5.1 Introduction

In the present chapter, three different problems with different types of failures have been proposed.

(a) Model I - Performance Evaluation of a Multi-State System Covering Imperfect Fault Coverage

In the present model, a three unit repairable system having three types of failure incorporating the coverage factor will be analyzed. All the three units are in parallel. The system can completely fail due to catastrophic failure, human failure and failure of all three units. The system goes to the degraded state due to the partial failure of its components or failure of any unit. The system recovers automatically from human failure. Some numerical examples are also presented to illustrate the model mathematically. System configuration and state transition diagram of the proposed model have been shown in Fig. 5.1 and 5.2 respectively.

(b) Model II - Performance Evaluation of a Multi-State System under Imperfect Fault Coverage and Reworking Strategy

In this model, a standby complex system with three types of failures by using the concept of coverage factor will be investigated. The system consists of two subsystems $A$ and $B$ connected in series. Subsystem $A$ consisting two units in which one is the main and the one is in standby mode. On the other hand subsystem $B$ consists of $n$ identical units in series. Initially the system is in good working condition. After the failure of the main unit of subsystem $A$, the standby unit takes over its functioning immediately through the switch over device. Failure of both units of subsystem $A$ results in complete failure of the system. The failure of any $j^{th}$ unit (where $j=1,...,n$) of subsystem $B$ brings the system to a degraded state from where system can be automatically repaired (perfect coverage). Failure of both the units of subsystem $A$, human failure and catastrophic failure brings the system to complete failure mode. Furthermore, the authors have assumed that the system cannot be automatically recovered (imperfect fault coverage) due to catastrophic failure. System configuration and state transition diagram of the proposed model have been shown in Fig. 5.3 and Fig. 5.4 respectively.
(c) Model III-Stochastic Modeling of a Multi-State Manufacturing System under Three Types of Failures with Perfect Fault Coverage

In the present model, a multi-state manufacturing system having three types of failures namely Type I, Type II, Type III, with two warm standby units incorporating perfect fault coverage will be developed. Failure type existence depends on the characteristic of the system operation during manufacturing. After the failure of primary unit, the warm standby unit immediately takes over replacing the primary unit. This work also included the reliability characteristics estimation of three units active parallel system and the effect of the time and deterioration rates on reliability and the effect of deterioration rates on MTTF. System configuration and state transition diagram of the proposed model have been shown in Fig. 5.5 and Fig. 5.6.

5.2 Assumptions and Notations

Including the assumptions (i) to (iv) as mentioned in section 3.2, we have followed more assumptions associated with these models:

(a) Model I

(v) The system can automatically repair in case of unit and human failure (covered faults).
(vi) The system cannot be repaired in case of catastrophic failure (uncovered fault).

(b) Model II

(v) System has completely failed due to the catastrophic failure, human failure and failure of standby unit of subsystem A.
(vi) System can automatically repair in case of unit failure of subsystem B (covered faults).
(vii) System cannot be repaired in case of catastrophic failure (uncovered fault).

(c) Model III

(v) The system consists of three identical units in which one operating and the other two in warm standby.
(vi) The system has four states namely good, degraded, under deterioration (slow, mild and fast) and failed.
(vii) All the failure of the operating unit, standby is switched into operation.
(viii) All units suffer three types of failures.
(ix) The system failed when all the three units are down.
(x) The system can automatically repair in case of Type I, Type II and Type III failures (covered faults).

Including the notations mentioned in abbreviations, there are some more notations associated with the respective models:

(a) Model I

\( \lambda / \lambda_h / \lambda_{c-f} \)  
Failure rates of the system due to unit failure/human failure/catastrophic failure.

\( P_i(t) \)  
Probability at time \( t \) of being in \( i^{th} \) state; \( i=0 \) to 5.

\( P_j(x \text{ or } y \text{ or } z, t) \)  
The probability that the system is in the state \( S_j \) for \( j=3, 4, 5 \); the system is running under repair and elapsed repair time \( x, y, z \) respectively.

\( \mu(x) / \phi(z) \)  
Repair rates throughout the system due to unit failure/human failure.

\( A_c(t) \)  
Availability of the system at different values of coverage factor at time \( t \).

\( R_c(t) \)  
Reliability of the system at different values of coverage factor at time \( t \).

(b) Model II

\( \lambda_A \)  
Constant failure rate of the main and standby unit of subsystem \( A \).

\( \lambda_j \)  
Constant failure rate of \( j^{th} \) unit of subsystem \( B \).

\( \lambda_h \)  
Human failure rate.

\( \lambda_c \)  
Catastrophic failure rate.

\( P_{0,0}(t) / P_{0,j}(t)/ P_{1,0}(t) / P_{1,j}(t) \)  
The probability at time \( t \) when the system is in good state/degraded state due to the failure of \( j^{th} \) unit of subsystem \( B \) / due to the failure of main unit of subsystem \( A \) / due to the failure of \( j^{th} \) unit of subsystem \( B \) with failure of main unit of subsystem \( A \) respectively.

\( \mu(x) / \psi(z) / \phi_j(y) \)  
Repair rates of the system from failed state due to standby unit failure/human error / \( j^{th} \) unit failure of subsystem \( B \) with the failure of the standby unit of subsystem \( A \).

\( P_{2,0}(x,t) \)  
The probability of state that both the units of subsystem \( A \) have failed and system is under repair, elapsed repair time is \( x \); \( t \).

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\[ P_F(z, t) \] The probability of state that the system has failed due to human error and under repair, elapsed repair time is \( z; t \).
\[ P_{2,j}(y, t) \] The probability of state that the system has failed due to the failure of subsystem \( A \) with subsystem \( B \) is degraded and under repair, elapsed repair time is \( y; t \).
\[ P_c(w, t) \] The probability of state that the system has failed due to catastrophic failure.

(c) Model III

\( c\lambda_1, c\lambda_2, c\lambda_3 \) Type I failure rate, Type II failure rate, Type III failure rate respectively.
\( \mu_1, \mu_2, \mu_3 \) Repair rates of Type I, Type II, Type III failures respectively.
\( \alpha_1, \alpha_2, \alpha_3 \) Slow deterioration rate, mild deterioration rate and fast deterioration rate respectively.
\( \phi_1, \phi_2 \) Minor maintenance rates.
\( \phi_3 \) Major maintenance rate.
\[ P_i(t) \] Transition state probabilities from state \( S_0 \) to \( S_{18} \), where \( i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 \).

5.3 State Description

(a) Model I

<table>
<thead>
<tr>
<th>State</th>
<th>State description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>The system is in good working condition.</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>The system is in the degraded state due to unit failure.</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>The system is in the degraded state due to unit failure.</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>The system is in the failed state due to unit failure.</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>The system is in the failed state due to catastrophic failure.</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>The system is in the failed state due to human failure.</td>
</tr>
</tbody>
</table>
(b) Model II

State description

- **S₀**: The system is in good working condition.
- **S₁**: The system is in good state due to failure of the main unit of subsystem A.
- **S₂**: The system is in failed state due to the failure of standby unit of subsystem A.
- **S₃**: The system is in degraded state due to the failure of any jᵗʰ unit subsystem B.
- **S₄**: The system is in degraded state due to failure of main unit of subsystem A and failure
of any \( j \)th unit of subsystem \( B \).

\( S_5 \) The system is in failed state due to failure of Subsystem \( A \) and failure of any unit of subsystem \( B \).

\( S_6 \) The system is in failed state due to human failure.

\( S_7 \) The system is in failed state due to the catastrophic failure.

---

Fig. 5.3: System configuration

Fig. 5.4: State transition diagram
(c) Model III

<table>
<thead>
<tr>
<th>State</th>
<th>State description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>When the system is in good working condition.</td>
</tr>
<tr>
<td>$S_1$</td>
<td>When the system is in degraded state due to Type I failure.</td>
</tr>
<tr>
<td>$S_2$</td>
<td>The system is in degraded state due to Type II failure.</td>
</tr>
<tr>
<td>$S_3$</td>
<td>The system is in failed due to Type III failure.</td>
</tr>
<tr>
<td>$S_4$</td>
<td>The system is in degraded state due to Type III failure.</td>
</tr>
<tr>
<td>$S_5$</td>
<td>The system is in failed due to Type II failure.</td>
</tr>
<tr>
<td>$S_6$</td>
<td>The system is in degraded state due to Type II failure.</td>
</tr>
<tr>
<td>$S_7$</td>
<td>The system is in degraded state due to Type III failure.</td>
</tr>
<tr>
<td>$S_8$</td>
<td>The system is in failed due to Type I failure.</td>
</tr>
<tr>
<td>$S_9$</td>
<td>The system is in degraded state due to Type I failure.</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>The system is in failed due to Type III failure.</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>The system is in degraded state due to Type III failure.</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>The system is in degraded state due to Type I failure.</td>
</tr>
<tr>
<td>$S_{13}$</td>
<td>The system is in failed due to Type II failure.</td>
</tr>
<tr>
<td>$S_{14}$</td>
<td>The system is in degraded state due to Type II failure.</td>
</tr>
<tr>
<td>$S_{15}$</td>
<td>The system is in failed due to Type I failure.</td>
</tr>
<tr>
<td>$S_{16}$</td>
<td>The system is in degraded state due to slow deterioration.</td>
</tr>
<tr>
<td>$S_{17}$</td>
<td>The system is in degraded state due to mild deterioration.</td>
</tr>
<tr>
<td>$S_{18}$</td>
<td>The system is in failed due to fast deterioration.</td>
</tr>
</tbody>
</table>

Fig. 5.5: System configuration
5.4 Formulation and Solution of the Mathematical Models

By the probability of considerations and continuity of arguments, the following set of differential equations governing the present mathematical models is obtained:
(a) Model I

\[
\frac{\partial}{\partial t} + 3c\lambda + c\lambda_n + (1-c)\lambda_{c_{of}} \] \[P_0(t) = \mu(x)P_1(t) + \int_0^\infty \phi(z)P_3(z,t)dz \] (5.1)

\[
\frac{\partial}{\partial t} + \mu(x) + 2c\lambda + (1-c)\lambda_{c_{of}} \] \[P_1(t) = \mu(x)P_2(t) + 3c\lambda P_0(t) \] (5.2)

\[
\frac{\partial}{\partial t} + \mu(x) + c\lambda + (1-c)\lambda_{c_{of}} \] \[P_2(t) = 2c\lambda P_1(t) + \int_0^\infty \mu(x)P_3(x,t)dx \] (5.3)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x) \] \[P_3(x,t) = 0 \] (5.4)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial y} \] \[P_4(y,t) = 0 \] (5.5)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \phi(z) \] \[P_3(z,t) = 0 \] (5.6)

Boundary conditions

\[P_3(0,t) = c\lambda P_1(t) \] (5.7)

\[P_4(0,t) = (1-c)\lambda_{c_{of}} \left[ P_0(t) + P_1(t) + P_2(t) \right] \] (5.8)

\[P_3(0,t) = c\lambda_n P_0(t) \] (5.9)

Initial condition

\[P_0(t) = 1 \text{ at } t=0 \text{ and all other state probabilities are zero initially} \] (5.10)

Taking Laplace transformation of equations (5.1-5.6) and (5.7-5.9), we get

\[
\left[ s + 3c\lambda + c\lambda_n + (1-c)\lambda_{c_{of}} \right] \overline{P}_0(s) = 1 + \mu(x)\overline{P}_1(s) + \int_0^\infty \phi(z)\overline{P}_3(z,s)dz \] (5.11)
After solving Equations (5.11) to (5.16) with the help of (5.17) to (5.19), one can get the Laplace transform state probabilities as given below:

\[
\begin{align*}
\left[ s + \mu(x) + 2c\lambda + (1-c)\lambda_{cf} \right] \bar{P}_1(s) &= \mu(x)\bar{P}_1(s) + 3c\lambda\bar{P}_0(s) \\
\left[ s + \mu(x) + c\lambda + (1-c)\lambda_{cf} \right] \bar{P}_2(s) &= 2c\lambda\bar{P}_1(s) + \int_0^\infty \mu(x)\bar{P}_2(x,s)dx \\
\left[ s + \frac{\partial}{\partial x} + \mu(x) \right] \bar{P}_3(x,s) &= 0 \\
\left[ s + \frac{\partial}{\partial y} \right] \bar{P}_4(y,s) &= 0 \\
\left[ s + \frac{\partial}{\partial z} + \phi(z) \right] \bar{P}_5(z,s) &= 0 \\
\bar{P}_3(0,s) &= c\lambda\bar{P}_2(s) \\
\bar{P}_4(0,s) &= (1-c)\lambda_{cf} \left[ \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) \right] \\
\bar{P}_5(0,s) &= c\lambda_{cf}\bar{P}_0(s)
\end{align*}
\]

After solving Equations (5.11) to (5.16) with the help of (5.17) to (5.19), one can get the Laplace transform state probabilities as given below:

\[
\begin{align*}
\bar{P}_0(s) &= \frac{\{s + d_2\} - \bar{D}_1(s) \{s + \phi\} \{s + \phi(y)\} \{s + \phi(y)\} - 3c\lambda\mu(x)\{s + \phi(y)\} - \phi(y)\lambda_{cf}\{s + d_2 - \bar{D}_1(s)\}}{(s + d_1)\{s + d_2 - \bar{D}_1(s)\} \{s + \phi(y)\} - 3c\lambda\mu(x)\{s + \phi(y)\} - \phi(y)\lambda_{cf}\{s + d_2 - \bar{D}_1(s)\}} \\
\bar{P}_1(s) &= \frac{3c\lambda}{s + d_2 - \bar{D}_1(s)} \bar{P}_0(s) \\
\bar{P}_2(s) &= \frac{6c^2\lambda^2}{(s + d_1 - \bar{D}_1(s))(s + d_2 - \bar{D}_1(s))} \bar{P}_0(s) \\
\bar{P}_3(s) &= \frac{6c^3\lambda^3}{(s + \mu(x))(s + d_1 - \bar{D}_1(s))(s + d_2 - \bar{D}_1(s))} \bar{P}_0(s)
\end{align*}
\]
The Laplace transformation of the probabilities that the system is in the up (either good or degraded state) and down (failed) state at any time is

\[
\bar{P}_u(s) = \frac{(1-c)\lambda_{cf}}{s}\left[1 + \frac{3c\lambda}{s+d_2 - \bar{D}_1(s)} + \frac{6c^2\lambda^2}{(s+d_1 - \bar{D}_2(s))(s+d_2 - \bar{D}_1(s))}\right]\bar{P}_0(s) \tag{5.24}
\]

\[
\bar{P}_d(s) = \frac{\lambda_y}{s+\phi(y)}\bar{P}_0(s) \tag{5.25}
\]

Where \(d_1 = \mu(x) + c\lambda + (1-c)\lambda_{cf},\) \(d_2 = \mu(x) + 2c\lambda + (1-c)\lambda_{cf},\) \(d_3 = \mu(x) + 2c\lambda + (1-c)\lambda_{cf},\)

\[
\bar{D}_1(s) = \frac{2c\lambda\mu(x)}{(s+d_1) - \bar{D}_2(s)}, \quad \bar{D}_2(s) = \frac{c\lambda\mu(x)}{(s+\mu(x))}
\]

The Laplace transformation of the probabilities that the system is in the up (either good or degraded state) and down (failed) state at any time is

\[
\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s)
\]

\[
= \left(1 + \frac{3c\lambda}{s+d_2 - \bar{D}_1(s)} + \frac{6c^2\lambda^2}{(s+d_1 - \bar{D}_2(s))(s+d_2 - \bar{D}_1(s))}\right)\bar{P}_0(s) \tag{5.26}
\]

\[
\bar{P}_{down}(s) = \bar{P}_3(s) + \bar{P}_4(s) + \bar{P}_5(s)
\]

\[
= \left[\frac{6c^3\lambda^3}{(s+\mu(x))(s+d_1 - \bar{D}_2(s))(s+d_2 - \bar{D}_1(s))}\right] + \frac{\lambda_y}{s+\phi(y)}
\]

\[
+ \frac{(1-c)\lambda_{cf}}{s}\left[1 + \frac{3c\lambda}{s+d_2 - \bar{D}_1(s)} + \frac{6c^2\lambda^2}{(s+d_1 - \bar{D}_2(s))(s+d_2 - \bar{D}_1(s))}\right]\bar{P}_0(s) \tag{5.27}
\]

(b) Model II

\[
\left[\frac{\partial}{\partial t} + \lambda_+ c \lambda_+ + \lambda_n + (1-c)\lambda_c\right] P_{0,0}(t) = \int_0^\infty P_{2,0}(x,t)\mu(x)dx + \int_0^\infty P_{2,j}(y,t)\phi_j(y)dy
\]

\[
+ \int_0^\infty P_{F}(z,t)\psi(z)dz + \phi_j(y)P_{0,j}(t) \tag{5.28}
\]

\[
\left[\frac{\partial}{\partial t} + \lambda_+ c \lambda_+ + \lambda_n + (1-c)\lambda_c\right] P_{i,0}(t) = \phi_j(y)P_{i,j}(t) + \lambda_+ P_{i,0}(t) \tag{5.29}
\]

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\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x) \right] P_{2,0}(x,t) = 0
\] (5.30)

\[
\frac{\partial}{\partial t} + \lambda_d + \phi_j(y) + (1-c)\lambda_c \right] P_{0,j}(t) = c \lambda_j P_{0,0}(t)
\] (5.31)

\[
\frac{\partial}{\partial t} + \lambda_d + \phi_j(y) + (1-c)\lambda_c \right] P_{1,j}(t) = c \lambda_j P_{0,0}(t) + \lambda_d P_{0,j}(t)
\] (5.32)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \phi_j(y) \right] P_{2,j}(y,t) = 0
\] (5.33)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \psi(z) \right] P_z(z,t) = 0
\] (5.34)

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial w} \right] P_w(w,t) = 0
\] (5.35)

**Boundary conditions**

\[
P_{2,0}(0,t) = \lambda_d P_{1,0}(t)
\] (5.36)

\[
P_{2,j}(0,t) = \lambda_d P_{1,j}(t)
\] (5.37)

\[
P_{\epsilon}(0,t) = \lambda_h \left[ P_{0,0}(t) + P_{1,0}(t) \right]
\] (5.38)

\[
P_{\epsilon}(0,t) = (1-c)\lambda_c \left[ P_{0,0}(t) + P_{1,0}(t) + P_{0,j}(t) + P_{1,j}(t) \right]
\] (5.39)

\[
P_{0,0}(t) = 1 \text{ and other state probabilities are zero}
\] (5.40)

Taking Laplace transformation of equations (5.28-5.35) and (5.36-5.40), we get

\[
\left[ s + \lambda_d + c \lambda_j + \lambda_h + (1-c)\lambda_c \right] \overline{P}_{0,0}(s) = 1 + \int_0^\infty \overline{P}_{2,0}(x,s) \mu(x)dx + \int_0^\infty \overline{P}_{2,j}(y,s) \phi_j(y)dy
\]

\[
+ \int_0^\infty \overline{P}_{\epsilon}(z,s) \psi(z)dz + \phi_j(y) \overline{P}_{0,j}(s)
\] (5.41)

\[
\left[ s + \lambda_d + c \lambda_j + \lambda_h + (1-c)\lambda_c \right] \overline{P}_{1,0}(s) = \phi_j(y) \overline{P}_{1,j}(s) + \lambda_d \overline{P}_{0,0}(s)
\] (5.42)

\[
\left[ s + \frac{\partial}{\partial x} + \mu(x) \right] \overline{P}_{2,0}(x,s) = 0
\] (5.43)
\[
\begin{align*}
[s + \lambda_4 + \phi_j (y) + (1-c) \lambda_c] \bar{P}_{0,0} (s) &= c \lambda_j \bar{P}_{0,0} (s) \quad (5.44) \\
[s + \lambda_4 + \phi_j (y) + (1-c) \lambda_c] \bar{P}_{1,0} (s) &= c \lambda_j \bar{P}_{0,0} (s) + \lambda_j \bar{P}_{0,1} (s) \quad (5.45) \\
[s + \frac{\partial}{\partial y} + \phi_j (y)] \bar{P}_{2,0} (y, s) &= 0 \quad (5.46) \\
[s + \frac{\partial}{\partial z} + \psi(z)] \bar{P}_{2,0} (z, s) &= 0 \quad (5.47) \\
[s + \frac{\partial}{\partial w}] \bar{P}_r (w, s) &= 0 \quad (5.48) \\
\bar{P}_{2,0} (0, s) &= \lambda_4 \bar{P}_{1,0} (s) \quad (5.49) \\
\bar{P}_{2,j} (0, s) &= \lambda_4 \bar{P}_{1,j} (s) \quad (5.50) \\
\bar{P}_r (0, s) &= \lambda_4 [\bar{P}_{0,0} (s) + \bar{P}_{1,0} (s)] \quad (5.51) \\
\bar{P}_c (0, s) &= (1-c) \lambda_4 [\bar{P}_{0,0} (s) + \bar{P}_{1,0} (s) + \bar{P}_{0,j} (s) + \bar{P}_{1,j} (s)] \quad (5.52)
\end{align*}
\]

Solving (5.41-5.48) with the help of (5.49-5.52) one may get various state probabilities as given below:

\[
\begin{align*}
\bar{P}_{0,0} (s) &= \frac{1}{d_2} \\
\bar{P}_{1,0} (s) &= \frac{1}{(s + d)} \left[ \phi_j (y) \left[ \frac{c \lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_4}{(s + d_1)} \right] + \lambda_4 \right] \bar{P}_{0,0} (s) \right] \quad (5.53) \\
\bar{P}_{2,0} (s) &= \left[ \frac{\lambda_4 \phi_j (y)}{(s + d)} \right] \left[ \frac{c \lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_4}{(s + d_1)} \right] \left[ \frac{1 - \bar{S}_\mu (s)}{s} \right] + \frac{1}{(s + d)} \lambda_4 \left[ \frac{1 - \bar{S}_\mu (s)}{s} \right] \right] \bar{P}_{0,0} (s) \quad (5.54) \\
\bar{P}_{0,j} (s) &= \frac{1}{(s + d_1)} \frac{c \lambda_j \bar{P}_{0,0} (s)}{\bar{P}_{0,0} (s)} \quad (5.55) \\
\bar{P}_{1,j} (s) &= \frac{C \lambda_j \bar{P}_{0,0} (s)}{(s + d_1)} \left[ 1 + \frac{\lambda_4}{(s + d_1)} \right] \quad (5.56) \\
\bar{P}_{2,j} (s) &= \frac{\lambda_4 C \lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_4}{(s + d_1)} \right] \left[ \frac{1 - \bar{S}_\mu (s)}{s} \right] \bar{P}_{0,0} (s) \quad (5.57)
\end{align*}
\]
\[ \bar{P}_e(s) = \lambda_n + \left[ \frac{\lambda_n}{s + d} \phi_j(y) \left( \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \right) \right] \bar{P}_{0,0}(s) \]  
(5.59)

\[ \bar{P}_c(s) = \frac{1}{s} (1 - c) \lambda_c \bar{P}_{0,0}(s) \left[ 1 + \frac{1}{s + d} \left[ \phi_j(y) \left( \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \right) \right] + \lambda_d \right] + \]
(5.60)

Where \( \bar{S}_\mu(s) = \int_0^\infty \mu(x) \left\{ \exp \left[ -sx - \int_0^x \mu(x)dx \right] \right\} dx \), \( \bar{S}_\psi(s) = \int_0^\infty \psi(z) \left\{ \exp \left[ -sz - \int_0^z \psi(z)dz \right] \right\} dx \)

\[ \bar{S}_\phi(s) = \int_0^\infty \phi(y) \left\{ \exp \left[ -sy - \int_0^y \phi(y)dy \right] \right\} dy \], \( d = \lambda_d + c\lambda_j + \lambda_n + (1 - c)\lambda_c \),

\[ d_1 = \lambda_d + (1 - c)\lambda_c + \phi_j(y) \] and

\[ \frac{-\lambda_n \phi_j(y)}{(s + d)} \left( \frac{c\lambda_j}{(s + d_1)} \right) \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \bar{S}_\psi(s) - \frac{\lambda^2_n}{(s + d)} \bar{S}_\phi(s) - \]

\[ d_2 = \lambda_n \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \bar{S}_\psi(s) - \lambda_n \lambda_c \bar{S}_\psi(s) - \phi_j(y) \frac{c\lambda_j}{(s + d_1)} \]

The Laplace transformations of the probabilities that the system is in the up (either good or degraded state) and down (failed state) at any time are as follows:

\[ \bar{P}_{up}(s) = \bar{P}_{0,0}(s) + \bar{P}_{1,0}(s) + \bar{P}_{0,1}(s) + \bar{P}_{1,1}(s) \]

\[ \begin{align*} 
\bar{P}_{up}(s) &= \left[ 1 + \frac{1}{s + d} \left[ \phi_j(y) \left( \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \right) \right] + \lambda_d \right] + \\
&= \left[ 1 + \frac{1}{s + d} \left[ \phi_j(y) \left( \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \right) \right] + \lambda_d \right] \bar{P}_{0,0}(s) \\
&+ \frac{1}{s + d_1} c\lambda_j + \frac{c\lambda_j}{(s + d_1)} \left[ 1 + \frac{\lambda_d}{(s + d_1)} \right] \\
\end{align*} \]

(5.61)

\[ \bar{P}_{down}(s) = \bar{P}_{2,0}(s) + \bar{P}_F(s) + \bar{P}_{2,1}(s) + \bar{P}_c(s) \]
(c) Model III

\[
\begin{align*}
\frac{\partial}{\partial t} + c\lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2 \] P_0(t) \\
= \mu_1 P_1(t) + \mu_2 P_6(t) + \mu_3 P_{11}(t) + \phi_1 P_{16}(t) + \phi_2 P_{17}(t) \quad (5.63)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} + c\lambda_2 + c\lambda_3 + \mu_1 \] P_1(t) = \mu_1 P_2(t) + \mu_3 P_4(t) + c\lambda_1 P_0(t) \quad (5.64)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} + c\lambda_3 + \mu_2 \] P_2(t) = \mu_2 P_3(t) + c\lambda_2 P_1(t) \quad (5.65)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} + \mu_3 \] P_3(t) = c\lambda_3 P_2(t) \quad (5.66)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} + c\lambda_2 + \mu_3 \] P_4(t) = c\lambda_2 P_1(t) + \mu_2 P_5(t) \quad (5.67)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} + \mu_5 \] P_5(t) = c\lambda_2 P_4(t) \quad (5.68)
\end{align*}
\]
\[
\left[\frac{\partial}{\partial t} + c\lambda_1 + c\lambda_3 + \mu_2\right]P_6(t) = c\lambda_2P_6(t) + \mu_7P_7(t) + \mu_9P_9(t) \tag{5.69}
\]

\[
\left[\frac{\partial}{\partial t} + c\lambda_1 + \mu_3\right]P_7(t) = \mu_1P_8(t) + c\lambda_3P_6(t) \tag{5.70}
\]

\[
\left[\frac{\partial}{\partial t} + \mu_1\right]P_8(t) = c\lambda_1P_7(t) \tag{5.71}
\]

\[
\left[\frac{\partial}{\partial t} + c\lambda_3 + \mu_4\right]P_9(t) = c\lambda_4P_6(t) + \mu_5P_{10}(t) \tag{5.72}
\]

\[
\left[\frac{\partial}{\partial t} + \mu_5\right]P_{10}(t) = c\lambda_3P_9(t) \tag{5.73}
\]

\[
\left[\frac{\partial}{\partial t} + c\lambda_1 + c\lambda_2 + \mu_3\right]P_{11}(t) = \mu_1P_{12}(t) + \mu_2P_{14}(t) + c\lambda_5P_9(t) \tag{5.74}
\]

\[
\left[\frac{\partial}{\partial t} + c\lambda_2 + \mu_4\right]P_{12}(t) = c\lambda_1P_{11}(t) + \mu_2P_{13}(t) \tag{5.75}
\]

\[
\left[\frac{\partial}{\partial t} + \mu_2\right]P_{13}(t) = c\lambda_2P_{12}(t) \tag{5.76}
\]

\[
\left[\frac{\partial}{\partial t} + c\lambda_4 + \mu_2\right]P_{14}(t) = \mu_1P_{15}(t) + c\lambda_2P_{11}(t) \tag{5.77}
\]

\[
\left[\frac{\partial}{\partial t} + \mu_5\right]P_{15}(t) = c\lambda_1P_{14}(t) \tag{5.78}
\]

\[
\left[\frac{\partial}{\partial t} + c\alpha_3 + \phi_1\right]P_{16}(t) = c\alpha_1P_6(t) + \phi_3P_{18}(t) \tag{5.79}
\]

\[
\left[\frac{\partial}{\partial t} + c\alpha_3 + \phi_2\right]P_{17}(t) = c\alpha_2P_0(t) + \phi_3P_{18}(t) \tag{5.80}
\]
\[
\frac{\partial}{\partial t} + \phi_3 + \phi_5 \right] P_{18}(t) = c\alpha_2 P_{16}(t) + c\alpha_3 P_{17}(t) \quad (5.81)
\]

\[P_0(t)=1 \text{ and other state probabilities are zero at } t=0 \quad (5.82)\]

Taking Laplace transformation of Equations (5.63-5.82), we get

\[\left[ s + c\lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2 \right] \bar{P}_0(s) = \mu_4 \bar{P}_1(s) + \mu_5 \bar{P}_6(s) + \mu_3 \bar{P}_{11}(s) + \phi_1 \bar{P}_{16}(s) + \phi_2 \bar{P}_{17}(s) \quad (5.83)\]

\[\left[ s + c\lambda_2 + c\lambda_3 + \mu_1 \right] \bar{P}_1(s) = \mu_2 \bar{P}_2(s) + \mu_4 \bar{P}_3(s) + \mu_1 \bar{P}_6(s) + c\lambda_2 \bar{P}_0(s) \quad (5.84)\]

\[\left[ s + c\lambda_3 + \mu_2 \right] \bar{P}_2(s) = \mu_4 \bar{P}_3(s) + c\lambda_3 \bar{P}_1(s) \quad (5.85)\]

\[\left[ s + \mu_3 \right] \bar{P}_3(s) = c\lambda_3 \bar{P}_2(s) \quad (5.86)\]

\[\left[ s + c\lambda_2 + \mu_3 \right] \bar{P}_4(s) = c\lambda_3 \bar{P}_1(s) + \mu_2 \bar{P}_3(s) \quad (5.87)\]

\[\left[ s + \mu_2 \right] \bar{P}_5(s) = c\lambda_2 \bar{P}_4(s) \quad (5.88)\]

\[\left[ s + c\lambda_1 + c\lambda_3 + \mu_2 \right] \bar{P}_0(t) = c\lambda_2 \bar{P}_0(s) + \mu_1 \bar{P}_1(s) + \mu_1 \bar{P}_6(s) \quad (5.89)\]

\[\left[ s + c\lambda_1 + \mu_4 \right] \bar{P}_1(s) = \mu_4 \bar{P}_2(s) + \mu_1 \bar{P}_3(s) + c\lambda_1 \bar{P}_0(s) \quad (5.90)\]

\[\left[ s + \mu_1 \right] \bar{P}_6(s) = c\lambda_1 \bar{P}_1(s) \quad (5.91)\]

\[\left[ s + c\lambda_3 + \mu_1 \right] \bar{P}_6(s) = c\lambda_1 \bar{P}_0(s) + \mu_2 \bar{P}_{10}(s) \quad (5.92)\]

\[\left[ s + \mu_5 \right] \bar{P}_{10}(s) = c\lambda_2 \bar{P}_5(s) \quad (5.93)\]

\[\left[ s + c\lambda_1 + c\lambda_2 + \mu_1 \right] \bar{P}_{11}(s) = \mu_4 \bar{P}_{12}(s) + \mu_2 \bar{P}_{14}(s) + c\lambda_1 \bar{P}_0(s) \quad (5.94)\]

\[\left[ s + c\lambda_2 + \mu_1 \right] \bar{P}_{12}(s) = c\lambda_1 \bar{P}_{11}(s) + \mu_2 \bar{P}_{13}(s) \quad (5.95)\]
\[ s + \mu_2 \] \[ P_{13}(s) = c \lambda_3 P_{15}(s) \] (5.96)

\[ s + c \lambda_1 + \mu_2 \] \[ P_{14}(s) = \mu_1 P_{15}(s) + c \lambda_2 P_{11}(s) \] (5.97)

\[ s + \mu_1 \] \[ P_{15}(s) = c \lambda_1 P_{14}(s) \] (5.98)

\[ s + c \alpha_3 + \phi_1 \] \[ P_{16}(s) = c \alpha_4 P_0(s) + \phi_2 P_{18}(s) \] (5.99)

\[ s + c \alpha_3 + \phi_2 \] \[ P_{17}(s) = c \alpha_2 P_0(s) + \phi_3 P_{18}(s) \] (5.100)

\[ s + \phi_3 + \phi_4 \] \[ P_{18}(s) = c \alpha_4 P_{16}(s) + c \alpha_5 P_{17}(s) \] (5.101)

Solving (5.83-5.101) one may get various state probabilities as given below:

\[ \bar{P}_0(s) = \frac{1}{D(s)} \] (5.102)

\[ \bar{P}_1(s) = \left[ \frac{(c \lambda_a)}{(A_a)} \right] \bar{P}_0(s) \] (5.103)

\[ \bar{P}_2(s) = \left[ \frac{(c \lambda_a c \lambda_2)}{(A_a A_b)} \right] \bar{P}_0(s) \] (5.104)

\[ \bar{P}_3(s) = \left[ \frac{(c \lambda_a c \lambda_2 c \lambda_3)}{(A_b (s + \mu_3))} \right] \bar{P}_0(s) \] (5.105)

\[ \bar{P}_4(s) = \left[ \frac{(c \lambda_a c \lambda_3)}{(A_a A_{ab})} \right] \bar{P}_0(s) \] (5.106)

\[ \bar{P}_5(s) = \left[ \frac{(c \lambda_a c \lambda_2 c \lambda_3)}{(A_a A_{ab} (s + \mu_3))} \right] \bar{P}_0(s) \] (5.107)

\[ \bar{P}_6(s) = \left[ \frac{(c \lambda_a)}{(A_d)} \right] \bar{P}_0(s) \] (5.108)

\[ \bar{P}_7(s) = \left[ \frac{(c \lambda_a c \lambda_3)}{(A_e A_d)} \right] \bar{P}_0(s) \] (5.109)

\[ \bar{P}_8(s) = \left[ \frac{(c \lambda_a c \lambda_2 c \lambda_3)}{(A_e A_d (s + \mu_1))} \right] \bar{P}_0(s) \] (5.110)

\[ \bar{P}_9(s) = \left[ \frac{(c \lambda_a c \lambda_2)}{(A_e A_d)} \right] \bar{P}_0(s) \] (5.111)
\[
\overline{P}_{10}(s) = \left[ (c\lambda_1c\lambda_2c\lambda_3)/(A_xA_y(s + \mu_5)) \right] \overline{P}_0(s) \tag{5.112}
\]

\[
\overline{P}_{11}(s) = \left[ (c\lambda_3)/(A_y) \right] \overline{P}_0(s) \tag{5.113}
\]

\[
\overline{P}_{12}(s) = \left[ (c\lambda_1c\lambda_3)/(A_yA_x) \right] \overline{P}_0(s) \tag{5.114}
\]

\[
\overline{P}_{13}(s) = \left[ (c\lambda_1c\lambda_2c\lambda_3)/(A_xA_y(s + \mu_2)) \right] \overline{P}_0(s) \tag{5.115}
\]

\[
\overline{P}_{14}(s) = \left[ (c\lambda_2c\lambda_3)/(A_yA_x) \right] \overline{P}_0(s) \tag{5.116}
\]

\[
\overline{P}_{15}(s) = \left[ (c\lambda_1c\lambda_2c\lambda_3)/(A_xA_y(s + \mu_4)) \right] \overline{P}_0(s) \tag{5.117}
\]

\[
\overline{P}_{16}(s) = \left[ (A_y/A_x) \right] \overline{P}_0(s) \tag{5.118}
\]

\[
\overline{P}_{17}(s) = \left[ \frac{c\alpha_2 + (\phi_2c\alpha_1/(s + 2\phi_2))(A_y/A_x)}{(s + d_1) - (\phi_2c\alpha_3/(s + 2\phi_3))} \right] \overline{P}_0(s) \tag{5.119}
\]

\[
\overline{P}_{18}(s) = \left[ \frac{c\alpha_1/(s + 2\phi_2)}{A_y/A_x + \overline{P}_{17}(s)} \right] \overline{P}_0(s) \tag{5.120}
\]

Where

\[
d = c\lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2, \quad d_1 = c\lambda_2 + c\lambda_3 + \mu_1, \quad d_2 = c\lambda_3 + \mu_2, \quad d_3 = c\lambda_2 + \mu_3,
\]

\[
d_4 = c\lambda_1 + c\lambda_3 + \mu_2, \quad d_5 = c\lambda_2 + \mu_3, \quad d_6 = c\lambda_3 + \mu_4, \quad d_7 = c\lambda_4 + c\lambda_2 + \mu_4, \quad d_8 = c\lambda_2 + \mu_5,
\]

\[
d_9 = c\lambda_1 + \mu_2, \quad d_{10} = c\alpha_2 + \phi_1, \quad d_{11} = c\alpha_3 + \phi_2
\]
\[ D(s) = \begin{pmatrix} (s + \lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2) - c\lambda_2\mu_1 / \{(s + c\lambda_2 + c\lambda_3 + \mu_1) \\
-c\lambda_2\mu_3 / (s + c\lambda_2 + \mu_2) - c\lambda_2\mu_2 / (s + \mu_2) \\
c\lambda_2\mu_2 / (s + c\lambda_3 + \mu_2 - c\lambda_3\mu_1 / (s + \mu_1)) \\
-c\lambda_2 / \{(s + c\lambda_1 + c\lambda_3 + \mu_2) - c\lambda_2\mu_1 / (s + c\lambda_2 + \mu_1) \\
-c\lambda_1\mu_1 / (s + c\lambda_3 + \mu_1 - c\lambda_3\mu_1 / (s + \mu_1)) \\
-c\lambda_3\mu_3 / \{(s + c\lambda_1 + c\lambda_2 + \mu_2) - c\lambda_1\mu_1 / (s + \mu_1) \\
-c\lambda_2\mu_2 / (s + \mu_2) - c\lambda_2\mu_1 / (s + \mu_1) \} - \\
\{s + c\alpha_3 + \phi_1\} \{c\alpha_3 + \phi_2\} / \{(s + c\alpha_2 + \phi_2) - c\alpha_3\phi_1 / (s + 2\phi_2)\} \} / \\
\{s + c\alpha_2 + \phi_1\} - c\alpha_3\phi_1 / (s + 2\phi_2) - c^2\alpha_3^2\phi_1^2 / (s + 2\phi_2) \} / \\
\{s + c\alpha_3 + \phi_2\} - c\alpha_3\phi_2 / (s + 2\phi_2) \} - \\
\{s + c\alpha_3 + \phi_1\} - c\alpha_3\phi_1 / (s + 2\phi_2) - c^2\alpha_3^2\phi_1^2 / (s + 2\phi_2) \} / \\
\{s + c\alpha_3 + \phi_2\} - c\alpha_3\phi_2 / (s + 2\phi_2) \} / \\
\{s + c\alpha_3 + \phi_1\} - c\alpha_3\phi_1 / (s + 2\phi_2) - c^2\alpha_3^2\phi_1^2 / (s + 2\phi_2) \} / \\
\{s + c\alpha_3 + \phi_2\} - c\alpha_3\phi_2 / (s + 2\phi_2) \} / \\
\{s + c\alpha_3 + \phi_1\} - c\alpha_3\phi_1 / (s + 2\phi_2) - c^2\alpha_3^2\phi_1^2 / (s + 2\phi_2) \} / \end{pmatrix} \]

\[ A_4 = \begin{pmatrix} (s + d_1) - \frac{c\lambda_3\mu_3}{(s + d_3)} - \frac{c\mu_3 \lambda_2}{(s + d_1)} - \frac{c\lambda_2 \mu_2}{(s + d_2)} - \frac{(c\lambda_2 \mu_2)}{(s + \mu_2)} \end{pmatrix}, \quad A_6 = \begin{pmatrix} (s + d_2) - \frac{c\lambda_3 \mu_3}{(s + \mu_2)} \end{pmatrix} \]

\[ A_{ab} = \begin{pmatrix} (s + d_3) - \frac{c\lambda_2 \mu_2}{(s + \mu_2)} \end{pmatrix}, \quad A_i = \begin{pmatrix} (s + d_4) - \frac{c\lambda_3 \mu_3}{(s + d_3)} - \frac{c\mu_3 \lambda_4}{(s + d_1)} - \frac{c\lambda_3 \mu_1}{(s + d_6)} - \frac{(c\lambda_3 \mu_1)}{(s + \mu_2)} \end{pmatrix}, \]

\[ A_4 = \begin{pmatrix} (s + d_6) - \frac{c\lambda_4 \mu_4}{(s + d_5)} \end{pmatrix}, \quad A_6 = \begin{pmatrix} (s + d_5) - \frac{c\lambda_4 \mu_4}{(s + \mu_1)} \end{pmatrix} \]

\[ A_j = \begin{pmatrix} (s + d_7) - \frac{c\lambda_4 \mu_4}{(s + d_6)} - \frac{c\mu_4 \lambda_2}{(s + d_5)} - \frac{c\lambda_2 \mu_2}{(s + d_4)} - \frac{c\lambda_2 \mu_2}{(s + \mu_2)} \end{pmatrix}, \quad A_8 = \begin{pmatrix} (s + d_8) - \frac{c\lambda_2 \mu_2}{(s + \mu_2)} \end{pmatrix} \]
The Laplace transformations of the probabilities that the system is in the up (either good or degraded state) and down (failed) state at any time are as follows:

\[
P_{up}(s) = P_0(s) + P_1(s) + P_2(s) + P_4(s) + P_6(s) + P_7(s) + P_9(s) \\
+ P_{11}(s) + P_{12}(s) + P_{14}(s) + P_6(s) + P_{17}(s)
\]

\[
P_{down}(s) = P_3(s) + P_5(s) + P_6(s) + P_{10}(s) + P_{13}(s) + P_{12}(s) + P_{18}(s)
\]

(5.121)

(5.122)

5.5 Numerical Illustrations

5.5.1 Availability Analysis

(a) Model I

Taking the values of different parameters as \( \lambda = 0.35, \lambda_{ef} = 0.25, \lambda_n = 0.20, \mu(x) = \phi(z) = 1, c=0.2, 0.4, 0.6, 0.8, 0.9 \) [2, 60, 61, 70, 82] in Equation (5.26) and then taking the inverse Laplace transform, we get the availability of the system for different values of the coverage factor.

\[
A_{c=0.2}(t) = 0.01768130182e^{(-1.824055766t)} + 0.09128243267e^{(-1.322443551t)} \\
+0.04547413964e^{(-1.083653666t)} - 0.01448835145e^{(-0.7729286156t)} \\
+0.860050477e^{(-0.05691840151t)}
\]

(5.123)
\[ A_{c=0.4}(t) = 0.02553899906e^{-1.137081317t} + 0.05526355948e^{-1.422651163t} \]
\[ + 0.05733063204e^{-1.080727258t} - 0.02694893186e^{0.6700251150t} \]
\[ + 0.8888157414e^{-0.05951514693t} \]  
(5.124)

\[ A_{c=0.6}(t) = 0.03563466705e^{-2.407768841t} + 0.01832193710e^{-1.52118271t} \]
\[ + 0.06848322349e^{-1.082280941t} - 0.01920327909e^{-0.619564258t} \]
\[ + 0.8967634510e^{-0.0491855217t} \]  
(5.125)

\[ A_{c=0.8}(t) = 0.04705159347e^{-2.657934944t} + 0.01991661638e^{-1.618634675t} \]
\[ + 0.08008446769e^{-1.085602683t} + 0.008886570321e^{-0.5996467898t} \]
\[ + 0.8838939850e^{0.0281890050t} \]  
(5.126)

\[ A_{c=0.9}(t) = 0.05306158699e^{-2.778117345t} - 0.03943141214e^{1.667035737t} \]
\[ + 0.08627460551e^{-1.087706678t} + 0.02779023962e^{-0.5973753307t} \]
\[ + 0.8723049803e^{-0.01476501866t} \]  
(5.127)

Varying time scale \( t \) from 0 to 15 in Equations (5.123) to (5.127), one can obtain Table 5.1 and correspondingly Fig. 5.7, representing the behavior of availability of the system with respect to time.
### Table 5.1: Availability as function of time

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<th>Time (t)</th>
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**Fig. 5.7: Availability as function of time**
(b) Model II

Taking the values of different parameters as \( \lambda_d = 0.020, \lambda_c = 0.025, \lambda_h = 0.030, \lambda_j = 0.035, \phi_j(y) = 1, c = 0.1, 0.2, 0.3, 0.4, 0.5 \) \[2, 60, 61, 70, 82\]. Putting all these values in equation (5.61), then after taking the inverse Laplace transform and varying time unit \( t \) from 0 to 15, we obtain Table 5.2 and correspondingly Fig. 5.8 which represents the behavior of availability of the system with respect to time.

<table>
<thead>
<tr>
<th>Time ((t))</th>
<th>(A(t))</th>
<th>(c=0.1)</th>
<th>(c=0.2)</th>
<th>(c=0.3)</th>
<th>(c=0.4)</th>
<th>(c=0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.96267</td>
<td>0.96837</td>
<td>0.97409</td>
<td>0.97981</td>
<td>0.98555</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.93788</td>
<td>0.94878</td>
<td>0.95971</td>
<td>0.97069</td>
<td>0.98171</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.91771</td>
<td>0.93338</td>
<td>0.94915</td>
<td>0.96501</td>
<td>0.98096</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.89935</td>
<td>0.91946</td>
<td>0.93973</td>
<td>0.96017</td>
<td>0.98078</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.88179</td>
<td>0.90695</td>
<td>0.93056</td>
<td>0.95532</td>
<td>0.98034</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.86470</td>
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<td>0.92134</td>
<td>0.95019</td>
<td>0.97942</td>
<td></td>
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<td>7</td>
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<tr>
<td>9</td>
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<td>0.89291</td>
<td>0.93288</td>
<td>0.97366</td>
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<tr>
<td>10</td>
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<td>0.88322</td>
<td>0.92655</td>
<td>0.97087</td>
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<td>11</td>
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<td>0.82805</td>
<td>0.87344</td>
<td>0.91999</td>
<td>0.96772</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.76843</td>
<td>0.81536</td>
<td>0.86361</td>
<td>0.91323</td>
<td>0.96425</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.75334</td>
<td>0.80278</td>
<td>0.85375</td>
<td>0.90631</td>
<td>0.96050</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.73852</td>
<td>0.79031</td>
<td>0.84387</td>
<td>0.89924</td>
<td>0.95650</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.72397</td>
<td>0.77797</td>
<td>0.83398</td>
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<td>0.95228</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Availability as a function of time
(c) Model III

Taking the values of different parameters as $\lambda_1 = 0.20$, $\lambda_2 = 0.20$, $\lambda_3 = 0.20$, $\alpha_1 = 0.10$, $\alpha_2 = 0.20$, $\alpha_3 = 0.30$, $\mu_1 = \mu_2 = \mu_3 = \phi_1 = \phi_2 = \phi_3 = 1$, $c = 0.3, 0.5, 0.7, 0.9$ [2, 60, 61, 70, 82]. Putting all these values in (5.121) then taking the inverse Laplace transform, and varying time unit $t$ from 0 to 10, we get Table 5.3 and Fig. 5.9 respectively.

| Time($t$) | $A(t)$ | $c=0.3$ | $c=0.5$ | $c=0.7$ | $c=0.9$ |
|-----------|--------|---------|---------|---------|
| 0         | 1.00000| 1.00000 | 1.00000 | 1.00000 |
| 1         | 0.99844| 0.99579 | 0.99198 | 0.98710 |
| 2         | 0.99709| 0.99217 | 0.98517 | 0.97632 |
| 3         | 0.99644| 0.99038 | 0.98175 | 0.97090 |
| 4         | 0.99614| 0.98952 | 0.98008 | 0.96826 |
| 5         | 0.99600| 0.98910 | 0.97927 | 0.96696 |
| 6         | 0.99593| 0.98890 | 0.97886 | 0.96631 |
| 7         | 0.99590| 0.98880 | 0.97866 | 0.96597 |
| 8         | 0.99589| 0.98875 | 0.97855 | 0.96579 |
| 9         | 0.99588| 0.98873 | 0.97850 | 0.96570 |
| 10        | 0.99588| 0.98871 | 0.97847 | 0.96565 |

Table 5.3: Availability as a function of time
5.5.2 Reliability Analysis

(a) Model I

Setting the repair rates equal to zero in Equation (5.26), the reliability of the system is given as

\[
R(t) = \frac{1 + \frac{3c\lambda}{s + 2c\lambda + (1-c)\lambda_{csf}} + \frac{6c^2\lambda^2}{(s + c\lambda + (1-c)\lambda_{csf})(s + 2c\lambda + (1-c)\lambda_{csf})}}{s + 3c\lambda + c\lambda_{csf} + (1-c)\lambda_{csf}} \tag{5.128}
\]

Taking the values of failure rates and coverage factor as mentioned in section 5.5.1 (a), in Equation (5.128) and taking the inverse Laplace transform, we get the reliability of the system as

\[
R_{c=0.2}(t) = 0.5757575758e^{(0.45t)} + 2.333333333e^{(0.27t)} - 1.909090909e^{(0.34t)} \tag{5.129}
\]

\[
R_{c=0.4}(t) = 2.333333333e^{(0.29t)} + 0.5757575758e^{(0.65t)} - 1.909090909e^{(0.43t)} \tag{5.130}
\]

\[
R_{c=0.6}(t) = 2.333333333e^{(0.31t)} + 0.5757575758e^{(0.85t)} - 1.909090909e^{(0.52t)} \tag{5.131}
\]

\[
R_{c=0.8}(t) = 2.333333333e^{(0.33t)} - 1.909090909e^{(0.61t)} + 0.5757575758e^{(1.05t)} \tag{5.132}
\]
Setting time unit \( t = 0 \) to 15 in Equations (5.129) to (5.133), one can obtain the Table 5.4 and Fig. 5.10 correspondingly, which represent how the reliability varies as the time increases?

\[
R_{c=0.9}(t) = -1.9090909e^{-0.655t} + 0.5757575758e^{-0.15t} + 2.3333333333e^{-0.34t}
\]  
(5.133)

<table>
<thead>
<tr>
<th>Time (( t ))</th>
<th>( R_c(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c=0.2 )</td>
</tr>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td>0.78950</td>
</tr>
<tr>
<td>2</td>
<td>0.62665</td>
</tr>
<tr>
<td>3</td>
<td>0.49885</td>
</tr>
<tr>
<td>4</td>
<td>0.39757</td>
</tr>
<tr>
<td>5</td>
<td>0.31681</td>
</tr>
<tr>
<td>6</td>
<td>0.25222</td>
</tr>
<tr>
<td>7</td>
<td>0.20048</td>
</tr>
<tr>
<td>8</td>
<td>0.15906</td>
</tr>
<tr>
<td>9</td>
<td>0.12593</td>
</tr>
<tr>
<td>10</td>
<td>0.09949</td>
</tr>
<tr>
<td>11</td>
<td>0.07843</td>
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<td>0.06170</td>
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<td>14</td>
<td>0.03795</td>
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<tr>
<td>15</td>
<td>0.02968</td>
</tr>
</tbody>
</table>

Table 5.4: Reliability as function of time
Taking all repairs equal to zero in (5.61) and then taking the inverse Laplace transform, the numerical values of the reliability of the system as in Table 5.5 and graphically shown by Fig. 5.11. It is calculated by using the failure rates as 5.5.1 (b) and setting time unit $t = 0$ to 15.
<table>
<thead>
<tr>
<th>Time ( (t) )</th>
<th>Reliability ( R(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c=0.1 )</td>
</tr>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td>0.95198</td>
</tr>
<tr>
<td>2</td>
<td>0.90591</td>
</tr>
<tr>
<td>3</td>
<td>0.86174</td>
</tr>
<tr>
<td>4</td>
<td>0.81945</td>
</tr>
<tr>
<td>5</td>
<td>0.77897</td>
</tr>
<tr>
<td>6</td>
<td>0.74026</td>
</tr>
<tr>
<td>7</td>
<td>0.70328</td>
</tr>
<tr>
<td>8</td>
<td>0.66797</td>
</tr>
<tr>
<td>9</td>
<td>0.63427</td>
</tr>
<tr>
<td>10</td>
<td>0.60213</td>
</tr>
<tr>
<td>11</td>
<td>0.57150</td>
</tr>
<tr>
<td>12</td>
<td>0.54232</td>
</tr>
<tr>
<td>13</td>
<td>0.51454</td>
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<tr>
<td>14</td>
<td>0.48809</td>
</tr>
<tr>
<td>15</td>
<td>0.46293</td>
</tr>
</tbody>
</table>

Table 5.5: Reliability as a function of time

Fig.5.11: Reliability as a function of time
(c) Model III

Taking all repairs rates zero in (5.121), we have reliability expression of the system as:

\[
R(s) = \frac{1}{(c\lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2 + s)} \left( 1 + \frac{c\lambda_1}{(c\lambda_2 + c\lambda_3 + s)} + \frac{c\lambda_2}{(c\lambda_2 + c\lambda_3 + s)(c\lambda_2 + s)} + \frac{c\lambda_3}{(c\lambda_2 + c\lambda_3 + s)(c\lambda_2 + s)} \right)
\]

Let us fix the values of failure rates and coverage factors as mentioned in section 5.5.1 (c). Now by putting these values in equation (5.134) and after taking Inverse Laplace of and setting \( t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \), one can obtain Table 5.6 and Fig. 5.12, which represents how reliability varies as the time increases?

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>( R(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c=0.3 )</td>
</tr>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td>0.99620</td>
</tr>
<tr>
<td>2</td>
<td>0.98579</td>
</tr>
<tr>
<td>3</td>
<td>0.97001</td>
</tr>
<tr>
<td>4</td>
<td>0.94995</td>
</tr>
<tr>
<td>5</td>
<td>0.92651</td>
</tr>
<tr>
<td>6</td>
<td>0.90046</td>
</tr>
<tr>
<td>7</td>
<td>0.87245</td>
</tr>
<tr>
<td>8</td>
<td>0.84302</td>
</tr>
<tr>
<td>9</td>
<td>0.81265</td>
</tr>
<tr>
<td>10</td>
<td>0.78171</td>
</tr>
</tbody>
</table>

Table 5.6: Reliability as a function of time
5.5.3 MTTF Analysis

Taking all repairs to zero in $\bar{P}_{wp}(s)$ as $s$ tends to zero, one can obtain MTTF as;

$$MTTF = \lim_{s \to 0} \bar{P}_{wp}(s)$$ (5.135)

(a) Model I

Using (5.26) in (5.135), we have

$$MTTF = \frac{1 + \frac{3c\lambda}{2c\lambda + (1-c)\lambda_{csf}} + \frac{6c^2\lambda^2}{(c\lambda + (1-c)\lambda_{csf})(2c\lambda + (1-c)\lambda_{csf})}}{3c\lambda + c\lambda_h + (1-c)\lambda_{csf}}$$ (5.136)

Taking the value of different parameters as mentioned in section 5.5.1 (a) and varying $\lambda$, $\lambda_{csf}$, $\lambda_h$ respectively as 0.1 to 0.9, one may obtain the variation of MTTF with respect to failure rates as given in Table 5.7 and graphically shown in Fig. 5.13.
Table 5.7: MTTF as function of failure rates
(b) Model II

Using (5.61) in (5.135)

\[
MTTF = \frac{1 + \frac{\lambda_s}{\lambda_s + c\lambda_j + \lambda_c + (1-c)\lambda_c}}{\lambda_s + c\lambda_j + \lambda_c + (1-c)\lambda_c} + \frac{c\lambda_j}{\lambda_s + (1-c)\lambda_c} + \frac{c\lambda_j}{\lambda_s + (1-c)\lambda_c} 
\]

Taking the values of different parameters as mentioned in section 5.5.1 (b) and varying \( \lambda_s, \lambda_c, \lambda_h, \lambda_j \) one by one respectively as 0.1, 0.2, 0.3, 0.4, 0.05, 0.6, 0.7, 0.8, 0.9 in Equation (5.137), one may obtain the variation of MTTF with respect to failure rates as mentioned in Table 5.8 and Fig. 5.14 respectively.
Using (5.121) in (5.135), we have

\[
MTTF = \frac{1}{(c\lambda_1 + c\lambda_2 + c\lambda_3 + c\alpha_1 + c\alpha_2)} \left[ 1 + \frac{c\lambda_1}{(c\lambda_1 + c\lambda_2)} + \frac{c\lambda_1 c\lambda_2}{(c\lambda_1 + c\lambda_2)(c\lambda_3)} + \frac{c\lambda_1 c\lambda_2 c\lambda_3}{(c\lambda_1 + c\lambda_2)(c\lambda_3)(c\lambda_4)} + \frac{c\lambda_1 c\lambda_2 c\lambda_3 c\lambda_4}{(c\lambda_1 + c\lambda_2)(c\lambda_3)(c\lambda_4)(c\lambda_5)} \right]
\]

Taking the values of different parameters as mentioned in section 5.5.1 (c) and varying \(\lambda_1, \lambda_2, \lambda_3, \alpha_1, \alpha_2, \alpha_3\) one by one respectively as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 in Equation (5.138), one may obtain the variation of MTTF with respect to failure rates, Table 5.9 and corresponding Fig. 5.15.
### Table 5.9: MTTF as a function of failure rates

<table>
<thead>
<tr>
<th>Variation in failure rates</th>
<th>MTTF w.r.t. failure rates ($c=0.3$)</th>
<th>MTTF w.r.t. failure rates ($c=0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_1$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>0.7</td>
<td>8.547</td>
<td>8.547</td>
</tr>
</tbody>
</table>

**Note:** The table shows the MTTF (Mean Time To Failure) for different values of failure rates with two different constants ($c=0.3$ and $c=0.5$).
5.5.4 Expected Profit

(a) Model I

Using (5.123) to (5.127) in (3.46), the expected profits for the same set of parameters as mentioned in section 5.5.1 (a) are given as

\[
E_{P_{\text{vol.2}}}(t) = (-0.009693399812e^{(-1.824055766t)} - 0.06902557966e^{(-1.322443551t)})
\]

\[-0.04196372057e^{(-0.10836953666t)} + 0.01874474713e^{(0.7729286156t)}
\]

\[-15.11023597e^{(0.05691840151t)} + 15.21217392)K_1-tK_2 \]

(5.139)
\[ E_{P=0.4}(t) = (-0.01195041052e^{-1.237081317} - 0.3884547450e^{-1.422651163}) \\
-0.05304819659e^{-1.080727258} + 0.04022077868e^{-0.6700251150}) \\
-14.93427786e^{0.8595151463} + 14.99790116)K_1 - rK_2 \]  
(5.140)

\[ E_{P=0.6}(t) = (-0.01479987050e^{-2.407768841} - 0.0120449012e^{-1.521188271}) \\
-0.06327616808e^{-1.082209941} + 0.03099470580e^{-0.6195664258}) \\
-18.23226490e^{-0.4918552117} + 18.29139072)K_1 - rK_2 \]  
(5.141)

\[ E_{P=0.8}(t) = (-0.01770231193e^{-2.657934944} + 0.01230457786e^{-1.618634675}) \\
-0.07376959264e^{-1.085602683} - 0.01481967464e^{-0.5996467898}) \\
-31.36499305e^{-0.0281890805} + 31.45898005)K_1 - rK_2 \]  
(5.142)

\[ E_{P=0.9}(t) = (-0.01909983647e^{-2.788117345} + 0.02365360938e^{-1.667035737}) \\
-0.07931790419e^{-1.087706570} - 0.04652056788e^{-0.597353307}) \\
-59.07916545e^{-0.0147650186} + 59.20045015)K_1 - rK_2 \]  
(5.143)

Setting \( K_1 = 1 \) and \( K_2 = 0.1, 0.3, 0.6 \), respectively in Equations (5.139) to (5.143), one can get the Table 5.10 and corresponding Fig. 5.16.

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>( c=0.2 )</th>
<th>( c=0.4 )</th>
<th>( c=0.6 )</th>
<th>( c=0.8 )</th>
<th>( c=0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.812</td>
<td>0.612</td>
<td>0.312</td>
<td>0.818</td>
<td>0.618</td>
</tr>
<tr>
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<td>1.121</td>
<td>0.521</td>
<td>1.531</td>
<td>1.131</td>
</tr>
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<td>0.772</td>
<td>2.208</td>
<td>1.608</td>
</tr>
<tr>
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<td>2.778</td>
<td>1.978</td>
<td>0.778</td>
<td>2.829</td>
<td>2.029</td>
</tr>
<tr>
<td>5</td>
<td>3.344</td>
<td>2.344</td>
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<td>2.408</td>
</tr>
<tr>
<td>6</td>
<td>3.873</td>
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<td>3.948</td>
<td>2.748</td>
</tr>
<tr>
<td>7</td>
<td>4.367</td>
<td>2.967</td>
<td>0.867</td>
<td>4.452</td>
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<td>8</td>
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<td>9</td>
<td>5.259</td>
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<td>0.799</td>
<td>5.357</td>
<td>3.557</td>
</tr>
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</table>

Table 5.10: Expected profit as a function of time
(b) Model II

Using availability equation, in (3.46) expected profit for the same set of parameters as mentioned in section 5.5.1 (b), is given by setting $K_1 = 1$ and $K_2 = 0.1, 0.3, 0.6$ respectively, one gets Table 5.11 and Fig. 5.17 respectively.
<table>
<thead>
<tr>
<th>Time(t)</th>
<th>$c=0.1$</th>
<th>$c=0.2$</th>
<th>$c=0.3$</th>
<th>$c=0.4$</th>
<th>$c=0.5$</th>
</tr>
</thead>
<tbody>
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<td>$K_2=0.1$</td>
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<td>$K_2=0.6$</td>
<td>$K_2=0.1$</td>
<td>$K_2=0.3$</td>
<td>$K_2=0.6$</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>0.67969</td>
<td>0.37969</td>
<td>0.88259</td>
<td>0.68259</td>
</tr>
<tr>
<td>2</td>
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<td>1.32938</td>
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Table 5.11: Expected profit as a function of time
(c) Model III

Using availability expressions in (3.46) of this model, for different values of $c$, expected profit for the same set of parameters as mentioned in section in section 5.5.1 (c) one can get Table 5.12 and Fig. 5.18 respectively.

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Table 5.12: Expected profit as function of time

Fig. 5.17: Expected profit as a function of time
5.6 Result Discussion

In this chapter, availability, reliability, MTTF and expected profit for the considered systems by using the coverage factor and employing Markov process have been evaluated. From the results as received from the analysis of the designed systems, it is concluded that:

(a) Model I

Fig. 5.7 shows the effect of the coverage factor on the availability of the system with respect to time. Critical examination of Fig. 5.7 yields that the values of the availability, decrease approximately in a constant manner with the increment in time. Furthermore, with the increment of coverage factor availability of the system increases.

The numerical value of reliability is always between one and zero. It is a non-increasing function between these limits. Fig. 5.10 shows the trends of reliability of the system with respect to the time when all the failures have some fixed values. From the graph, it is concluded that the reliability of the system decreases with the passage of time sharply. Moreover, with increment in the coverage factor reliability of the system increases.

Because of the model used, the MTTF of the system decreases with increment in catastrophic, human and unit failure. Also, from Fig. 5.13, it is observe that MTTF increases with increment in the coverage factor.
Fig. 5.16 represents the graph of the expected profit versus time. From the analysis of the graph it is concluded that increasing service cost leads decrement in expected profit. In the context of the coverage factor, expected profit increases with increment in coverage factor.

(b) Model II

The effect of the coverage factor on the availability of the system has been shown in Fig. 5.8. From the figure, it is observed that the availability of the system decreases as the time increases. Also with the increment in the coverage factor the availability of the system increases.

Fig. 5.11 shows the trends of reliability of the system with respect to the time when all the failure and repair rates have some fixed values. From the graph, it is concluded that the reliability of the system decreases approximately in a constant manner with increment in time. Also, the reliability of the system increases as the coverage factor increases.

From Fig. 5.14, it is observed that the MTTF of the system increase with respect to the failure of the \( j^{th} \) unit of subsystem \( B \) and decreases with respect to the failure rate of main and standby unit of subsystem \( A \), human failure and catastrophic failure. From this it is concluded that failure of subsystem \( A \), human failure and catastrophic failure much more controlled during the system operation while it seems to be uncontrolled with respect to failure of any \( j^{th} \) unit of subsystem \( B \). Also with the increment in coverage factor MTTF of the system increases.

Keeping the revenue cost per unit time at value 1 with coverage factor values at 0.1, 0.2, 0.3, 0.4, 0.5 and varying service cost as 0.1, 0.3 and 0.6 table 5.11 has been obtained. By critical examination of Fig. 5.17 it is observed that increasing service cost leads decrement into expected profit. Also the Fig. 5.17, shows that expected profit increases with increment in coverage factor.

(c) Model III

Table 5.3 shows availability of the stated system with respect to time \( t \). Critical examination of corresponding Fig. 5.9 yields that the values of the availability decreases approximately in an even manner with the increment in time.

Fig. 5.12 shows the trends of reliability of the designed system with respect to the time when all the failure and repair rates have some fixed values. From the graph, it is concluded that the
reliability of the system decreases more sharply with the passage of time when all repairs follow an exponential time distribution. Reliability can be improved by introducing duplicate paths at the component or sub-system level, simplicity of expression, proper design, and other familiar means. However, item analysis is measured as the most effective way to increase reliability and performance of the system.

Fig. 5.15 shows that MTTF of the above stated system with respect to various failure rates. A critical examination of graph shows that the MTTF decreases with increment in $\lambda_1, \lambda_2, \lambda_3, \alpha_1, \alpha_2,$ and $\alpha_3$. It is also concluded that MTTF of the system decreases with the increment in coverage factor.

Fig. 5.18 represents the cost function vs. time. Here, it is observed that increasing service cost leads decrement into expected profit. The results show that minimum service cost leads to maximum expected profit on the other hand maximum service cost leads to minimum profit. From this work it is concluded that by controlling service cost, high profit could be attained.

### 5.7 Conclusion

The Markov process based approach for reliability evaluation of a multi-state system with combinatorial performance requirements subject to imperfect fault coverage has been discussed within model I. Although, the previous authors have developed the various models for complex systems with imperfect fault coverage, but they did not include human failure in their research, which is an essential factor in reference to the failure of any system.

Model II is the analysis of a two unit standby system with three types of failures incorporating coverage factor. In the context of this model, previously numerous mathematical models have been developed for a parallel repairable system with degradation and common cause failure. But in this work, various reliability measures with human and catastrophic failure, incorporating coverage factor have been evaluated. The results indicate that the performance of the stated system is different from those of the system without coverage.

The model III analyzed the effect of coverage factor and deterioration in a complex system. In context to this model, previous authors have analysed the reliability of such type of systems under the concept of deterioration, but they did not analyze the impact of perfect coverage on such systems.
The present study shows that incorporation of fault detection technique (coverage factor) is beneficial for excellence in the availability of the system, reduction of the cost