiating the \( x \) and \( y \) components of the current density (obtained due to perturbation) with respect to \( \xi \) and substituting it in the first order Lorentz force equation ((2.21a) and (2.21b)) give

\[
E_x^{(1)} = \frac{1}{k_p^2} \frac{\partial^2 E_z^{(1)}}{\partial \xi \partial x} + \frac{m \omega_p}{e \omega_p} v_z^{(0)}
\]

(2.35a)

and

\[
E_y^{(1)} = \frac{1}{k_p^2} \frac{\partial^2 E_z^{(1)}}{\partial \xi \partial y}
\]

(2.35b)

Further, substituting \( E_z^{(1)} \) generated within (Eq. (2.33a)) and behind (Eq. (2.33b)) the pulse and \( v_z^{(0)} \) (Eqs. (2.19b) & (2.20b)) gives the transverse components of the first order electric wakefields, generated within and behind the laser pulse as,

\[
E_x^{(1)} = -\frac{\omega_p \epsilon f}{2e} \left\{ \frac{1}{4} + \left( 1 - \frac{4x^2}{r_0^2} \right) \frac{f}{k_p^2 r_0^2} \right\} \cos k_p (L - \xi) - \left( \frac{1}{4} + \frac{f - 1}{k_p^2 r_0^2} \right) \cos \frac{2\pi \xi}{L}, \quad (0 \leq \xi \leq L)
\]

(2.36a)

\[
E_y^{(1)} = \frac{2 \omega_p \epsilon f \chi_{xy}}{c k_p^2 r_0^4} \left[ f \cos k_p (L - \xi) - (f - 1) \cos \frac{2\pi \xi}{L} \right], \quad (0 \leq \xi \leq L)
\]

(2.36b)
The total transverse electric wakefields generated in magnetized plasma are given by the sum of the zeroth (Eqs. (2.17a) & (2.18a)) and the first (Eqs. (2.36) & (2.37)) order fields. From Eqs. (2.36b) & (2.37b), it may be concluded that the amplitude of \( E_y \), generated by the external magnetic field will be insignificant, in the broad beam limit. Hence the amplitudes of the wakes generated along the \( y \)-direction will have almost the same amplitude in magnetized plasma as in the unmagnetized case.

The first order perturbed transverse magnetic wakefields generated within and behind the laser pulse, are obtained by substituting Eqs. (2.36) & (2.37) in Maxwell’s Equations ((2.12a) & (2.12b)), as

\[
\begin{align*}
E_x^{(1)} &= -\frac{\omega_L e f}{4ck_pr_o}(1-\frac{4x^2}{r_o^2})[\sin k_x(L-\xi) + \sin k_x \xi] \\
& \quad + \frac{\omega_L e f}{8c}[\cos k_x(L-\xi) - \cos k_x \xi], \quad (2.37a) \\
E_y^{(1)} &= \frac{\omega_L e xyf}{ck_pr_o^4}[\sin k_y(L-\xi) + \sin k_y \xi]. \quad (2.37b)
\end{align*}
\]

\(( \xi < 0 \) )
\[ B_s^{(1)} = -\frac{\varepsilon_0 c f}{8c} \left[ \cos k_p (L - \zeta) - \left( \frac{f - 1}{f k_p r_0^2} + 1 \right) \cos \frac{2\pi \zeta}{L} \right] \tag{2.38b} \]

\[(0 \leq \zeta \leq L)\]

and

\[ B_x^{(1)} = 0, \tag{2.39a} \]

\[ B_y^{(1)} = \frac{\varepsilon_0 c f}{8c} \left[ \cos k_p (L - \zeta) - \cos k_p \zeta \right]. \tag{2.39b} \]

\[(\zeta < 0)\]

Eqs. (2.38a) and (2.39a) show that no magnetic wakefields are generated along the direction of polarization of the laser pulse, in magnetized plasma. However, from Eqs. (2.38b) and (2.39b) it is seen that the perturbation due to the external magnetic field will contribute to the generation of magnetic wakefields along the \( y \)-direction. The net magnetic wakefield generated along the \( y \)-direction, within and behind the pulse, will be given by Eqs. (2.38b) and (2.39b) respectively, since \( B_y \) in the zeroth order (absence of magnetic field) is zero.

The dashed and solid curves in Fig. (2.2) represent respectively, the variation of the normalized transverse electric wakefield \( E_{nt} (= eE_x / mc\omega_p) \) with respect to \( \zeta / L \), generated behind the laser pulse, in unmagnetized and magnetized plasma. The laser and plasma parameters used for plotting Fig. (2.2) are the same.
Fig. 2.2 Variation of the normalized transverse electric wakefield $E_{\text{nx}}(=eE_z/mc\omega_p)$ generated in unmagnetized (dotted curve) and magnetized (solid curve) plasma behind the pulse with respect to $\xi/L$. For $a_0^2 = 0.1$ ($I = 2.13 \times 10^{17} W/cm^2$), $\lambda_0 = 1 \mu m$, $r_0 = 20.0 \mu m$, $x = y = 6 \mu m$, $\lambda_p = L = 15.0 \mu m$, $n_0 = 4.958 \times 10^{18} cm^{-3}$ and $\omega_c/\omega_p = 0.1$ ($b = 71.378 Tesla$).
as those in Fig. (2.1). The amplitude of the wakes generated in magnetized plasma is about ten times more in comparison to the unmagnetized case.

### 2.3 Generation of Terahertz radiation

Considering the broad beam limit $r_0 >> L - \lambda_p$ (or $k_pr_0 >> 1$), the net electric and magnetic wakefields (given by the superposition of wakefields in unmagnetized and magnetized plasma), generated within and behind the laser pulse, respectively, reduce to

\[ E_z = B_y = -\frac{\omega_ef}{8c}[\cos k_p (L - \xi) - \cos \frac{2\pi\xi}{L}], \quad (2.40a) \]

\[ E_y = B_z = 0, \quad (2.40b) \]

\[ E_x = \frac{\epsilon k_p f}{8}[\sin k_p (L - \xi) + \frac{k_p L}{2\pi} \sin \frac{2\pi\xi}{L}] \quad (0 \leq \xi \leq L) \quad (2.40c) \]

and

\[ E_z = B_y = \frac{\omega_ef}{8c}[\cos k_p (L - \xi) - \cos k_p \xi], \quad (2.41a) \]
\[ E_z = B_z = B_x = 0, \quad (2.41b) \]

\[ E_z = \frac{\varepsilon k_p f}{8} [\sin k_p (L - \xi) + \sin k_p \xi]. \quad (2.41c) \]

\[ (\xi < 0) \]

From Eqs. (2.40c) and (2.41c) it is concluded that the external magnetic field does not contribute significantly towards the enhancement of wakefield amplitude in magnetized plasma along the longitudinal direction and therefore would not be beneficial for particle acceleration schemes. However, the mutually perpendicular transverse wakefields \( E_x \) and \( B_y \) (Eqs. (2.40a) and (2.41a)) are seen to have the same amplitude. This leads to the generation of an electromagnetic radiation oscillating at the plasma frequency. Maximizing the transverse wakefields in the limit \( L \to \lambda_p \), within and behind the laser pulse, gives

\[ E_{xm} = B_{ym} = \frac{\omega_p}{16c} (2\pi - k_p \xi) \sin k_p \xi \quad (0 \leq \xi \leq L) \quad (2.42a) \]

\[ E_{sn} = B_{yn} = -\frac{\omega_p \varepsilon}{8c} \sin k_p \xi. \quad (\xi < 0) \quad (2.42b) \]
Since the intensity of radiation having a transverse Gaussian profile is maximum on-axis, the variation of maximum, normalized, transverse electric (magnetic) field amplitude \( \left( \frac{eE_{\text{rms}}}{mc\omega_p} \right) \) with respect to \( \frac{\zeta}{L} \), is plotted in Fig. (2.3). The laser and plasma parameters are the same as used in Fig. (2.1). Dashed and solid curves in Fig. (2.3) show respectively, the maximum transverse electric wakefield generated within and behind the laser pulse. It is seen that the electric (magnetic) field oscillates with a peak normalized amplitude of 0.0039 (0.0029) behind (within) the laser pulse. This shows that the peak amplitude of the wakes generated behind the laser pulse is about 34% more in comparison to that generated within the pulse. The linear frequency \( (f_r) \) and intensity \( (I_r) \) of radiation obtained behind the laser pulse for the given parameters is 19.98 THz (lying in the THz frequency range) and \( 9.38 \times 10^{10} W/cm^2 \), respectively. However, the peak intensity of radiation within the laser pulse is \( 5.27 \times 10^{10} W/cm^2 \). It is therefore concluded that the electromagnetic radiation generated behind the laser pulse lies in the THz frequency range thereby leading to the generation of THz radiation.

In Fig. (2.4), the dotted and solid curves respectively show the variation of peak normalized on-axis transverse electric (magnetic) field amplitude with the applied magnetic field, within (at \( \frac{\zeta}{L} = 0.25 \)) and behind (at \( \frac{\zeta}{L} = -0.25 \)) the laser pulse. Straight line graphs are obtained, as only the first order interaction with the magnetic field has been considered. This graph can be used to obtain the external field required to generate THz fields of various intensities, within the limit
Fig. 2.3 Variation of maximum normalized on-axis transverse electric (magnetic) wakefields within (dashed curve) & behind (solid curve) the laser pulse of intensity $I = 2.13 \times 10^{17} W/cm^2$, with respect to $\xi / L$ for $a_0^2 = 0.1$ (corresponding to $I = 2.13 \times 10^{17} W/cm^2$), $\lambda_0 = 1 \mu m$, $\lambda_p = L = 15.0 \mu m$, $n_0 = 4.958 \times 10^{18} cm^{-3}$ and $\omega_c / \omega_p = 0.1$ ($b = 71.378$ Tesla).
Fig. 2.4 Variation of maximum normalized on-axis transverse electric (magnetic) field amplitude within (dashed curve for $\zeta/L = 0.25$) & behind (solid curve for $\zeta/L = -0.25$) the laser pulse with the applied magnetic field (0 to 250 Tesla) for $a_0^2 = 0.1$ (corresponding to $I = 2.13 \times 10^{17} W/cm^2$), $\lambda_0 = 1 \mu m$, $\lambda_p = L = 15.0 \mu m$ and $n_0 = 4.958 \times 10^{18} cm^{-3}$.
It is seen that the peak amplitude within the laser pulse is always less than the peak amplitude behind the laser pulse for different values of the magnetic field. This study will be significant for the generation of THz radiation through laser-magnetized plasma interaction.
Chapter 3

Electron acceleration by laser wakefields generated in plasma

In this chapter, a one-dimensional (1-D) numerical model has been set up to study the generation of longitudinal electrostatic wakefields via propagation of (a) a circularly polarized Gaussian laser pulse through axially magnetized plasma [135] and (b) a linearly polarized super-Gaussian pulse through homogeneous plasma [136]. Further, for both the cases, two-dimensional (2-D) particle-in-cell simulations have been performed to study the generated wakefields. Trapping and energy gain of a test electron, externally injected into the electrostatic longitudinal wakefields, are studied by plotting the separatrix curves. Optimum gain in energy of the test electron is determined for the two configurations, and compared respectively with unmagnetized plasma and linearly polarized Gaussian cases.

3.1 Numerical model for the propagation of a circularly polarized laser pulse through magnetized plasma

Consider a cold, homogeneous, underdense plasma having ambient electron density $n_0$, embedded in a constant external magnetic field ($\vec{b} = \sigma b\hat{z}$, where $\sigma = \pm 1$). A circularly polarized laser pulse, represented by the vector potential
\[
\tilde{A} = A(z,t)(\dot{x}\sin(k_0 z - \omega_0 t) - \dot{y}\cos(k_0 z - \omega_0 t))
\]  \hspace{1cm} (3.1)

(where \( k_0 \) (or \( \omega_0 \)) is the wavenumber (frequency) and \( A \) is the amplitude of the laser pulse), is considered to be propagating through the plasma along the positive \( z \)-direction. The electric and magnetic fields are of the form

\[
\tilde{E} = -\frac{1}{c} \frac{\partial \tilde{A}}{\partial t} - \tilde{\nabla} \Phi
\]

and

\[
\tilde{B} = \tilde{\nabla} \times \tilde{A},
\]

where \( \Phi \) is the scalar potential of the generated field.

The set of basic nonlinear fluid equations describing the interaction of the laser pulse with cold, relativistic, uniformly magnetized plasma are,

\[
\frac{d(\tilde{v})}{dt} = \frac{e}{m} \left( \frac{1}{c} \frac{\partial \tilde{A}}{\partial t} + \tilde{\nabla} \Phi - \frac{\tilde{v}}{c} \times (\tilde{\nabla} \times \tilde{A} + \sigma \tilde{\nabla} \tilde{v}) \right),
\]

\hspace{1cm} (3.2)

\[
\frac{\dot{n}_e}{\partial t} + \tilde{\nabla}.(n_e \tilde{v}) = 0
\]

\hspace{1cm} (3.3)

and

\[
\nabla^2 \Phi = 4\pi e(n_e - n_0)
\]

\hspace{1cm} (3.4)

where \( \gamma = (1 - v^2/c^2)^{-1/2} \) is the relativistic factor and \( \tilde{v} \& n_e \) are the plasma electron velocity and density respectively. The plasma electron velocities may be
considered to be a superposition of the slow and fast components \((\vec{v} = \vec{v}_s + \vec{v}_f)\),

oscillating at the plasma \((\omega_p = (4\pi n_0 e^2/m)^{1/2})\) and laser \((\omega_l)\) frequencies respectively. The components of the 1-D Lorentz force equation (3.2), comprising of slow and fast velocities, are therefore written as,

\[
\frac{\partial (\gamma(v_{zs} + v_{sf}))}{\partial t} + (v_{zs} + v_{sf}) \frac{\partial (\gamma(v_{zs} + v_{sf}))}{\partial z} = \frac{e}{mc} \frac{\partial A_x}{\partial t} + \frac{e}{mc} (v_{zs} + v_{sf}) \frac{\partial A_y}{\partial z} - \sigma \omega_c (v_{ys} + v_{sf}), \tag{3.5a}
\]

\[
\frac{\partial (\gamma(v_{ys} + v_{sf}))}{\partial t} + (v_{zs} + v_{sf}) \frac{\partial (\gamma(v_{ys} + v_{sf}))}{\partial z} = \frac{e}{mc} \frac{\partial A_y}{\partial t} + \frac{e}{mc} (v_{zs} + v_{sf}) \frac{\partial A_y}{\partial z} + \sigma \omega_c (v_{ys} + v_{sf}) \tag{3.5b}
\]

and

\[
\frac{\partial (\gamma(v_{zs} + v_{sf}))}{\partial t} + (v_{zs} + v_{sf}) \frac{\partial (\gamma(v_{zs} + v_{sf}))}{\partial z} = \frac{e}{m} \frac{\partial \Phi}{\partial z} - \frac{e}{mc} (v_{zs} + v_{sf}) \frac{\partial A_x}{\partial z} + (v_{ys} + v_{sf}) \frac{\partial A_y}{\partial z}. \tag{3.5c}
\]

While writing the left side of Eqs. (3.5), the convective derivative

\[
\left( \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{dt} + (\vec{v} \cdot \vec{V}) \vec{v} \right) \text{ has been used.}
\]
3.1(a) Fast plasma electron velocities

The equations governing the evolution of the fast plasma electron velocities are obtained by separating the fast terms from Eq. (3.5) as,

\[
\frac{dv_{sf}}{dt} = \frac{e}{\gamma mc} \frac{DA}{dt} - \frac{\sigma_0 \omega v_{sf}}{\gamma}, \tag{3.6a}
\]

\[
\frac{dv_{sf}}{dt} = \frac{e}{\gamma mc} \frac{DA}{dt} + \frac{\omega \nu_{sf}}{\gamma} \tag{3.6b}
\]

and

\[
\frac{dv_{sf}}{dt} = 0 \tag{3.6c}
\]

where \( \omega_c (= eb/mc) \) is the cyclotron frequency. Assuming the plasma to be cold, simultaneous solution of Eqs. (3.6a) and (3.6b) give the normalized (by \( c \)), fast transverse velocities as

\[
u_{sf} = \frac{a \omega_0}{(c \omega + \nu_{sf})} \sin (k_0 z - \omega_0 t) \tag{3.7a}
\]

and
\[ u_{sf} = -\frac{a\omega_0}{(\sigma\omega_e + \gamma\omega_b)} \cos(k_0 z - \omega_0 t) \]  

(3.7b)

where \( a(= \frac{eA}{mc^2}) \) is the normalized amplitude of the vector potential. Eqs. (3.7) give the relativistic quiver velocity of a plasma electron in the presence of a circularly polarized laser pulse and an axial magnetic field. In the absence of the magnetic field, Eqs. (3.7a) & (3.7b) reduce to the standard quiver velocity of the plasma electron in unmagnetized plasma. It may be noted that the direction of the magnetic field causes an increase (\( \sigma = -1 \)) or decrease (\( \sigma = +1 \)) in the quiver amplitude.

The transverse quiver velocities can be used to obtain the dispersion relation for a laser beam propagating in magnetized plasma. Substituting Eqs. (3.7) into the standard relativistic wave equation (Eq. 1.2, Chapter 1), gives the dispersion relation as

\[ c^2 k_0^2 = \omega_0^2 - \frac{\omega_p^2 \omega_0}{(\gamma\omega_b + \sigma\omega_e)}. \]  

(3.8)

In the absence of the external magnetic field, Eq. (3.8) reduces to the usual dispersion relation in unmagnetized plasma. The group velocity \( v_g (= \frac{d\omega_0}{dk}) \) of the laser pulse propagating in magnetized plasma is thus given by
\[ v_s = \frac{2c(\omega_h^2 - \omega_s \omega_p^2)/(\gamma \omega_h + \sigma \omega_s))^{1/2}}{(2\omega_h - \sigma \omega_s \omega_p^2/(\gamma \omega_h + \sigma \omega_s)^2)}. \]

### 3.1(b) Slow plasma electron velocities

The equations governing the evolution of the slow plasma electron velocities are obtained from Eqs. (3.5), as

\[ \frac{\partial(\mathbf{v}_{\perp})}{\partial t} + v_{zs} \frac{\partial(\mathbf{v}_{\perp})}{\partial z} + v_{zf} \frac{\partial(\mathbf{v}_{\perp})}{\partial z} = -\sigma \omega_s v_{ss}, \quad (3.10a) \]

\[ \frac{\partial(\mathbf{v}_{\parallel})}{\partial t} + v_{zs} \frac{\partial(\mathbf{v}_{\parallel})}{\partial z} + v_{zf} \frac{\partial(\mathbf{v}_{\parallel})}{\partial z} = \sigma \omega_s v_{ss}, \quad (3.10b) \]

and

\[ \frac{\partial(\mathbf{v}_{\perp})}{\partial t} + v_{zs} \frac{\partial(\mathbf{v}_{\perp})}{\partial z} = \frac{e}{m} \frac{\partial \Phi}{\partial z} - \frac{e}{mc} v_{zf} \frac{\partial A_x}{\partial z} - \frac{e}{mc} v_{zf} \frac{\partial A_y}{\partial z}. \quad (3.10c) \]

It is seen from Eq. (3.10c) that the slow longitudinal velocity is driven by the laser field. However, the transverse velocities (Eqs. (3.10a) and (3.10b)) are independent of the laser field due to the fact that the fast longitudinal velocity is zero. Thus, the radiation-dependent term \(((v_{zf} \times \vec{B})_{x,y})\) does not appear on the right hand side of
Eqs. (3.10a) and (3.10b). Therefore the slow transverse velocities oscillate at the
cyclotron frequency \( \omega_c \) and not the plasma frequency.

In order to study the generation of longitudinal electrostatic wakefields, the
slow components of the Lorentz force equation (3.10) are first transformed to a
frame moving with the group velocity \( (v_g) \) of the laser pulse. These equations are
written in terms of independent variables \( \xi = z - v_g t \) and \( \tau = t \). Hence, \( \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi} \)
and \( \frac{\partial}{\partial t} = -v_g \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau} \). Further, quasistatic approximation \( \frac{\partial}{\partial \tau} = 0 \) is applied to
the set of transformed fluid equations to study the slow evolution of wakefields.

The components of the transformed, normalized Lorentz force equation governing
the generation of slow plasma electron velocities, under QSA, are given by,

\[
\frac{\partial (u_{sx})}{\partial (k_p \xi)} = -\sigma \frac{\alpha_p}{\alpha_p \cdot \gamma(u_{sx} - \beta_y)} u_{sx} - u_{sx} \frac{\partial \gamma}{\gamma \cdot \partial (k_p \xi)}, \quad (3.11a)
\]

\[
\frac{\partial (u_{sy})}{\partial (k_p \xi)} = \sigma \frac{\alpha_p}{\alpha_p \cdot \gamma(u_{sy} - \beta_y)} u_{sy} - u_{sy} \frac{\partial \gamma}{\gamma \cdot \partial (k_p \xi)}, \quad (3.11b)
\]

and

\[
\frac{\partial (u_{sz})}{\partial (k_p \xi)} = \frac{1}{\gamma(u_{sz} - \beta_z)} \frac{\partial \phi}{\partial (k_p \xi)} - \frac{\alpha_0}{2(\gamma \omega_0 + \sigma \omega_0)(u_{sz} - \beta_z)} \frac{\partial u_0^2}{\partial (k_p \xi)} - \frac{u_{sz}}{\gamma \cdot \partial (k_p \xi)} \quad (3.11c)
\]
where $\phi = e\Phi/mc^2$ and $\beta_g (= v_g/c)$ are respectively the normalized scalar potential of the generated wakefield and group velocity of the laser pulse. It may be noted that while deriving Eqs. (3.11), the fast velocities (Eqs. 3.7) have been used and the second harmonics of the laser frequency have been neglected. Further, transforming the continuity and Poisson’s equations (3.3 and 3.4) and applying QSA, gives

$$\frac{\partial n}{\partial (k_{\rho} \xi)} = -\frac{n}{(u_{\infty} - \beta_g)} \frac{\partial u_{\infty}}{\partial (k_{\rho} \xi)}$$

(3.12)

and

$$\frac{\partial^2 \phi}{\partial (k_{\rho} \xi)^2} = n - 1.$$  

(3.13)

where $n (= n_e/n_o)$ is the normalized plasma electron density.

3.2 Numerical analysis and simulation study

The wake potential and hence the longitudinal electrostatic wakefields ($E_{nz} = -\partial \phi/\partial (k_{\rho} \xi)$) are obtained by simultaneously solving Eqs.(3.11)-(3.13), using the fourth order Runge-Kutta algorithm, for appropriate laser and plasma
parameters. It may be noted that these equations are valid for arbitrary pulse profiles and pump strengths.

The laser pulse profile is considered to be Gaussian
\[ a = a_0 \exp\left(-\left(\xi - \xi_0\right)^2 / L^2\right), \]
where \( a_0, \xi_0 \) and \( L \) are respectively the laser strength parameter, arbitrary pulse centre and laser pulse length. The amplitude of the axial wakefield generated behind the trailing edge of the Gaussian laser pulse will tend to be maximum when the pulse length is approximately equal to \( \frac{1}{\pi \sqrt{2}} \) times the plasma wavelength. Therefore, for a plasma wavelength \( \lambda_p = 32\mu m \) \((n_0 = 1.09 \times 10^{18} / cm^3)\), the pulse length is taken to be \( L = 7.2\mu m \) (pulse duration of 24 fs), for attaining maximum wakefield amplitude. The laser frequency is considered to be \( \omega_l = 2.355 \times 10^{15} Hz \) (corresponding to laser wavelength of 800 nm) and \( \omega_l / \omega_p = 6 \) \((B = 2000T)\). Two regimes of laser intensity have been studied viz. the highly \((a_0 = 1.0 \) corresponding to intensity \( I = 2.14 \times 10^{18} W/cm^2 \)\) and mildly relativistic regimes \((a_0 = 0.3 \) corresponding to intensity \( I = 1.926 \times 10^{17} W/cm^2 \)\). Since \( \omega_l \) has been considered to be greater than \( \omega_p \), the transverse electron velocities and hence the transverse wakefields, oscillating at the cyclotron frequency, do not arise in the laser pulse frame and thus, only longitudinal electrostatic wakefields are generated.
In Figs. 3.1 and 3.2, curves $a$, $b$ and $c$ show the variation of normalized wake potential with respect to $k_p \xi_k$ for $\sigma = -1, 0$ and $+1$ for $a_0 = 1.0$ and $0.3$ respectively. Comparing curves $a$, $b$ and $c$ in Fig. 3.1 (3.2), it is seen that there is an enhancement of $24.5\%$ ($18.3\%$) in the wake potential obtained in presence of a reversed magnetic field as compared to the unmagnetized case. However, a decrease of $16.7\%$ ($13.4\%$) is observed with a forward magnetic field. Further, the comparison of the wake potential curves given in Figs. 3.1 and 3.2, show that the potentials of the generated wakes in the relativistic regime ($a_0 = 1.0$) are enhanced tenfold with respect to the mildly relativistic case ($a_0 = 0.3$). Similarly, curves $a$, $b$ and $c$ in Fig. 3.3 (Fig. 3.4) depict the variation of the normalized, longitudinal, electrostatic wakefield ($E_{mz}$) with $k_p \xi_k$ for $\sigma = -1, 0$ and $+1$ respectively, for $a_0 = 1.0$ ($0.3$); while curve $d$ depicts the profile of the Gaussian pulse centred at $k_p \xi_k = 16.0 (14.0)$. An increase of $18.2\%$ ($17.6\%$) in the amplitude of the generated wakefield for $\sigma = -1$ and a decrease of $13.4\%$ ($13.1\%$) for $\sigma = +1$ in comparison to the unmagnetized case, is seen. This shows that a reversed (forward) magnetic field, increases (decreases) the amplitude of the generated axial wake potential and hence the wakefields. From Figs. 3.3 and 3.4, it may be noted that the sinusoidal wakefields are obtained in the mildly relativistic regime, whereas steepening of the wakefields is observed in the relativistic regime.
Fig. 3.1 Variation of normalized longitudinal electrostatic potential ($\phi$) with $k_p\xi$

for $\sigma = -1$ (curve a), $\sigma = 0$ (curve b) and $\sigma = +1$ (curve c), $a_0 = 1.0$

(corresponding to intensity $I = 2.14 \times 10^{18} \text{ W/cm}^2$), $\lambda_0 = 800 \text{ nm}$,

$\lambda = 32 \mu \text{m}$, $n_0 = 1.09 \times 10^{18} / \text{cm}^3$, $L = 7.2 \mu \text{m}$ and $\omega / \omega_p = 6$. 

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Fig. 3.2 Variation of normalized longitudinal electrostatic potential ($\phi$) with $k_p \xi$

for $\sigma = -1$ (curve a), $\sigma = 0$ (curve b) and $\sigma = 1$ (curve c), $a_0 = 0.3$

(corresponding to intensity $I = 1.926 \times 10^{17} W/cm^2$), $\lambda_0 = 800 nm$,

$\lambda_p = 32 \mu m$, $n_0 = 1.09 \times 10^{16} / cm^2$, $L = 7.2 \mu m$ and $\omega / \omega_p = 6$. 

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Fig. 3.3  Variation of normalized longitudinal electrostatic wakefield ($E_{\text{w}}$) with $k_p \xi$ for $\sigma = -1$ (curve a), $\sigma = 0$ (curve b) and $\sigma = +1$ (curve c). $a_0 = 1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} \text{W/cm}^2$), $\lambda_0 = 800 \text{nm}$, $\lambda_p = 32 \mu\text{m}$, $n_0 = 1.09 \times 10^{18} / \text{cm}^3$, $L = 7.2 \mu\text{m}$ and $\omega_c / \omega_p = 6$. Curve d depicts the profile of the Gaussian pulse centered at $k_p \xi_0 = 16.0$. 
Fig. 3.4 Variation of normalized longitudinal electrostatic wakefield \( E_{\text{nl}} \) with \( k_p \xi \) for \( \sigma = -1 \) (curve a), \( \sigma = 0 \) (curve b) and \( \sigma = +1 \) (curve c), \( a_0 = 0.3 \) (corresponding to intensity \( I = 1.926 \times 10^{17} \text{ W/cm}^2 \)), \( \lambda_0 = 800 \text{ nm} \), \( \lambda_p = 32 \mu\text{m} \), \( n_0 = 1.09 \times 10^{18} / \text{cm}^3 \), \( L = 7.2 \mu\text{m} \) and \( \omega_p / \omega = 6 \). Curve d depicts the profile of the Gaussian pulse centered at \( k_p \xi_0 = 14.0 \).
In order to validate the results of the numerical study, simulations of the present configuration have been performed, using 2-D PIC code, XOOPIC [129]. A circularly polarized laser pulse, having Gaussian spatial and radial profile was launched in a homogeneous magnetized plasma. The full width at half maximum (FWHM) pulse length \( L_{\text{FWHM}} = \sqrt{2\ln 2} L \) was considered to be 8.47 \( \mu \text{m} \) (FWHM pulse duration of 28.25 fs), corresponding to \( L = 7.2 \mu \text{m} \). The size of the simulation domain was 80 \( \mu \text{m} \) (this included 20 \( \mu \text{m} \) vacuum distance) in the laser propagation direction \((x)\) and 400 \( \mu \text{m} \) in the transverse direction \((y)\). The domain has been divided into 2048×512 meshes. The time step (satisfying the Courant condition according to which the time step chosen must be less than the time it takes for light to cross a cell-size) was taken to be 0.07 fs. The transverse laser spot size \((r_0)\) was assumed to be 80 \( \mu \text{m} \), in order to fulfil the broad beam \((k_pr_0 \gg 1)\) condition so that the 1-D numerical results presented earlier are compatible with the simulation study. The ions form an immobile neutralizing background fluid. All other laser and plasma parameters are the same as those used for the numerical study.

In Fig. 3.5 (3.6), curves \( a, b \) and \( c \) are the simulation results showing the variation of normalized longitudinal electrostatic wakefields with propagation distance \((x)\) for reverse, zero and forward magnetic fields, respectively in the highly (mildly) relativistic regime. The simulation results show the same trend in the wakefield evolution curves as predicted via numerical study. However, the
Fig. 3.5 2-D XOOPIC simulation plot for the variation of normalized longitudinal electrostatic wakefield ($E_{nx}$) with propagation distance for

$\sigma = -1$ (curve a), $\sigma = 0$ (curve b), $\sigma = +1$ (curve c), $a_0 = 1.0$

($I = 2.14 \times 10^{18} \text{ W/cm}^2$), $\lambda_0 = 800 \text{ nm}$, $L_{\text{FWHM}} = 8.47 \mu \text{m}$, $r_0 = 80 \mu \text{m}$,

$\lambda_p = 32 \mu \text{m}$, $n_0 = 1.09 \times 10^{18} \text{ /cm}^3$ and $\omega_c / \omega_p = 6$. 

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Fig. 3.6 2-D XOOPIC simulation plot for the variation of normalized longitudinal electrostatic wakefield ($E_{nx}$) with propagation distance for

$\sigma = -1$ (curve $a$), $\sigma = 0$ (curve $b$), $\sigma = +1$ (curve $c$), $a_0 = 0.3$

($I = 1.926 \times 10^{17} \text{W/cm}^2$), $\lambda_0 = 800 \text{nm}$, $L_{\text{FWHM}} = 8.47 \mu\text{m}$, $r_0 = 80\mu\text{m}$,

$\lambda_p = 32\mu\text{m}$, $n_0 = 1.09 \times 10^{18} / \text{cm}^3$ and $\omega_p / \omega = 6.$
normalized peak values of the generated wakefield \((E_{\text{nx}})\) via simulation are seen to be suppressed as compared to the maximum wakefield amplitude obtained numerically. This may be attributed to 2-D nonlinear effects. Another important 2-D effect that may be noted is that the plasma wavelength (plasma density) increases (decreases) in the relativistic regime (Fig.3.5) as compared to the mildly relativistic case (Fig. 3.6). This is due to the enhanced ponderomotive force that dominates in the relativistic regime.

3.3 Trapping and acceleration of a test electron

The acceleration mechanism of a laser wakefield can be studied by injecting a test electron behind the laser pulse. The exchange of energy between the generated axial wakefield and the test electron can be obtained with the help of Hamiltonian dynamics [124]. Following the same procedure as mentioned in Section 1.4 of Chapter 1, the relativistic factor associated with the peak energy \((\gamma_e)\) of the test electron can be obtained from Eq. (1.41) as

\[
\gamma_e = \gamma'_p (1 + \gamma'_p \Delta \phi) \pm \gamma'_p \beta'_p [(1 + \gamma'_p \Delta \phi)^2 - 1]^{1/2}
\]

(3.14)

where \(\Delta \phi = \phi - \phi_{\text{min}}\) and + & - signs respectively give the maximum and minimum values of \(\gamma_e\). In Eq. (3.14), \(\beta'_p = v'_p / c\), where \(v'_p\) is the phase velocity
of the plasma wave) is taken to be equal to \( \beta_g (= v_g / c) \), since the generated plasma wave follows the laser pulse (propagating with the velocity \( v_g \)). Also, 
\[
\gamma_p' = \left( 1 - \frac{v_p'^2}{c^2} \right)^{-1/2}
\]
is considered to be equal to the relativistic factor corresponding to the injection energy of the test electron (\( \gamma_i \)).

The separatrices of the test electron in the phase space bucket of the generated axial wakefield, for unmagnetized (\( \sigma = 0 \)) and magnetized (\( \sigma = \pm 1 \), \( \omega_e / \omega_p = 6 \)) plasma, are plotted by simultaneously solving Eqs.(3.11)-(3.13) alongwith Eq. (3.14) using the fourth order Runge-Kutta technique. The laser and plasma parameters required for plotting the separatrix curves, as shown in Fig. 3.7, for unmagnetized and magnetized plasma, are the same as those used for plotting Figs. 3.1-3.4. In Fig.3.7 (for \( a_0 =1.0 \)), the point of injection (\( \phi_{\text{min}}, \psi_{\text{min}} \)) of the test electron for tracing the separatrix for unmagnetized plasma (curve \( b \)) and magnetized plasma \{ \( \sigma = -1 \) (curve \( a \)) & \( \sigma = +1 \) (curve \( c \)) \} are, respectively (-1.07560, 10.631), (-1.28996, 10.376) and (-0.92472, 10.801). The injection energy necessary for an electron to trace the separatrices \( a \), \( b \) and \( c \) in the phase space are respectively 22.5 MeV, 23.78 MeV, 25.69 MeV, while the maximum energies attained after acceleration are 4.5 GeV, 4.13 GeV and 4.09 GeV respectively. Therefore, as compared to the unmagnetized case, a decrease (increase) of 5.68% (8.03%) in the injection energy is observed for reversed (forward) magnetic field, whereas, an enhancement (reduction) of 8.9% (0.96%) in
Fig. 3.7  Separatrix plots for a test electron for $\sigma = -1$ (curve $a$), $\sigma = 0$ (curve $b$) and $\sigma = +1$ (curve $c$), with injection energies 22.5 MeV, 23.78 MeV and 25.69 MeV respectively, for $a_0 = 1.0$. 
the peak energy of the accelerated test electron is seen. It may be concluded that for intense laser pulses, application of a reversed magnetic field leads to two advantages, an increment in the peak energy of an externally injected test electron alongwith a lower injection energy requirement. On the contrary, a magnetic field applied along the forward direction suppresses the wakefield and hence the peak energy of the test electron.

Similarly, Fig. 3.8 shows the separatrices plots in the mildly relativistic regime \((a_0 = 0.3)\) for magnetized plasma with \(\sigma = +1\) (curve \(c\)), \(\sigma = -1\) (curve \(a\)) and unmagnetized plasma (curve \(b\)). For tracing the separatrices \(a\), \(b\) and \(c\), the test electron is injected respectively at \((-0.07613, 9.454)\), \((-0.06504, 9.454)\) and \((-0.05676, 9.454)\) with the injection energies 17.32 MeV, 20.43 MeV & 23.47 MeV respectively. It is observed that the maximum final energies of the accelerated test electron attained while tracing the separatrices \(a\), \(b\) and \(c\) are respectively 209.1 MeV, 245.8 MeV & 282.5 MeV. Hence, a decrease (increase) of 17.5% (14.9%), is seen in the maximum final energy of the accelerated test electron for magnetized plasma with \(\sigma = -1(\sigma = +1)\) as compared to unmagnetized case. However, the injection energy for \(\sigma = +1\) remains higher than that required for \(\sigma = -1\).

It can be seen from Eq. (3.14), that the relativistic factor associated with the maximum final energy \(\gamma_e\) of the accelerated test electron depends upon the injection energy \(\gamma'_{\text{inj}}\) of the test electron as well as on the magnitude of the generated wake potentials. Therefore, for low magnitude of wake potential (as in
Fig. 3.8  Separatrix plots for a test electron with $\sigma = -1$ (curve $a$), $\sigma = 0$ (curve $b$) and $\sigma = +1$ (curve $c$), with injection energies 17.32 MeV, 20.43 MeV & 23.47 MeV respectively, for $a_0 = 0.3$. 
the case of mildly relativistic regime), $\gamma_e$ is predominantly determined by the relativistic factor of injection energy ($\gamma'_p$). Now, since the group velocity of the laser pulse (and hence $v'_p$ & $\gamma'_p$) in magnetized plasma is greater for $\sigma = +1$ as compared to $\sigma = -1$ (Eq. 3.9), higher energy of the accelerated test electron is obtained for forward as compared to the reversed magnetization case when $a_0 << 1$. On the contrary, for $a_0 \sim 1$, the final test electron energies are governed mainly by the high wake potential which is about ten-fold greater than that in the mildly relativistic case. This leads to enhanced electron energy for the reversed magnetic field as compared to the forward field case.

In order to evaluate the effective gain in electron energy for each of the six cases under consideration, the gain is defined as $g = \frac{\gamma_e - \gamma'_p}{\gamma'_p}$. For $a_0 = 1.0$, the respective values of gain for $\sigma = -1, 0, +1$ are 199.0, 172.6 and 158.2. However, for $a_0 = 0.3$, the gain is given by 11.06, 11.03 and 11.03 for $\sigma = -1, 0, +1$ respectively. Thus, it is clear that effective enhancement of gain can be achieved for the highly relativistic regime with a reversed magnetic field. However, for the mildly relativistic regime, the presence of the magnetic field does not significantly change the net gain in electron energy. This study will be significant for the analysis of trapping and acceleration of a test electron in magnetized plasma, in the relativistic regime.
3.4 1-D numerical model and simulation of wakefields generated by super-Gaussian laser pulses

Consider a linearly polarized laser pulse, represented by the vector potential

$$\vec{A} = \hat{x} A_0(z,t) \cos(k_0 z - \omega_0 t) \quad (3.15)$$

propagating through pre-ionized plasma, having ambient density $n_0$ along the positive $z$-direction. Wakefield generation due to the interaction of the laser pulse with cold and homogeneous plasma is governed by basic nonlinear fluid equations (3.2 - 3.4). Following the same procedure as mentioned in Sec. (3.1), the transverse and longitudinal components of the Lorentz force equation (3.2) in the absence of external magnetic field, comprising of the slow and the fast plasma electron velocities, are respectively given by

$$\frac{\partial(\gamma(v_{ss} + v_{sf}))}{\partial t} + (v_{ss} + v_{sf}) \frac{\partial(\gamma(v_{ss} + v_{sf}))}{\partial z} = \frac{e}{mc} \frac{\partial A_z}{\partial t} + \frac{e}{mc} (v_{ss} + v_{sf}) \frac{\partial A_z}{\partial z}$$

(3.16a)

and

$$\frac{\partial(\gamma(v_{ss} + v_{sf}))}{\partial t} + (v_{ss} + v_{sf}) \frac{\partial(\gamma(v_{ss} + v_{sf}))}{\partial z} = \frac{e}{m} \frac{\partial \Phi}{\partial z} - \frac{e}{mc} ((v_{ss} + v_{sf}) \frac{\partial A_x}{\partial z}) .$$

(3.16b)
The $y$-component of the velocity does not arise since the laser pulse is linearly polarized along the $x$-direction.

Further, separating the equations governing the evolution of the fast plasma electron velocities the normalized (by $c$), transverse fast plasma electron velocity is obtained as,

$$u_{sf} = \frac{a}{\gamma} \sin(k_0 z - \omega_0 t). \quad (3.17)$$

The fast velocity along the longitudinal direction occurs at the second harmonic of the laser frequency which is neglected in the current study, therefore, $u_{sf} = 0$.

The slow plasma electron velocity driving the longitudinal plasma waves is evaluated from Eq. (3.16b), by separating the slow equation as,

$$\frac{\partial (\gamma v_x)}{\partial t} + v_x \frac{\partial (\gamma v_x)}{\partial z} = \frac{e}{m} \frac{\partial \Phi}{\partial z} - \frac{e}{mc} v_{sf} \left( \frac{\partial A_z}{\partial z} \right). \quad (3.18)$$

However, due to the absence of the driving force, the slow transverse velocity ($v_{x\alpha}$) will not exist.

Transforming Eq. (3.18) in terms of independent variables $\tau(=t)$ and $\xi(=z - v_s t)$ and applying QSA, the normalized, slow, longitudinal component of the Lorentz force equation (3.18) becomes
\[
\frac{\partial u_{\gamma\xi}}{\partial (k_p \xi)} = \frac{1}{\gamma(u_{\gamma\xi} - \beta_\xi)} \left\{ \frac{\partial \phi}{\partial (k_p \xi)} - u_{\gamma\xi}\frac{\partial a_0}{\partial (k_p \xi)} \right\} - u_{\gamma\xi} \frac{\partial \gamma}{\partial (k_p \xi)}.
\]

(3.19)

The transformed Poisson’s and continuity equations (3.12 & 3.13) are solved simultaneously with Eq. (3.19) using the fourth order Runge-Kutta numerical technique to give the wake potential and hence the electrostatic longitudinal wakefields \((E_{nc} = -\vec{V}\phi)\) generated behind the laser pulse.

A super-Gaussian pulse has the form \(a = a_0 \exp(-\frac{(\xi - \xi_0)^2}{L^{2m}})\), where \(\xi_0\) defines the arbitrary centre of the laser pulse; \(m = 2, 3, 4\ldots\) represents a super-Gaussian profile of various orders while \(m=1\) represents a Gaussian pulse. In order to perform the numerical study, consider \(m=2\) super-Gaussian pulse having pulse length \(L=12.0 \mu m\). All other laser-plasma parameters being the same as given in Section 3.2(a). This study has been performed in the relativistic regime of the laser pulse intensity \(I = 2.14 \times 10^{18} W/cm^2\) (corresponding to \(a_0 = 1.0\)).

In Fig. 3.9, solid and dotted curves show respectively the variation of normalized plasma electron density with respect to \(k_p \xi\), for super-Gaussian and Gaussian laser pulses. It is seen that there is an enhancement of 21.9% in the perturbed plasma electron density due to a super-Gaussian pulse as compared to the density curve obtained for Gaussian laser pulse. Similarly, Fig. 3.10 depicts the variation of normalized, longitudinal, electrostatic wakefield \((E_{nc})\) with \(k_p \xi\).
Fig. 3.9  Variation of normalized plasma electron density ($n$) for super-Gaussian (solid curve) and Gaussian (dotted curve) laser pulses with respect to $k_p\xi$ for $a_0 = 1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} \text{W/cm}^2$),

$\lambda_0 = 800 \text{nm}$, $\lambda_p = 32\mu\text{m}$, $n_0 = 1.09 \times 10^{18} \text{ / cm}^3$ and $L = 12.0\mu\text{m}$. 


Fig. 3.10 Variation of normalized longitudinal electrostatic wakefield ($E_{nw}$) for super-Gaussian (curve $c$) and Gaussian (curve $d$) laser pulses with respect to $k_p \xi$ for $a_0 = 1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} W/cm^2$), $\lambda_0 = 800 nm$, $\lambda_p = 32 \mu m$, $n_0 = 1.09 \times 10^{18} / cm^3$ and $L = 12.0 \mu m$. Curve $a$($b$) shows the evolution of the super-Gaussian (Gaussian) laser pulse.
Curve \( a (b) \) depicts the profile of the super-Gaussian (Gaussian) pulse centered at \( k_p \xi_0 = 15.0 \) while curve \( c (d) \) represents the longitudinal wakefield generated behind the super-Gaussian (Gaussian) pulse. Comparing curves \( c \) and \( d \), an increase of 20.4\% is seen in the amplitude of the generated wakefield via super-Gaussian laser pulse as compared to the Gaussian pulse case. The increase occurs because the gradient for super-Gaussian pulse is more in comparison with the Gaussian pulse case.

In order to validate the results obtained through 1-D numerical study, 2-D PIC simulations have been conducted, using XOOPIC code. The full width at half maximum (FWHM) pulse length \( L_{FWHM} = \sqrt{2 \ln 2L} \) was considered to be 14.2 \( \mu m \), corresponding to \( L = 12.0 \mu m \). A broad laser beam \( (k_p r_0 >> 1) \) has been assumed in order to reduce the transverse effects so that the results obtained are compatible with the numerical study. Therefore, the transverse laser spot size \( (r_0) \) was taken to be 80 \( \mu m \). The physical dimensions of the simulation box as well as the other laser-plasma parameters, required to perform the simulation study, are the same as that considered in Section 3.2 (b).

The three-dimensional plots obtained via PIC simulations, showing the variation of generated electric field amplitude with respect to the longitudinal and transverse distances for super-Gaussian and Gaussian laser pulses are shown in Figs. 3.11 and 3.12 respectively. It may be noted that the wake amplitude is maximum on-axis and decreases with increase in transverse distance. Comparing
Fig. 3.11 3-D PIC simulation plots showing the variation of normalized longitudinal electrostatic wakefield ($E_{nx}$) for super-Gaussian laser pulse with propagation distance for $a_0 = 1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} \text{ W/cm}^2$), $\lambda_0 = 800 \text{ nm}$, $\lambda_p = 32 \mu\text{m}$, $n_0 = 1.09 \times 10^{18} \text{ /cm}^3$ and $L = 12.0 \mu\text{m}$.
Fig. 3.12 3-D PIC simulation plots showing the variation of normalized longitudinal electrostatic wakefield ($E_{nw}$) for Gaussian laser pulse with propagation distance for $a_0=1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} \text{ W/cm}^2$), $\lambda_0 = 800 \text{ nm}$, $\lambda_p = 32 \mu \text{m}$, $n_0 = 1.09 \times 10^{18} / \text{cm}^3$ and $L = 12.0 \mu \text{m}$. 
Figs. 3.11 and 3.12, it is seen that the peak amplitude of the wakes obtained with the super-Gaussian laser pulse is more than the wakefield amplitude generated by a Gaussian pulse. Further, the 2-D curves depicting the variation of the normalized longitudinal electrostatic wakes with normalized distance \( k_p x \), are plotted in Fig. 3.13 for \( y = 0 \) (i.e. on-axis). The curves \( a \) and \( b \) depict the wakefield amplitudes generated via super-Gaussian and Gaussian laser pulses, respectively. An increase of 23.3% in the normalized amplitude of the wakes generated by super-Gaussian pulse as compared to the Gaussian pulse case has been observed. Comparing Figs. 3.10 and 3.13, it is seen that the results obtained via numerical studies and simulations are found to be in good agreement with each other. However, the wakefield amplitudes obtained via simulation are found to be slightly suppressed in comparison with the 1-D numerical results. This may be attributed to 2-D nonlinear effects.

### 3.5 Acceleration of an externally injected test electron

With the help of Hamiltonian dynamics (Section 1.4, Chapter 1), the energy exchanged between the generated axial wakefield and the test electron can be obtained. The peak energy attained by a test electron injected into the wake potential generated by a super-Gaussian pulse is obtained by following the same procedure as mentioned in Section 3.3.
Fig. 3.13  2-D XOOPIC simulation plots showing the variation of normalized longitudinal electrostatic wakefield ($E_{nx}$) for super-Gaussian (curve $a$) and Gaussian (curve $b$) laser pulses with respect to normalized distance for $a_0 = 1.0$ (corresponding to intensity $I = 2.14 \times 10^{18} W/cm^2$),

$\lambda_0 = 800 \text{ nm}$, $\lambda_p = 32 \mu m$, $n_0 = 1.09 \times 10^{18} / cm^3$ and $L = 12.0 \mu m$. 
The separatrix (curve $a$) traced by the test electron, accelerated by the longitudinal wake generated by a super-Gaussian laser pulse is plotted in Fig. 3.14. Separatrix (curve $b$) is also plotted for a Gaussian pulse case for the sake of comparison. The injection positions $(\phi_{\text{min}}, k_p, \xi)$ of the test electron for tracing the separatrix curves $a$ and $b$ are respectively $(-0.48527, 12.97)$ and $(-0.43964, 12.93)$. The injection energies necessary for trapping the test electron excited by the wakes generated by super-Gaussian and Gaussian laser pulses is the same ($=20.47$ MeV), while the maximum energies attained after acceleration are 2.46 GeV, and 2.08 GeV for curves $a$ and $b$ respectively. Hence, an enhancement of 15.4% in the maximum energy of the accelerated test electron is seen for super-Gaussian pulse as compared to Gaussian pulse case.

The gain in energy of the test electron is calculated to be respectively 119.22 and 100.75 for super-Gaussian and Gaussian laser pulses. Therefore, an increase of 15.5% in the energy gain of the test electron being accelerated by wakes generated by super-Gaussian laser pulse is observed as compared to the gain of the accelerated test electron by wakes driven by Gaussian laser pulse.

Thus, it may be concluded that the super-Gaussian laser pulse is capable of enhancing the wake amplitude and hence the peak energy of the test electron driven by electrostatic longitudinal wakefield as compared to the Gaussian pulse case of same intensity.
Fig. 3.14 Separatrix plots for a test electron driven by wakes generated by super-Gaussian (curve \(a\)) and Gaussian (curve \(b\)) laser pulses for \(a_0 = 1\) (corresponding to intensity \(I = 2.14 \times 10^{18} \text{ W/cm}^2\)), \(\lambda_0 = 800 \text{ nm}\), \(\lambda_p = 32\mu\text{m}\), \(n_0 = 1.09 \times 10^{18} \text{ /cm}^3\) and \(L = 12.0\mu\text{m}\).
Chapter 4

Wakefield generation by two-colour laser pulses propagating in homogeneous plasma

This chapter deals with the study of the generation of longitudinal as well as transverse electric wakefields, via passage of two-colour, sinusoidal laser pulses in uniform plasma [137]. The frequency difference between the two laser pulses is considered to be equal to the plasma frequency. The two laser pulses are linearly polarized along arbitrary directions. The relative angle between the two directions of polarization is varied and the amplitudes of the generated wakefields are compared. Further, VORPAL code has been used to perform two-dimensional PIC simulations [138], in order to validate the results reported via analytical study.

4.1 Formulation

Two linearly polarized laser pulses of frequencies $\omega_1$ and $\omega_2$, are considered to be copropagating along the positive $z$-direction through cold, underdense and homogeneous plasma. One of the laser pulses, having frequency $\omega_1$, is polarized along the $x$-direction while the second pulse, of frequency $\omega_2$, is polarized along an arbitrary angle $\theta$ with respect to the direction of polarization of
the first laser pulse. The synthesized electric field of the two laser pulses is represented as

\[
\bar{E} = \hat{x}\{E_1(r, z, t) \cos(k_1 z - \omega_1 t) + E_2(r, z, t) \cos(k_2 z - \omega_2 t) \cos \theta\} + \\
\hat{y}\{-E_2(r, z, t) \cos(k_2 z - \omega_2 t) \sin \theta\} \tag{4.1}
\]

where \( E_1, k_1 \) and \( \omega_1 \) represent respectively, the amplitude, wavenumber and angular frequency of the first \((i = 1)\) and second \((i = 2)\) laser pulse.

The propagation of the two laser pulses through homogeneous plasma is governed respectively by the Lorentz force equation, continuity equation

\[
\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m}\left[\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right] + \vec{v} \times \nabla \times \vec{v} - \frac{1}{2} \nabla \left(\vec{v} \cdot \vec{v}\right), \tag{4.2a}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0 \tag{4.2b}
\]

and the time dependent Maxwell’s equations

\[
\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \tag{4.3a}
\]

&

\[
\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \tag{4.3b}
\]
Where \( J(= -n_e e \vec{v}) \) is the current density. In the mildly relativistic regime, a perturbative technique is used to expand various parameters in orders of the laser strength parameter \( a_i(= eE_i/mc \omega_i << 1) \). The first order (of the radiation field) quiver velocity components are obtained by substituting the radiation field (Eq. 4.1), into the components of the first order Lorentz force equation

\[
\frac{\partial v^{(1)}_{x,y}}{\partial t} = -\frac{e}{m} E^{(1)}_{x,y},
\]  

(4.4)

to give

\[
v^{(1)}_x = c a_1 \sin(k_x z - \omega_1 t) + c a_2 \sin(k_z z - \omega_2 t) \cos \theta
\]  

(4.5a)

and

\[
v^{(1)}_y = -c a_2 \sin(k_z z - \omega_2 t) \sin \theta.
\]  

(4.5b)

The superscript on the left hand side of Eqs. (4.5) represents the order of the radiation field. It may be noted that in the absence of the second laser pulse, Eq. (4.5a) reduces to the quiver velocity of the plasma electrons for a linearly polarized laser pulse while \( v^{(1)}_y = 0 \).

The second order perturbative expansion of Eq. (4.2a) gives,
where \( \vec{E}^{(2)} (= E_{x,y,z}^{(2)}) \) represents the components of the electric wakefields generated via interaction of the two laser pulses with homogeneous plasma. The second order terms \( \vec{v}^{(1)} \times \vec{B}^{(1)} \) and \( \vec{v}^{(1)} \times (\vec{\nabla} \times \vec{v}^{(1)}) \), representing vortex motion cancel each other and therefore do not appear in the second order Lorentz force equation (4.6). Substituting the first order velocity components (Eq. 4.5a and 4.5b) into Eq. (4.6) gives

\[
\frac{\partial \vec{v}^{(2)}}{\partial t} = -\frac{e}{m} \vec{E}^{(2)} - \frac{1}{2} [\vec{\nabla} \{ v_x^{(1)} v_y^{(1)} + v_y^{(1)} v_z^{(1)} \}]
\]

Further, in order to study the slow evolution of electric wakefields, the fast terms oscillating at \( (\omega_1 + \omega_2) \) frequency are neglected. However, the terms with oscillations at \( (\omega_1 - \omega_2) \) frequency are retained in the analysis that follows due to the assumption that the frequency difference between the two laser pulses is equal to the plasma frequency i.e. \( \omega_1 - \omega_2 = \omega_p \).

In the present study, the two laser pulses are assumed to have spot size \( r_{0_i} \) and pulse length \( L_i \). The longitudinal profile of the two pulses, considered to be
sinusoidal, is of the form $a_i = a_{ni} \sin(\pi \xi / L_i)$ (where $a_{ni}$ represents the transverse profile and $\xi$ is an arbitrary position within the pulse frame). The transverse Gaussian profile is represented by $a_i = a_{0i} \exp\{-(x^2 + y^2) / r_{0i}^2\}$ (where $(x^2 + y^2)^{1/2}(<r_{0i})$ represents an arbitrary transverse distance from the common centroid of the two laser beams). In order to study the generation of wakefields, the second order slow components of the Lorentz force equation (4.7) are written in terms of independent variables $\zeta(=z-ct)$ and $\tau(=t)$. Further, applying quasistatic approximation, the components of the transformed, second order Lorentz force equation, governing the generation of slow plasma electron velocities, are given by,

$$\frac{\partial v^{(2)}_{x,y}}{\partial \xi} = \frac{e}{mc} (E^{(2)}_{x,y}) - \frac{c(x,y)}{r_{01}} a_1^2 - \frac{c(x,y)}{r_{02}} a_2^2 - a_1 a_2 \left\{ \frac{c(x,y)}{r_{01}^2} + \frac{c(x,y)}{r_{02}^2} \right\} \cos(k_p \xi) \cos \theta$$

(4.8a)

and

$$\frac{\partial v^{(2)}_z}{\partial \xi} = \frac{e}{mc} E^{(2)}_z + \frac{c}{2} \left[ a_1 \frac{\partial a_1}{\partial \xi} + a_2 \frac{\partial a_2}{\partial \xi} + \frac{\partial a_1}{\partial \xi} a_1 + \frac{\partial a_2}{\partial \xi} a_2 \right] \cos(k_p \xi) \cos \theta$$

$$- a_1 a_2 k_p \sin(k_p \xi) \cos \theta$$

(4.8b)
where \( k_p \xi = (k_1 - k_2)z - (\omega_1 - \omega_2)t \) has been substituted since \( k_1 - k_2 = k_p \) and \( (\omega_1 - \omega_2) = \omega_p \) has been assumed.

The ponderomotive force \( F_{pc} = -\frac{mc^2}{2} \left\{ \frac{\partial}{\partial \xi} (a_1 + a_2)^2 \right\} \), driving the longitudinal wakefields, may be obtained from Eq. (4.8b) as

\[
F_{pc} = -mc^2 \left[ a_1 \frac{\partial a_1}{\partial \xi} + a_2 \frac{\partial a_2}{\partial \xi} + \left( \frac{\partial a_1}{\partial \xi} a_2 + \frac{\partial a_2}{\partial \xi} a_1 \right) \cos \theta \cos k_p \xi \right. \\
- \left. a_1 a_2 k_p \cos \theta \sin k_p \xi \right]
\]  

(4.9)

This force arises due to two contributions: the first is the sum of the ponderomotive forces due to the two individual laser pulses (the first and the second terms on the right hand side of Eq. (4.9)), while the second contribution arises due to the beating of the two pulses (the third and the fourth terms on RHS of Eq. (4.9)). For long laser pulses, only the fourth term contributes to the excitation of plasma waves.

In the present analysis, the two laser pulses of wavelengths 1.064 \( \mu m \) \( (\omega_1 = 1.77 \times 10^{15}\ Hz) \) and 1.1006 \( \mu m \) \( (\omega_2 = 1.71 \times 10^{15}\ Hz) \), are considered to be copropagating through uniform plasma of density \( n_0 = 1.09 \times 10^{18}/cm^3 \) \( (\omega_p = 5.89 \times 10^{13}\ Hz) \). The pulse lengths \( L \) of the two laser pulses, respectively are considered to be 30 \( \mu m \) and 31 \( \mu m \) and the plasma wavelength \( (\lambda_p) \) is 32 \( \mu m \) so that \( L \sim \lambda_p \), satisfies the condition for generation of maximum amplitude
wakefields for sinusoidal laser pulses [30]. The laser intensities of the two laser pulses are assumed to be the same i.e. $I_{0i} = 4.84 \times 10^{16} \text{W/cm}^2$ which corresponds to $a_{01} = 0.2$ and $a_{02} = 0.207$ respectively. Also, for the sake of convenience, the waist size of the two laser beams is considered to be the same ($r_{0i} = 40 \mu\text{m}$).

The evolution of the normalized (by $mc\omega_p$) longitudinal ponderomotive force ($F_{npz}$) generated by the two laser pulses polarized in the same ($\theta = 0^\circ$) and opposite directions ($\theta = 180^\circ$) respectively, has been presented in Figs. 4.1 and 4.2. The ponderomotive force has been plotted within one cycle of the sinusoidal laser pulses. The solid curves in both the figures indicate the net normalized ponderomotive force driving the longitudinal electric wakefields. The dotted curves depict the contributions due to the envelopes of the two individual laser pulses, while the dashed curves indicate the contributions due to the beating of the two laser pulses, to the net ponderomotive force. Comparing the dashed and dotted curves in Fig. 4.1 (4.2), it is seen that the ponderomotive force generated by beating of the two laser pulses is out of phase (in phase) with the ponderomotive force generated individually by the two laser pulses when they have the same (opposite) directions of polarization. Hence, the net ponderomotive force (solid curve) is seen to be suppressed (enhanced) if the two laser pulses have the same (opposite) polarization directions. This suggests that the wakefields generated by oppositely polarized lasers should be larger than those generated by pulses having the same polarization directions.
Fig. 4.1 Variation of normalized net ponderomotive force (solid curve), contribution due to the driving force by individual laser pulses (dotted curve) and beating of the two pulse envelopes (dashed curve) with \( k_p \xi \) for two laser pulses with the same directions of polarization for \( a_{01} = 0.2, \quad a_{02} = 0.207 \) (corresponding to intensities \( I_{01} = I_{02} = 4.84 \times 10^{16} \text{ W/cm}^2 \), \( \lambda_1 = 1.064 \mu m, \quad \lambda_2 = 1.1006 \mu m, \quad L_1 = 30 \mu m, \quad L_2 = 31 \mu m \) and \( \lambda_p = 32 \mu m \).
Fig. 4.2 Variation of normalized net ponderomotive force (solid curve), contribution due to the driving force by individual laser pulses (dotted curve) and beating of the two pulse envelopes (dashed curve) with $k_p \xi$ for two laser pulses with opposite directions of polarization for $a_{01} = 0.2$, $a_{02} = 0.207$ (corresponding to intensities $I_{01} = I_{02} = 4.84 \times 10^{16} \text{W/cm}^2$), $\lambda_1 = 1.064 \mu m$, $\lambda_2 = 1.1006 \mu m$, $L_1 = 30 \mu m$, $L_2 = 31 \mu m$ and $\lambda_p = 32 \mu m$. 
4.2 Generation of electric wakefields

In order to derive the electric wakefields, quasistatic approximation is applied to the time dependent Maxwell’s equations (4.3a and 4.3b) which transform them as,

\[
(\nabla_\perp + \frac{\partial}{\partial \xi} \hat{z}) \times \vec{E} = \frac{\partial \vec{B}}{\partial \xi}
\]

(4.10a)

and

\[
(\nabla_\perp + \frac{\partial}{\partial \xi} \hat{z}) \times \vec{B} = \frac{4\pi}{c} \vec{J} - \frac{\partial \vec{E}}{\partial \xi}
\]

(4.10b)

where \( \nabla_\perp \) represents the transverse gradients of the generated fields. Eqs. (4.10a and 4.10b) are the slow field equations which can be simultaneously solved with Eqs. (4.8a and 4.8b) to yield the transverse and longitudinal electric wakefields.

4.2(a) Longitudinal electric wakefields

Longitudinal wakefields may be obtained from Eq. (4.10) with the assumption that for broad laser beams \( r_{0i} > L_{0i} \), where \( i = 1, 2 \), the transverse
gradients of transversely generated fields are negligible. Therefore, the longitudinal components of the Maxwell’s equations (4.10a and 4.10b) give,

\[ B_z^{(2)} = 0 \]  \hspace{1cm} (4.11a)

and

\[ \frac{\partial E_z^{(2)}}{\partial \xi} = \frac{4\pi}{c} J_z^{(2)} \]  \hspace{1cm} (4.11b)

Eq. (4.11a) indicates that no magnetic wakefield is generated in the longitudinal direction. However, axial electric wakefield driven by current density \( J_{z}^{(2)} = n_0\varepsilon_0 v_{z}^{(2)} \), can be determined by first differentiating Eq. (4.11b) with respect to \( \xi \) and then combining with Eq. (4.8b) to yield,

\[
\frac{\partial^2 E_z^{(2)}}{\partial \xi^2} + k_p^2 E_z^{(2)} = -\frac{mc^2 k_p^2}{2e} a_1 \frac{\partial a_1}{\partial \xi} + a_2 \frac{\partial a_z}{\partial \xi} + \left( \frac{\partial a_1}{\partial \xi} a_z + \frac{\partial a_z}{\partial \xi} a_1 \right) \cos \theta \cos k_p \xi \\
- a_1 a_z k_p \cos \theta \sin k_p \xi
\]  \hspace{1cm} (4.12)

Substituting \( a_i = a_{0i} \exp\left( -\frac{x^2 + y^2}{r_{0i}^2} \right) \sin(\pi x/L_i) \) (where \( i = 1, 2 \) respectively, for the first and the second laser pulses) in Eq. (4.12) gives
\[
\frac{\partial^2 E^{(2)}_z}{\partial \xi^2} + k_p^2 E^{(2)}_z = -\frac{mc^2 k_p^2 \pi}{4e} \left\{ \frac{1}{L_1} a_{01}^2 \exp(-2r_1^2 / r_{01}^2) \sin(2\pi \xi) + \frac{1}{L_2} a_{02}^2 \exp(-2r_2^2 / r_{02}^2) \sin(2\pi \xi) \right\} - \cos \theta \cos (k_p \xi)
\]

\[
\frac{mc^2 k_p^2 \pi}{2e} a_{01} a_{02} \exp \left\{ -\left( \frac{r_1^2}{r_{01}^2} + \frac{r_2^2}{r_{02}^2} \right) \right\} \left[ \frac{1}{L_1} \cos \frac{\pi \xi}{L_1} \sin \frac{\pi \xi}{L_1} + \frac{1}{L_2} \sin \frac{\pi \xi}{L_2} \cos \frac{\pi \xi}{L_2} \right]
\]

\[
+ \frac{mc^2 k_p^3}{2e} a_{01} a_{02} \exp \left\{ -\left( \frac{r_1^2}{r_{01}^2} + \frac{r_2^2}{r_{02}^2} \right) \right\} \left( \frac{\pi \xi}{L_1} \sin \frac{\pi \xi}{L_1} \cos \theta \cos (k_p \xi) \right).
\]

(4.13)

The wakefields generated behind the laser pulses are obtained by integrating Eq. (4.13) between the limits \(0\) to \(-L\), where \(L \sim L_2\) and \(L_2 > L_1\). Thus,

\[
E^{(2)}_z = \frac{mc^2 k_p \pi}{4e} \frac{a_{11}^2}{L_1} \int_{-L}^{0} \sin \frac{2\pi \xi'}{L_1} \sin k_p (\xi' - \xi) d\xi' + \frac{a_{12}^2}{L_2} \int_{-L}^{0} \sin \frac{2\pi \xi'}{L_2} \sin k_p (\xi' - \xi) d\xi' - \frac{mc^2 a_{11} a_{12} k_p \cos \theta}{4e} \sin k_p \xi \int_{-L}^{0} \left( \frac{\pi}{L_1} \cos \frac{\pi \xi'}{L_1} \sin \frac{\pi \xi'}{L_1} + \frac{\pi}{L_2} \sin \frac{\pi \xi'}{L_2} \cos \frac{\pi \xi'}{L_2} \right) d\xi'
\]

\[
- \frac{mc^2 a_{11} a_{12} k_p^2 \cos \theta \cos k_p \xi}{4e} \sin \frac{\pi \xi}{L_1} \sin \frac{\pi \xi}{L_2} d\xi'.
\]

(4.14)

Solving Eq. (4.14), the normalized (by \(e/mc \omega_p\)) axial component of electric wakefield \(E^{(2)}_{wz}\), obtained behind the laser pulses, is given by

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\begin{align*}
E_{nc}^{(2)} &= \left(\frac{a_{11}^2 f_1 + a_{22}^2 f_2}{8}\right) (\sin k_p (L - \xi) + \sin k_p \xi) + \frac{a_{12} a_{12}}{8} \cos \theta \\
&\quad \left\{-\sin k_p \xi \left[\{1 - \cos \pi\left(\frac{L}{L_1} - \frac{L}{L_2}\right)\} - \{1 - \cos \pi\left(\frac{L}{L_1} + \frac{L}{L_2}\right)\}\right] + \\
&\quad \cos k_p \xi \left[\frac{k_p}{\pi\left(1 - \frac{1}{L_1}ight)} \sin \pi\left(\frac{L}{L_1} - \frac{L}{L_2}\right) - \frac{k_p}{\pi\left(1 + \frac{1}{L_2}\right)} \sin \pi\left(\frac{L}{L_1} + \frac{L}{L_2}\right)\right]\right\} \\
\end{align*}

(4.15)

where \( f_i = (1 - k_i^2 L_i^2 / 4\pi^2)^{-1} \). Eq. (4.15) indicates that the longitudinal electric wakefield is dependent on the angle \( \theta \). Varying the value of \( \theta \) leads to different peak values of amplitude of \( E_{nc}^{(2)} \). The first term on the right hand side of Eq. (4.15) indicates the sum of the longitudinal wakefields generated by the first and the second laser pulses individually while the second term occurs due to the combined effect of both the laser pulses. This term is dependent on \( \theta \). Therefore, the peak amplitude of the wakes generated by two colour laser fields tends to increase or decrease with the angle between the polarization directions of the two laser pulses. It may be noted that if either of the two pulses is switched off, Eq. (4.15) gives the standard wakefield generated by a single sinusoidal laser pulse.

For purposes of particle acceleration, the wakefields obtained behind both the laser pulses have to be studied. Therefore, the wakes generated by the two colour laser pulses are evaluated behind an arbitrary distance \( L (= 33 \mu m) \), which is slightly greater than the longer pulse length \( L_2 \). Since the generated wakefields
are dependent on the angle between the polarization direction of the two laser pulses, the variation of on-axis wake amplitudes for $\theta = 180^\circ, 90^\circ$ and $0^\circ$ are plotted in Fig. 4.3 as curves $a$, $b$ and $c$ respectively. The dotted curve shows the wakefields generated by a single laser pulse of length $30 \mu m$ and a much higher intensity ($I_{o1} = 3.3 \times 10^{17} W/cm^2$) compared to the two colour pulse system. It is seen that for a two laser pulse system, maximum amplitude wakefields are obtained when the two laser pulses are considered to be oppositely polarized. The same wakefield amplitude can be obtained by a single laser pulse of much higher intensity. Thus, it can be concluded that two colour laser pulses of lower intensities can generate longitudinal wakefields having amplitude that is comparable with the wake amplitude generated by a single laser pulse, with intensity greater than the sum of the intensities of the constituent pulses of the two colour pulse system.

4.2(b) Transverse electric wakefields

In order to study the generation of transverse wakefields ($E_x$ and $E_y$), the transverse components of transformed Maxwell’s equations (4.10a and 4.10b) are simultaneously solved to yield

$$\frac{\partial E^{(2)}_z}{\partial (x, y)} = -\frac{4\pi n_0 eV^{(2)}_{x,y}}{c}. \quad (4.16)$$

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Fig. 4.3 Variation of normalized on-axis longitudinal electrostatic wakefield ($E_{\text{nr}}$) with $k_p\xi$ for $\theta=180^\circ$ (curve a), $90^\circ$ (curve b) and $0^\circ$ (curve c) for $I_{01} = I_{02} = 4.84 \times 10^{16} \text{ W/cm}^2$ (corresponding to $a_{01}=0.2$, $a_{02}=0.207$) $\lambda_1 = 1.064 \mu m$, $\lambda_2 = 1.1006 \mu m$, $L_1 = 30 \mu m$, $L_2 = 31 \mu m$ and $\lambda_p = 32 \mu m$. Dotted curve shows the wakes generated by a single laser pulse with $\lambda_0 = 1.064 \mu m$, $a_{01}=0.53$ ($I_{01} = 3.3 \times 10^{17} \text{ W/cm}^2$) and $L = 30 \mu m$. 
Differentiating Eq. (4.16) with respect to $\xi$ and combining it with second order, slow transverse component of Lorentz force Equation (4.8a), gives the normalized (by $e/mc\omega_p$) transverse electric wakefields ($E_{n(x,y)}^{(2)}$) as,

$$E_{n(x,y)}^{(2)} = \frac{1}{k_p} \left[ a_1^2 \frac{(x,y)}{r_{01}^2} + a_2^2 \frac{(x,y)}{r_{02}^2} + a_1a_2 \left( \frac{(x,y)}{2r_{01}^2} + \frac{(x,y)}{2r_{02}^2} \right) \cos \theta \cos k_p \xi \right] +$$

$$- \frac{1}{k_p} \frac{\partial^2 E_{n(x,y)}^{(2)}}{\partial (k_p \xi) \partial x}.$$  \hspace{1cm} (4.17)

Substituting the normalized longitudinal electric wakefield (Eq. 4.15) into Eq. 4.17 gives

$$E_{n(x,y)}^{(2)} = \frac{1}{k_p} \left[ a_1^2 \frac{(x,y)}{2r_{01}^2} + a_2^2 \frac{(x,y)}{2r_{02}^2} + a_1a_2 \left( \frac{(x,y)}{2r_{01}^2} + \frac{(x,y)}{2r_{02}^2} \right) \cos \theta \cos k_p \xi \right]$$

$$+ \frac{1}{2k_p} \left[ a_1^2 f_1(x,y) + a_2^2 f_2(x,y) \right] \left( -\cos k_p (L - \xi) + \cos k_p \xi \right)$$

$$+ a_1a_2 \frac{\cos \theta (x,y)}{2} \left( -\frac{(x,y)}{r_{01}^2} - \frac{(x,y)}{r_{02}^2} \right) \left[ \frac{1}{k_p} \left( 1 - \cos \pi \left( \frac{L}{L_1} - \frac{L}{L_2} \right) \right) - \right]$$

$$\left\{ \frac{1}{k_p} \left( 1 - \cos \pi \left( \frac{L}{L_1} + \frac{L}{L_2} \right) \right) \right\}$$

$$+ \sin k_p \xi \left\{ \frac{1}{\pi \left( \frac{L}{L_1} - \frac{L}{L_2} \right)} \sin \pi \left( \frac{L}{L_1} - \frac{L}{L_2} \right) - \frac{1}{\pi \left( \frac{L}{L_1} + \frac{L}{L_2} \right)} \sin \pi \left( \frac{L}{L_1} + \frac{L}{L_2} \right) \right\}.$$  \hspace{1cm} (4.18)
Eq. (4.18) represents the normalized transverse electric wakefields generated behind the two laser pulses. As in the case of the longitudinal wakefields, the generated transverse wakes comprise of the wakes generated by two individual laser pulses as well as the wakes generated due to the synthesized field of both the laser pulses. These wakefields excited by the combined laser fields are found to be dependent on the relative polarization angle $\theta$. By varying the value of $\theta$, the polarization direction of the second laser pulse is varied with respect to the first laser pulse, which in turn is responsible for increase or decrease in the peak value of amplitude of the generated wakefields. 

For the same laser and plasma parameters as used in Fig. 4.1, the variation of the transverse electric wakefields with respect to $k_p \xi$, for different values of $\theta$ are plotted in Fig. 4.4. Curves $a$, $b$ and $c$ depict the wakefield at $x = y = 6 \mu m$, taking polarization angles as $180^\circ$, $90^\circ$ and $0^\circ$ respectively. The transverse wakes generated by a single laser pulse of length $30 \mu m$ and intensity $I_{01} = 2.1 \times 10^{17} W/cm^2$ (greater than the sum of the intensities of two colour laser pulses) has been shown by the dotted curve. The maximum amplitudes attained by wakefields while tracing curves $a$, $b$, and $c$ are respectively 0.0031, 0.0009 and 0.0013. It is seen that maximum amplitude wakes are generated when the two lasers are oppositely polarized. Further, comparing the transverse wake amplitude generated by two oppositely polarized laser pulses with the wakes generated by a single laser pulse, it is observed that the wake amplitudes generated in both the
Fig. 4.4  Variation of normalized transverse wakefield ($E_{n(x,y)}$) with $k_p\xi$ for

$\theta = 180^\circ$ (curve a),  $90^\circ$ (curve b),  $0^\circ$ (curve c) for $I_{01} = I_{02} =$

$4.84 \times 10^{16}\ \text{W/cm}^2$ (corresponding to $a_{01}=0.2$, $a_{02}=0.207$)

$\lambda_1=1.064\ \mu\text{m}$, $\lambda_2=1.1006\ \mu\text{m}$, $L_1=30\ \mu\text{m}$, $L_2=31\ \mu\text{m}$, $r_o=40\ \mu\text{m}$, $x=y=6\ \mu\text{m}$ and $\lambda_p=32\ \mu\text{m}$. Dotted curve shows the wakes generated by a single laser pulse with $\lambda_0=1.064\ \mu\text{m}$, $a_{01}=0.41$

($I_{01}=2.1 \times 10^{17}\ \text{W/cm}^2$ ) and $L=30\ \mu\text{m}$.
above cases is the same. Thus, it can be concluded that the transverse wakefields of same amplitude are generated by two lower intensity laser pulses as that generated by a single laser pulse of higher intensity.

4.3 Two-dimensional PIC simulation study

In order to validate the analytical results, two-dimensional PIC simulations, using VORPAL code, have been performed [131]. The simulations were conducted in the relativistic regime, where the intensity of the two linearly polarized laser pulses of wavelengths $\lambda_1 = 1.064 \mu m$ ($\omega_1 = 1.77 \times 10^{15}$ Hz) and $\lambda_2 = 1.1006 \mu m$ ($\omega_2 = 1.71 \times 10^{15}$ Hz) respectively, is considered to be equal $I_1 = I_2 = 1.21 \times 10^{18}$ W/cm$^2$ (which corresponds to $a_1 = 1.0$ and $a_2 = 1.0344$). The two laser pulses were considered to have a Gaussian spatial and radial profile. The full width at half maximum (FWHM) pulse lengths of the two laser pulses $L_{\text{FWHM}} = \sqrt{8 \ln 2} L_{\text{rms}}$ was taken to be 12.0 $\mu m$. The minimum laser spot size of the two laser pulses (assumed to be equal) was 50 $\mu m$. The plasma electron density ($n_0$) is the same as that considered while plotting Fig. 4.1. The size of the simulation domain was 78.9 $\mu m$ (this included 10 $\mu m$ vacuum distance) in the laser propagation direction ($x$) and 366 $\mu m$ in the transverse directions ($y$ and $z$), respectively. The simulation used one particle per cell along $x$ as well as $y$ direction to represent the plasma electrons.
The time step, satisfying the Courant condition was considered to be $177.14 \times 10^{-18}$ seconds while the total propagation time of the two laser pulses was $2.612 \times 10^{-12}$ seconds. It may be noted that each of the two laser pulses comprise of several (approximately 12) cycles. Therefore, the carrier envelope phase effect did not influence the propagation of the pulses.

Figs. 4.5 and 4.6, respectively represent the simulation curves depicting the generation of the longitudinal electric wakefields, via passage of two colour laser pulses, having same and opposite polarization directions, through homogeneous plasma. The peak values of the generated wakes are respectively $4.6 \times 10^{10} V/m$ and $1.255 \times 10^{10} V/m$ when the laser pulses have opposite and same polarization directions. Comparing the amplitudes of the generated wakes, it is seen that the amplitude of wakefield generated by two laser pulses having opposite directions of polarization is about 3.7 times more in comparison to the amplitude of the wakes generated by two laser pulses having the same polarization direction. The observations of the 2-D simulation study are compatible with the results already reported in the analytical study (Section 4.2 (a)), and therefore validate the theoretical analysis presented therein. The same trend of the generated electric wakefields as seen in the relativistic regime is observed in the mildly relativistic regime. Fig. 4.7 (4.8) represents the surface plot of the longitudinal electric wakefield generated by two laser pulses with same (opposite) polarization directions.
Fig. 4.5 Variation of longitudinal electric wakefield ($E_x$) with propagation distance for two laser pulses with same polarization directions for $I_1 = I_2 = 1.21 \times 10^{18}$ W/cm$^2$ (corresponding to $a_1 = 1.0$ and $a_2 = 1.0344$), $\lambda_1 = 1.064 \mu m$, $\lambda_2 = 1.1006 \mu m$, $L_{FWHM} = 12.0 \mu m$, $r_{0i} = 50 \mu m$ and $\lambda_r = 32 \mu m$ using 2-D VORPAL simulation code.
Fig. 4.6  Variation of longitudinal electric wakefield ($E_x$) with propagation distance for two laser pulses with opposite polarization directions for $I_1 = I_2 = 1.21 \times 10^{18} \text{ W/cm}^2$ (corresponding to $a_1 = 1.0$ and $a_2 = 1.0344$), $\lambda_1 = 1.064 \mu m$, $\lambda_2 = 1.1006 \mu m$, $L_{FWHM} = 12.0 \mu m$, $r_{oi} = 50 \mu m$ and $\lambda_p = 32 \mu m$ using 2-D VORPAL simulation code.
Fig. 4.7    Surface plot of the longitudinal electric wakefield \( (E_z) \) generated by two laser pulses with same polarization direction, using 2-D VORPAL simulation code.
Fig. 4.8 Surface plot of the longitudinal electric wakefield ($E_z$) generated by two laser pulses with opposite polarization direction, using 2-D VORPAL simulation code.
This study will be significant for the generation of large amplitude longitudinal as well as transverse electric wakefields, produced by the beating of short, two-colour laser pulses which can be utilized respectively for the purposes of acceleration and focusing of the test electron.
CHAPTER 5

Conclusions

5.1 Conclusions

The present thesis deals with a detailed analytical as well as simulation study of wakefield generation due to the interaction of short (~fs), intense laser pulses with preformed homogeneous plasma. The study includes (i) terahertz radiation generation due to the interaction of linearly polarized laser pulses with transversely magnetized plasma (ii) enhancement of wakefields and electron acceleration by circularly polarized laser pulses propagating in axially magnetized plasma as well as super-Gaussian pulses propagating in homogeneous plasma and (iii) generation of wakefields by two-colour laser pulses interacting with homogeneous plasma.

The effect of external static magnetic field on the electric and magnetic wakefields generated due to laser-plasma interaction process has been studied in the mildly relativistic regime, using a perturbative approach. The direction of the applied magnetic field is considered to be perpendicular to the propagation and polarization directions of the linearly polarized laser pulse. The axial and transverse electric and magnetic wakefields are derived with the help of time dependent Maxwell’s equations. The source driving these equations is obtained using the Lorentz force and continuity equations. Quasistatic approximation is used to study the evolution of the slow plasma electron velocities and wakefields.
It is observed that in the broad beam limit, the mutually perpendicular transverse electric and magnetic wakefields have the same amplitude and oscillate with the plasma frequency both within & behind the laser pulse, thus generating THz radiation. This field is generated due to the coupling of the slow, longitudinal plasma electron velocities with the externally applied transverse magnetic field. For typical laser and plasma parameters, the peak amplitude of the transverse wakes generated within and behind the laser pulse is compared graphically. It is seen that the wakefield amplitude within the laser pulse is small as compared to that generated behind the pulse. Further, the graphical depiction of the variation of the peak transverse electric field amplitude with the applied magnetic field can be used to evaluate the external magnetic field required to generate THz fields of various intensities. This study will be significant for the generation of THz radiation through laser-magnetized plasma interaction.

Further, comparing the axial wakefields generated in unmagnetized and magnetized plasma shows that the peak amplitude of the axial wakes is nearly the same. Thus, it can be concluded that for the present configuration, the presence of external transverse magnetic field has significant (negligible) effect on the transverse (longitudinal) wakes generated.

Enhancement of longitudinal wakes in magnetized plasma is possible by considering the propagation of a circularly polarized laser pulse in longitudinally magnetized plasma. This configuration has been studied in the relativistic regime.
with laser intensity beyond $10^{18} \text{W/cm}^2 (\alpha_0 \geq 1)$, for which the relativistic factor $\gamma$ plays a dominant role in determining the propagation characteristics of a laser pulse propagating in plasma. Since a perturbative technique is not valid in this regime, a one-dimensional numerical model has been developed to study the generation of longitudinal electrostatic wakefields by the propagation of a circularly polarized laser pulse in magnetized plasma. The direction of the external magnetic field is considered to be along as well as opposite to the direction of propagation of the laser pulse. The nonlinear fluid equations, describing the interaction of the laser pulse with uniformly magnetized plasma are transformed to the laser pulse frame. These equations are reduced to a time independent form using the quasistatic approximation and solved numerically to study the generation of longitudinal, electrostatic wakefields in the highly as well as mildly relativistic regimes.

It is seen that the slow transverse velocities and hence transverse wakefields do not arise for such a configuration of the laser and external magnetic field. However, the longitudinal wakefields are affected by the presence of externally applied field. The variation of the potential and electric field amplitude of the generated wakefield, when the magnetic field is directed opposite to and along the direction of laser pulse propagation has been studied and compared with the unmagnetized case. It is observed that the potential and field of the longitudinal electrostatic wakefields for reversed (forward) magnetic field increases (decreases)
as compared to the unmagnetized case. The numerically predicted results when compared with 2-D PIC simulation results, obtained using XOOPIC code, show the same trend of the wakefield amplitudes as predicted via numerical study.

Numerical methods are used to evaluate the relativistic factor ($\gamma_e$) associated with the maximum energy attained by an externally injected test electron, due to the generated electrostatic wakefield, with reversed and forward applied magnetic fields, for the two laser intensity regimes. This energy depends on the injection energy ($\gamma'_p$) of the test electron as well as on the magnitude of the generated wake potentials. It may be noted that $\gamma'_p$ is proportional to the phase velocity of the plasma wake (or the group velocity of the laser pulse). Therefore, for low magnitude of wake potential (as in the case of mildly relativistic regime), $\gamma_e$ is predominantly determined by the injection energy ($\gamma'_p$). Now, since the group velocity (and hence $v'_p$ & $\gamma'_p$) of the laser pulse in magnetized plasma is greater for the forward magnetic field case as compared to the reverse field, higher energy of the accelerated test electron is obtained for forward as compared to the reversed magnetization case when $a_0 << 1$. On the contrary, in the highly relativistic regime ($a_0 \sim 1$), the final test electron energies are governed mainly by the high wake potential which is about ten-fold greater than that in the mildly relativistic case, which thereby enhances (reduces) the maximum energy attained by the test electron, when the magnetic field is applied along the reverse (forward)
direction. In addition, the injection energy (proportional to $\gamma'_p$) required for trapping and accelerating a test electron reduces (enhances) when the reversed (forward) magnetic field is applied due to decreased (increased) phase velocity of the plasma wake.

It is important to note that in the mildly relativistic regime, the external magnetic field does not cause any enhancement in the effective energy gain by the test electron. However, for the highly relativistic regime, a significant enhancement in effective gain in energy by the test electron is obtained when a reversed magnetic field is applied. This study will be significant for the analysis of trapping and acceleration of a test electron by the longitudinal wakefields generated in magnetized plasma.

The 1-D numerical model is further used to analyze the longitudinal electrostatic wakefields generated by the interaction of a super-Gaussian laser pulse with homogeneous plasma. The generated wakefields are compared with the wakes driven by a Gaussian laser pulse. It is observed that the generated wake amplitude for super-Gaussian laser pulse shows a remarkable increase as compared to that generated by a Gaussian laser pulse. Further, 2-D PIC simulations, using XOOPIC code, are conducted to validate the analytical results. The simulation results are found to be in close agreement with the numerically obtained results. The trapping and acceleration of an externally injected test electron by the generated longitudinal wakes, has also been studied. It is observed that the energy gain is appreciably
enhanced for test electron accelerated by the wakes generated by a super-Gaussian laser pulse as compared to the energy gain for Gaussian pulse case.

The abovementioned studies for enhancement of wake amplitude have been conducted by considering the propagation of a single laser pulse through magnetized plasma. However, in the present thesis, possibility of enhancement in amplitude of the generated wakefields by propagation of two-colour laser pulses through homogeneous plasma has also been reported. A perturbative approach is employed to study the generation of longitudinal as well as transverse electrostatic wakefields in the mildly relativistic regime. The two linearly polarized laser pulses (having frequencies $\omega_1$ and $\omega_2$) are polarized in different arbitrary directions. The difference in frequencies of the two laser pulses is assumed to be equal to the plasma frequency i.e. $\omega_1 - \omega_2 = \omega_p$. The lengths of the two laser pulses are taken to be of the order of the plasma wavelength. The net ponderomotive force driving the longitudinal wakefields arises due to two contributions. The first is the sum of the ponderomotive force due to the envelopes of the two individual laser pulses, while the second contribution arises due to the combined effect of both the pulses. These contributions to the net ponderomotive force are compared graphically. It is seen that the former ponderomotive force is out of (in) phase with respect to the latter, when the two laser pulses are polarized in the same (opposite) directions, thereby reducing (enhancing) the net ponderomotive force. The electrostatic wakefields driven by this ponderomotive force also show the same trend of results, when
plotted for same and oppositely polarized laser pulses. It is seen that the wakefield amplitude generated by two colour, oppositely polarized laser pulses is the same as that generated by a single laser pulse having intensity greater than the sum of the intensities of the constituent pulses, in a two colour pulse system.

In order to validate the analytical results, two-dimensional PIC simulations, using VORPAL code, have been conducted, to study the propagation of two-colour linearly polarized Gaussian laser pulses through homogeneous, cold plasma, in the relativistic regime. It is seen that the longitudinal electric wakefields show the same trend of results as reported analytically. This study may be looked upon as a combination of the laser wakefield as well as plasma beat wave mechanisms. This study will be significant for the generation of large amplitude longitudinal wakefields for the purpose of particle acceleration, by two colour laser pulses.

5.2 Recommendations for future work

In the present thesis, the generation of wakefields due to the interaction of intense laser pulses with homogeneous plasma, has been studied analytically as well as via simulation. The enhancement of the wake amplitude has been studied for normally incident laser pulses. The wakefield amplitude can be modified by considering the propagation of obliquely incident laser pulses in preformed plasma.
Therefore, the study of s and p-polarized, obliquely incident laser pulses interacting with homogeneous plasma, can be an interesting proposal for future work.

The generated longitudinal wakefields with enhanced amplitudes are utilized for developing laser wakefield accelerators which are capable of accelerating electrons to ultrarelativistic energies. However, problems like diffraction spreading, electron dephasing and pump depletion limit the performance of LWFA’s. This would require a detailed analysis of the evolution of the laser pulse. Techniques for efficient reduction of these problems will be taken up as future work.

One of the main drawbacks of the laser-plasma based electron accelerators is the large energy spread. Various methods for improving the quality of the electron beam, accelerated by the generated longitudinal wakes, will be studied in future. Also, an insight will be developed into the physics of the bubble regime of the laser-plasma accelerators. Simulation studies of injection, trapping and acceleration of electrons, using PIC codes, can be undertaken for future research.
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