Chapter 1
Introduction

Cosmology is the scientific study of origin and evolution of the universe. In modern sense, it deals with the study of overall dynamical and physical behaviour of billions of galaxies spread across vast distances and of the evolution of this enormous system over several billion years [1]. Modern cosmology began when Einstein’s general theory of relativity applied to understand the universe [2]. A remarkable progress in understanding of the universe has been made over several years since Friedmann obtained the solutions to the Einstein field equations [2]. When combined the Friedmann solutions with the Hubble redshift distance relation, it is interpreted that we live in an expanding universe [3]. Thus extrapolate back in time, investigators deduced that the universe emerged with great explosion, from unbelievable dense, hot region, popularly known as big bang. The hot big bang or standard cosmology [3] is considered as a very successful theory to describe many observed features of the universe [4]. One of the strong predictions of hot big bang model is the existence of a particular kind of radiation that would preserve its black body form even today, known as the cosmic microwave background radiation.

The discovery of the microwave background radiation [5, 6], together with the fact that the abundances of Helium isotopes and Deutirium has led to the wide acceptance of the standard cosmology. The standard cosmology is
spectacularly successful. It provides reliable and tested account of the history of the universe from about 0.01 sec after the big bang until today, some 15 billions years later. Thus in short, the primary pieces of evidence that support the standard model of cosmology are the expansion of the universe, the cosmic microwave background radiation and abundance of the light elements.

Despite its success, the hot big bang model leaves many features of the universe unexplained. The most important of these are horizon problem, singularity problem, flatness problem, homogeneity problem, structure formation problem, monopole problem and so on. All these problems are very difficult and defy solutions within the standard cosmology theory. Most of these problems have, in the past couple of decades, been either completely resolved or considerably relaxed in the context of one complete scenario, called the inflationary scenario \[7\]. At present there are several versions \(7, 8, 9\) of the inflationary model, called the inflationary paradigm. According to the simplest version of the inflationary scenario \(10\), the universe in the past expanded almost exponentially with time, while its energy density was dominated by the effective potential energy density of a homogeneous scalar field, called the inflaton. Just after the inflationary period the universe was devoid of particles, that is, cold. Therefore a mechanism is needed to understand that how the universe reheated after the inflation because the temperature of universe was insufficient for the nucleosynthesis and moreover the universe was devoid of particles.

It is believed that at the end of the inflation, the inflaton field started quasi-periodic motion with slowly decreasing amplitude and that led to creation of particles of various kinds, after thermalization of which due to collisions and decays, the universe became hot again. Thus, the temperature of the universe raised to the extent that it was sufficient to trigger thermonuclear reactions. From then on, it can be described by the usual hot big bang theory. Therefore the oscillatory phase of the inflaton and its related issues are important to understand particle creation and further evolution of the
Most of the inflationary scenario and related issues are described on the basis of classical Friedmann equations with scalar field on the Friedmann-Robertson-Walker (FRW) universe, assuming its validity even at very early stage of the universe. However, it is believed that quantum effects of matter fields and quantum fluctuations played significant role in the early universe, though quantum gravity effects were negligible. Therefore, the proper description of a cosmological model can be studied in terms of the semiclassical gravity of the Friedmann equations with quantized matter fields as source for gravity. Recently, study of inflation in the semiclassical theory of gravity has received much attention [11, 12]. There are works in which quantum properties of the inflaton were investigated in the inflationary scenario. In the new inflation scenario quantum effects of the inflaton were partially taken into account by using one-loop effective potential with an initial thermal condition [13]. In the stochastic inflationary [14] scenario the inflaton is studied quantum mechanically by dealing with the phase-space quantum distribution function and the probability distribution [15]. The semiclassical quantum gravity seems to be a viable method throughout the whole non-equilibrium quantum process from the pre-inflation period of hot plasma in thermal equilibrium to the inflation period and finally to the matter dominated era.

The studies aforementioned show that results obtained in classical gravity are quite different from those in semiclassical gravity. Even though both classical and quantum inflaton in the oscillatory phase leads to the same power law expansion, the correction to expansion does not show any oscillatory behaviour in semiclassical gravity in contrast with the oscillatory behaviour seen in classical gravity. It is to be noted that coherently oscillating inflaton suffers from particle production. Such studies revealed that quantum effects played role in the inflation and its related phenomenon. Recently, it has been found that nonclassical state formalisms of quantum optics, such as coherent and squeezed states, are quite useful to deal with quantum effects in
cosmology [16] - [27]. At a glance, it may appear that coherent and squeezed state formalisms and cosmology are two different branches of Physics having no connections. However, the mathematical and physical properties of these states find much use to deal many quantum issues in cosmology. The squeezed and coherent states are being used as probes for studying quantum effects in cosmology such as cosmological particle creation [17], inflationary scenario [23], entropy generation [28], detection of gravitational waves [29], etc., [30]-[32]. It is believed that the relic graviton and other primordial perturbations created from zero-point quantum fluctuations, in the process of cosmological evolution, should be now in a strongly squeezed states [17]. The squeezed vacuum states under consideration are the many particle states and hence the resulting field can be called classical, but the statistical properties of the field differ greatly from those of the corresponding coherent state and from that point of view, the produced field is highly nonclassical. In the present work we have made use of the squeezed state formalism to calculate the expectation value of the energy momentum tensor of the scalar field. The vacuum expectation values of the energy momentum tensor defined prior to any dynamics in the gravitational field give us all the information about the particle production and vacuum polarization and hence it may be argued that the squeezed state representation of the scalar field can account for particle creation.

We examine particle production in the semiclassical Friedmann equation and obtain their solutions in the aforementioned states in the flat, open and closed FRW cosmological models. Also, we study density fluctuations and hence examine validity of the semiclassical theory in the oscillatory phase of inflaton in the coherent and squeezed states. Since, we use the quantum optical states to study the inflaton, it would be useful to study its classical or nonclassical nature in the cosmological context. In quantum optics context such study is carried out by using a quantity known as the Mandel’s $Q$ parameter [33]. The $Q$ parameter is a measure to examine classical or
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The nonclassical nature of the field under investigation.

In the semiclassical approach of gravity, the back reaction can play an important role in determining the dynamics of the universe [34]. Thus the back reaction also be taken into account where matter field treated as quantum mechanical, even though the background metric is classical. Many authors have studied the back reaction problem in the semiclassical gravity [35] - [37]. However, we have not included the effects of back reaction in the present work.

1.1 Motivation and Thesis Outline

At the end of inflation the universe became cold. The temperature was not sufficient to trigger thermonuclear reactions or nucleosynthesis. So, we need a mechanism to understand, how the temperature of the universe enhanced to initiate nucleosynthesis process. One of the mechanisms that reheat the universe after the inflation is due to particle production and its related phenomenon. After the inflation substantial amount of particles production occurred that helped to repopulate the universe with matter and radiation. The produced particles moved randomly and collided among themselves. Due to those collisions the temperature raised in such a way that the universe became hot again and nucleosynthesis started. Therefore, end of the inflationary period is crucial for the further evolution of the universe because the temperature attained after the reheating was not too large and not too small. Thus, study of the oscillatory phase of inflaton and its associated phenomenon are important to understand the universe.

Usually, the classical Friedmann equations with unquantized homogeneous scalar field are used to describe the inflation and its related phenomenon. Quantum effects in such model can be investigated in terms of the semiclassical Friedmann equations with the quantized inflaton. Such studies show that corresponding results are quite different from its classical
counterpart. One interesting issue related to these kind of studies is particle creation due to quantum effects [38, 39]. Thus, the present work is to study the oscillatory phase of the inflaton after inflation in semiclassical theory of gravity by representing the inflaton in the coherent and squeezed state formalisms of the quantum optics. We examine, particle creation in the coherent and squeezed states, in the semiclassical theory of gravity for the flat FRW universe. We, also study density fluctuations and validity of the semiclassical theory in the quantum optical states, during the oscillatory phase of inflaton. We examine classical or nonclassical behaviour of the inflaton in the coherent and squeezed state formalisms, with a quantity called the cosmological $Q$ parameter. Finally, we study the oscillatory phase of nonclassical inflaton and its related issues in the thermal counterparts of the coherent and squeezed states.

The primary aim of present study is to consider the flat Friedmann model of the universe in the oscillatory phase of the quantized inflaton in the semiclassical theory of gravity. However the solutions to the Friedmann equations imply the open and closed FRW universe also. Therefore it is interesting to see how the semiclassical gravity and its related phenomenon play role in determining dynamics of the open and closed FRW models. Thus, the general goal of the present work is to study a massive minimal nonclassical scalar field in the flat, open and closed FRW universe by representing it in terms of the coherent and squeezed states formalism of quantum optics as well as in their thermal counterparts.

In chapter 2, we present a brief discussion of the Einstein field equations. Scalar field in curved spacetime and the basic mechanism of particle creation for the quantized scalar field is explained briefly. The basic setup of the scalar field in the FRW models is also discussed. In the last section of the chapter 2, quantum optical states such as the coherent and squeezed state formalisms are introduced and their basic properties are also discussed briefly, in view of application of these states in the cosmological context.
Chapter 3, describes the semiclassical Friedmann equations and their solutions in the flat, open and closed FRW universe models in terms of representation of the scalar field in the coherent and squeezed state formalisms. A comparative study of the solutions in the three different models of the universe is also done.

Chapter 4, contains the study of the particle production of the nonclassical inflaton during its oscillatory phase in the flat FRW universes in the coherent and squeezed state formalisms. The chapter contains the study of particle creation in the other two Friedmann models and a comparative discussion of it with the flat FRW model is also carried out.

In chapter 5, we examine validity of the semiclassical Friedmann equation during the oscillatory phase of inflaton. This is done by computing the density fluctuations of energy momentum tensor in the semiclassical theory of gravity, in the coherent and squeezed states with the help of a dimensionless quantity. The chapter also deals with the cosmological $Q$ parameter to examine classical or nonclassical nature of inflaton field, in the coherent and squeezed states.

Chapter 6, studies solutions of the semiclassical Friedman equation in the thermal coherent, squeezed vacuum and squeezed states in the three FRW models of the universe. We compare the solutions of the closed, open and flat FRW models of the universe in the thermal nonclassical states. We study, particle production at the end of inflation in the oscillatory regime in the semiclassical theory of gravity by using the thermal coherent and squeezed state formalisms. Density and quantum fluctuations are also studied in the thermal coherent and squeezed vacuum states. Using the density fluctuations, we examine validity of the semiclassical theory of gravity, in the flat FRW universe. A comparative study of classical gravity with semiclassical gravity in the thermal coherent state is also presented.

In chapter 7, we summarize the results.