Chapter 5

Projected properties of triaxial modified Hubble mass model: inclusion of extra radial functions with spherical harmonics of second orders

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Summary Triaxial modified Hubble mass models are studied. These models are flattened versions of the spherical modified Hubble model and are constructed by adding additional radial functions, multiplied by spherical harmonics of second order. The models are more general than those studied by earlier workers. The potential and the projected surface density of the mass models can be calculated analytically. Profiles of ellipticity, position angles and correlation between axial ratios in asymptotic radii are studied. The inclusion of additional terms in density function, allows one to produce a large variety of these profiles, in particular, profiles with small scale variations, which can be compared with observations. We present the profiles of elliptical galaxy NGC 661 for comparison, with our model.
5.1 Introduction

Triaxial modified Hubble mass models were proposed by Schwarzschild (1979) and cast into analytical form by deZeeuw and Merritt (1983). Following the approach of deZeeuw and Carollo (1996), Chakraborty and Thakur (2000, hereafter CT00), studied the projected properties of these models. The triaxial models of these type shows ellipticity variations and isophotal twists in their projections. On projecting these models the radial profiles of the parameters of the elliptical isophotes are found to be smooth functions of the radius. But, many elliptical galaxies, devoid of any features, indicating the absence of shells or dust, are found to exhibit small scale variations in the radial profiles of the parameters of the elliptical isophotes. It indicates that density function of such galaxies are more complex than that studied by CT00. With this aim in mind we modified the models of CT00 by adding extra radial terms to the functions each multiplied by second order spherical harmonics and studied the projected properties of the model. The potential and the projected surface density $\Sigma$ of the resultant mass model can be calculated analytically. This makes us possible to investigate some of the projected properties analytically. We (Das & Chakraborty, 2001) calculated the profiles of the surface density $\Sigma$, the axis ratio $b/a$ and the position angles $\Theta_\ast$ of the major axis as functions of radial distance. These can be compared with photometric data of real galaxies. The analytical expressions of the surface density is very useful for the calculations of $b/a$ and $\Theta_\ast$. In the asymptotic regions analytical expressions can be derived, while in intermediate regions simple numerical methods can be adopted for these calculations. The inclusion of large number of terms in the density function, allows us to produce a large variety of the profiles of projected parameters.

Another purpose which this resultant model can serve is in building up of ensembles of model for the study of intrinsic shapes of triaxial mass models. It was shown by Thakur & Chakraborty (2001a, b, hereafter TC01a,b) that intrinsic shapes of triaxial mass models can be estimated using photometric data only and results of shape estimation would be insensitive to the choice of the models, if one consider ensembles of models. So, before applying their method to real elliptical galaxies for constraining shapes, one has to build a large ensembles of models showing isophotal twists and ellipticity variations.

In section 5.2, we describe the mass model and in section 5.3, we present the projected properties. Section 5.4 is devoted to results and a discussion.
5.2 The mass model

We modified the triaxial potential of the fgh model (de Zeeuw and Merritt, 1983)

\[ V(r, \theta, \phi) = u(r) - v(r)Y_2^0(\theta) + w(r)Y_2^2(\theta, \phi) \] (5.1)

by adding \( v_e(r) \) and \( w_e(r) \) respectively to \( v(r) \) and \( w(r) \). The modified form of potential is

\[ V(r, \theta, \phi) = u(r) - (v + v_e)(r)Y_2^0(\theta) + (w + w_e)(r)Y_2^2(\theta, \phi) \] (5.2)

where \((r, \theta, \phi)\) are usual spherical coordinates defined such that \( x = r \sin \theta \cos \phi \), \( y = r \sin \theta \sin \phi \) and \( z = r \cos \theta \) with the functions \( Y_2^0(\theta) = \frac{3}{2} \cos^2 \theta - \frac{1}{2} \) and \( Y_2^2(\theta, \phi) = 3 \sin^2 \theta \cos 2\phi \) are usual spherical harmonics and \( u(r) \), \( v(r) \), \( v_e(r) \), \( w(r) \) and \( w_e(r) \) are five radial functions. We take \( u(r) \) to be the potential of the spherical modified Hubble mass model, defined by

\[ u(r) = -GM\frac{\ln[r + \sqrt{b_0^2 + r^2}]}{r} \] (5.3)

where \( M \) is the mass of the model and \( b_0 \) is the scale length. We choose \( v(r) \), \( w(r) \) same as adopted in (de Zeeuw and Merritt, 1983) reproduced below with slightly different notations.

\[ u(r) = -GM \frac{b_0^3 r^2}{(b_0^2 + r^2)^{3/2}} \]
\[ w(r) = -GM \frac{b_0^3 r^2}{(b_0^2 + r^2)^{3/2}} \] (5.4)

In chapter 4, we presented a modification of potential (5.1), by including additional radial functions multiplied by fourth order spherical harmonics. We chose forms of radial functions which lead to positive radial functions, at all \( r \), in density \( \rho \). Here, we proceed with a different motivation. The extra radial functions \( v_e \) and \( w_e \) are chosen such that the potential and the density remains same as in the original fgh model, at asymptotic radii. These relations are also re derived below, in a slightly different form.

A suitable choice of \( v_e \) and \( w_e \) are

\[ v_e(r) = -GM \frac{a_1^4 r^6}{(a_1^2 + r^2)^4} \]
\[ w_e(r) = -GM \frac{a_2^4 r^6}{(a_2^2 + r^2)^4} \] (5.5)

We note that \( v(r) \) and \( w(r) \) go as \(-r^2\) at small radii and as \(-1/r\), at large radii. On the other hand, \( v_e \) and \( w_e \) go as \(-r^6\) at small radii and as \(-1/r^2\) at large radii.
From Poisson's equation the associated density distribution $\rho(r, \theta, \phi)$ for the above potential is

$$\rho(r, \theta, \phi) = f(r) - (g + g_\text{e})(r)Y_2^0(\theta) + (h + h_\text{e})(r)Y_2^1(\theta, \phi)$$  \hspace{1cm} (5.6)$$

where

$$f(r) = \frac{M}{4\pi} \frac{1}{(b_0^2 + r^2)^{3/2}},$$
$$g(r) = \frac{3M}{4\pi} \frac{b_1}{b_0^3} 2r^4 + 7b_2^2 r^2,$$
$$g_\text{e}(r) = \frac{M}{4\pi} \frac{4a_1^4}{b_0^3} r^6 + 12a_2^6 + 9a_3^4 r^4,$$
$$h(r) = \frac{3M}{4\pi} \frac{b_2^3}{b_0^3} 2r^4 + 7b_2^2 r^2,$$
$$h_\text{e}(r) = \frac{M}{4\pi} \frac{4a_4^6}{b_0^3} r^8 + 12a_5^{10} + 9a_6^{12} r^4.$$

where $b_1, ... , b_4$ and $a_1, ..., a_4$ are all constants. The radial dependence of the extra terms $g_\text{e}, h_\text{e}, v_\text{e}$ and $w_\text{e}$ are such that they are effective in intermediate range of $r$ only and not at small and at large radii. At large radii $g_\text{e}, h_\text{e}$ decrease as $r^{-4}$ whereas $g(r), h(r)$ decrease as $r^{-3}$. Likewise, at small radii, $g_\text{e}$ and $h_\text{e}$ decrease as $r^4$ whereas $g$ and $h$ goes as $r^3$. Therefore, at small and at large radii, the terms $f(r), g(r)$ and $h(r)$ are dominating and the constant $b_1, b_2, b_3, b_4$ appearing in the radial functions are expressed in terms of axis ratios of the approximate ellipsoidal constant $\rho$ surfaces, at small and at large radii in the same way as in fgh model. Note that $g_\text{e}(r)$ and $h_\text{e}(r)$ may become negative for large values of $a_2$ and $a_4$. Therefore, the parameters $a_1, ..., a_4$ are chosen such that $|g_\text{e}(r)| \ll |g(r)|$ and $|h_\text{e}(r)| \ll |h(r)|$, at all $r$. We have chosen $a_1, a_3 \sim 0.1b_0$ and $a_2, a_4 \sim 10b_0$ for the study of projected properties.

The value of axis ratios of constant $\rho$ surfaces at very large and at very small radii can be expressed as $(p_{\infty}, q_{\infty})$ and $(p_0, q_0)$ respectively. At very large $r$, constant density $\rho$ surfaces can be written as

$$K \left( x^2 + y^2 + z^2 \right)^{5/2} = z^2 \left( 1 - A \right) + \left( 1 + A/2 + 3B \right) x^2 +$$
$$+ \left( 1 + A/2 - 3B \right) y^2,$$

where $\rho = K = \text{constant},$

$$A = 6 \left( \frac{b_1}{b_0} \right)^3,$$
$$B = 6 \left( \frac{b_2}{b_0} \right)^3.$$  \hspace{1cm} (5.8)\hspace{1cm} (5.9)
If \( a, \ b, \ c \) are the intercepts of this ellipsoid at \( x, y, z \) axis respectively, then
\[
K_a^3 = \left(1 + \frac{A}{2} + 3B\right),
\]
\[
K_b^3 = \left(1 + \frac{A}{2} - 3B\right),
\]
\[
K_c^3 = (1 - A),
\]
(5.10)
and, therefore, axis ratios of constant \( \rho \) surface at very large \( r \) i.e., \((p_{\infty}, q_{\infty})\) are given by
\[
p_{\infty}^3 = \left(\frac{b}{a}\right)^3 = \frac{1 + \frac{2}{3} - 3B}{1 + \frac{2}{3} + 3B},
\]
\[
p_{\infty}^3 = \frac{1 + 3\left(\frac{b_1}{b_3}\right)^3 - 18\left(\frac{b_2}{b_3}\right)^3}{1 + 3\left(\frac{b_1}{b_3}\right)^3 + 18\left(\frac{b_2}{b_3}\right)^3},
\]
(5.11)
and
\[
q_{\infty}^3 = \left(\frac{c}{a}\right)^3 = \frac{1 - 6\left(\frac{b_1}{b_3}\right)^3}{1 + 3\left(\frac{b_1}{b_3}\right)^3 + 18\left(\frac{b_2}{b_3}\right)^3}.
\]
(5.12)
Solving the above equations (5.11) and (5.12) for \((b_1/b_o)\) and \((b_3/b_o)\) we find
\[
6 \left(\frac{b_1}{b_o}\right)^3 = \frac{(1 + p_{\infty}^3 - 2q_{\infty}^3)}{(1 + p_{\infty}^3 + q_{\infty}^3)},
\]
\[
6 \left(\frac{b_3}{b_o}\right)^3 = \frac{(1 - p_{\infty}^3)}{2(1 + p_{\infty}^3 + q_{\infty}^3)}.
\]
(5.13)
Similarly at very small value of \( r \), the constant density \( \rho \) surfaces can be written as
\[
K = a^2 \left\{ -\frac{1}{2} - 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 + \frac{1}{2} \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 + 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right\}
\]
\[
+ b^2 \left\{ -\frac{1}{2} + 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 - 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right\}
\]
\[
+ c^2 \left\{ -\frac{1}{2} - 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 \right\}.
\]
(5.14)
If \( a, \ b, \ c \) are intercept of this ellipsoid at \( x, y, z \) axes respectively, then
\[
K = a^2 \left\{ -\frac{1}{2} + 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 + 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right\},
\]
\[
K = b^2 \left\{ -\frac{1}{2} + 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 - 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right\},
\]
\[
K = c^2 \left\{ -\frac{1}{2} - 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 \right\}.
\]
(5.15)
Therefore axis ratios of constant \( \rho \) surface at very small \( r \) i.e., \((p_o, q_o)\) are given by
\[
p_o^2 = \frac{\left[ -\frac{1}{2} + 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 + 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right]}{\left[ -\frac{1}{2} + 7 \left(\frac{b_o}{b_2}\right)^2 \left(\frac{b_1}{b_2}\right)^3 - 21 \left(\frac{b_o}{b_4}\right)^2 \left(\frac{b_3}{b_4}\right)^3 \right]},
\]
(5.16)
5.3 PROJECTED PROPERTIES

5.3.1 Projected surface density

Projected surface density $\Sigma$ of the model of form (5.6) is calculated by projecting the model along the line of sight. We adopted the procedure of ZC96 and CT00 for calculating the projected surface density $\Sigma$. Projected surface density of the model is found analytically. While projecting we adopted the convention of de Zeeuw & Franx (1989) [see chapter 2] and choose coordinates $(x', y', z')$ such that $z'$-axis runs along the line-of-sight, and $x'$-axis lies in the $(x, y)$-plane. We considered $(\theta', \phi')$ as the standard spherical coordinates of the line-of-sight and $(R, \Theta)$ as the polar coordinates in $(x', y')$-plane. Then the projected surface density $\Sigma(x', y')$ or $\Sigma(R, \Theta)$ is obtained by integrating the model density (5.6) along the line-of-sight, i.e., $z'$-axis,

$$\Sigma(R, \Theta) = \int_{-\infty}^{+\infty} \rho dz'.$$

By means of equations (2.4) and (2.5), we have transformed the galaxy coordinate $(x, y, z)$ to observers coordinate $(x', y', z')$ and integrated over $z'$, we found that the projected surface density $\Sigma(R, \Theta)$ can be written as

$$\Sigma(R, \Theta) = \Sigma_0(R) + \Sigma_2(R) \cos 2(\Theta - \Theta_*),$$

where

$$\Sigma_0(R) = 2P_1 + \left(1 - 3 \cos^2 \theta\right) \left[(G_1 + G_{1e}) - \frac{3}{2}(G_2 + G_{2e})\right] +$$

$$+ [6(H_1 + H_{1e}) - 9(H_2 + H_{2e})] \sin^2 \theta' \cos 2\phi',$$

$$\Sigma_2(R) = \left[6(H_2 + H_{2e}) \cos \theta' \sin 2\phi\right]^2 + \left[\frac{3}{2}(G_2 + G_{2e}) \sin^2 \theta' -$$

$$- 3(H_2 + H_{2e}) \left(1 + \cos^2 \theta'\right) \cos 2\phi\right]^2.$$
We have defined the integrals
\[
G_1(R) = \int_R^\infty \frac{r \, g(r) \, dr}{\sqrt{r^2 - R^2}},
\]
\[
G_{1e}(R) = \int_R^\infty \frac{r \, g_e(r) \, dr}{\sqrt{r^2 - R^2}},
\]
\[
G_2(R) = R^2 \int_R^\infty \frac{g(r) \, dr}{r \sqrt{(r^2 - R^2)}},
\]
\[
G_{2e}(R) = R^2 \int_R^\infty \frac{g_e(r) \, dr}{r \sqrt{(r^2 - R^2)}},
\]
and similarly for \( F_1, H_1, H_{1e}, H_2 \) and \( H_{2e} \) in terms of functions \( f(r), h(r) \) and \( h_e(r) \) respectively.

The major axis of the projected surface density (5.19) is at the position angle \( \Theta_a \), given by
\[
\tan 2\Theta_a = \frac{T \, h_3}{h_1 + (1 - T) \, h_2},
\]
where
\[
h_1 = \sin^2 \phi' - \cos^2 \phi' \, \cos^2 \theta',
\]
\[
h_2 = \cos^2 \phi' - \sin^2 \phi' \, \cos^2 \theta',
\]
\[
h_3 = \sin 2\phi' \, \cos \theta',
\]
which depend only on the viewing angles \( (\theta', \phi') \). The quantity \( T = T(R) \) in equation (5.23) is the triaxiality parameter, given by
\[
T = T(R) = \frac{4 \, (H_2 + H_{2e})(R)}{(G_2 + G_{2e})(R) + 2 \, (H_2 + H_{2e})(R)}. \tag{5.25}
\]
The equations of the projected surface density, position angle and triaxiality parameter are similar to that of the ZC96 and CTOO. Here we calculate the integrals analytically like that of CTOO except for the contributions from the additional radial functions, introduced here. We obtain
\[
F_1(R) = \frac{M}{4\pi b_0^3} \frac{b_0^3}{b_0^2 + R^2},
\]
\[
G_1(R) = \frac{M}{4\pi b_0^3} \frac{2b_0^3}{(b_0^2 + R^2)^3} \left[ 2b_0^2 + 9b_0 R^2 + 3R^4 \right],
\]
\[
G_{1e}(R) = \frac{M}{4\pi b_0^3} \frac{4b_0^3}{(b_0^2 + R^2)^3} \left[ 3b_0^2 + R^2 \right],
\]
\[
G_2(R) = \frac{M}{4\pi b_0^3} \frac{4b_0^3}{(b_0^2 + R^2)^3} R^2 \left[ 3b_0^2 + R^2 \right],
\]
\[
G_{2e}(R) = \frac{M}{4\pi b_0^3} \left[ 4a_1^2 R^2 I_2 + 12a_2 I_1 - 9a_4 I_4 \right]. \tag{5.26}
\]
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where
\[ a^2 + R^2 = \alpha^2 \]
and \[ \beta = \frac{1}{512 \alpha^4} \]

\[
I_1 = \beta(63R^6 + 14R^2\alpha^2 + 3\alpha^4),
I_2 = \beta(63R^6 + 21R^4\alpha^2 + 9R^2\alpha^4 + 3\alpha^6),
I_3 = \beta(63R^8 + 28R^6\alpha^2 + 18R^4\alpha^4 + 12R^2\alpha^6 + 7\alpha^8),
I_4 = \beta(63R^2 + 7\alpha^2).
\]

and for \( H_{1e}(R) \) and \( H_{2e}(R) \) in terms of \( a_3 \) and \( a_4 \), in place of \( a_1 \) and \( a_2 \) respectively.

Position angle \( \Theta_* \) (5.23) and the axis ratio \( b/a \) can be expressed in terms of above expressions. At small and at large radii \( \Theta_* \) and \( b/a \) can be expressed in terms of elementary functions. Equation (5.23) of the position angle, has two solutions \( \Theta_* \) and \( \Theta_* - \frac{\pi}{2} \) for a given model and viewing angles.

The observed axis ratio \( b/a \) of the projected surface density can be calculated by using the relation

\[
\Sigma(b, \Theta_* - \frac{\pi}{2}) = \Sigma(a, \Theta_*),
\]
therefore,

\[
\Sigma_0(a) + \Sigma_2(a) = \Sigma_0(b) - \Sigma_2(b). \tag{5.27}
\]

At a finite radial distance from the centre, the axis ratio \( b/a \) has to be calculated from equation (5.27) numerically. However, at large and at small radii, the analytical expressions of \( b/a \) can be obtained from equation (5.27).

The term \( (G_2 + G_{2e})/(H_2 + H_{2e}) \) in the expression (5.23) of position angle \( \Theta_* \) and triaxiality parameter \( T \) equation (5.25) can be written analytically using the equation (5.26). This in turn enables us to write an analytical expressions for \( \Theta_* \) and \( T \), at asymptotic limits, which are presented below.

At large \( R \), the ratio \( [(G_2 + G_{2e})/(H_2 + H_{2e})]_\infty \) and the triaxiality parameter \( T_\infty \) are given by

\[
\left( \frac{G_2 + G_{2e}}{H_2 + H_{2e}} \right)_\infty = \frac{b_3^2}{b_3^2}, \tag{5.28}
\]
and

\[
T_\infty = \frac{4 \frac{b_3^2}{b_1^2 + 2 b_3^2}}{1 - \frac{b_3^2}{b_1^2 + 2 b_3^2}} = \frac{1 - p_{2e}^2}{1 - p_{2e}^2}. \tag{5.29}
\]
Likewise, at small $R$, the ratio $\left(\frac{G_2 + G_{2e}}{H_2 + H_{2e}}\right)_o$ and the triaxiality $T_o$ are

$$
\left(\frac{G_2 + G_{2e}}{H_2 + H_{2e}}\right)_o = \frac{12b_4^2}{b_4^2} - \frac{33.13a_4}{a_4^{3/2}},
$$

(5.30)

and

$$
T_o = 4 \left[ \frac{\frac{12b_6^2}{b_6^2} - \frac{33.13a_6}{a_6^{3/2}}}{2 + \frac{\frac{12b_4^2}{b_4^2} - \frac{33.13a_4}{a_4^{3/2}}}{2}} \right].
$$

(5.31)

We find that the expression for $\left(\frac{G_2 + G_{2e}}{H_2 + H_{2e}}\right)_\infty$ and therefore the position angle $\Theta_\ast$ is the same as $(G_2/H_2)_\infty$ that of CT00 but at small radii the expression for $\left(\frac{G_2 + G_{2e}}{H_2 + H_{2e}}\right)_o$ and $\Theta_\ast$ changes from that of CT00.

Further, using the function $h_1, h_2, h_3, h_4$ of $(\theta', \phi')$ given by equation (2.39) and defining

$$
Z \equiv \left[ h_4 + h_5 \left( \frac{G_2 + G_{2e}}{H_2 + H_{2e}} \right)_o + h_6 \left( \frac{G_2 + G_{2e}}{H_2 + H_{2e}} \right)_o \right]^{1/2},
$$

(5.32)

We calculate the axis ratios at asymptotic radii. At asymptotic large radii, by putting the value of $\left(\frac{G_2 + G_{2e}}{H_2 + H_{2e}}\right)_\infty$ from (5.28) in (5.32), we obtain the value of $Z$ as

$$
Z^\infty = \left[ h_4 + h_5 \left( \frac{b_4^2}{b_3^2} \right)_o + h_6 \left( \frac{b_4^2}{b_3^2} \right)_o \right]^{1/2},
$$

(5.33)

Solving equation (5.27) we obtained axial ratio at very large $R$ as

$$
\left( \frac{b}{a} \right)_\infty^2 = \frac{b_3^2}{b_3^2} - \frac{2b_3^2Z^\infty}{\frac{1}{b_3^2} + \frac{2b_3^2Z^\infty}}.
$$

(5.34)

Comparing (5.34) with the (2.42) we found that the axial ratios at very large $R$ remains same as that of the CT00. For very small $R$ the axial ratio is given by

$$
\left( \frac{b}{a} \right)_o^2 = \frac{A - 12Bb_3^3}{A + 12Bb_3^3},
$$

(5.35)

where

$$
A = -\frac{2}{b_0} + \frac{2b_3^2h_4}{b_3^2} + \frac{72t_5^2h_5}{b_5^4} + \frac{2n_1C}{a_2^{3/2}} + \frac{12b_3^3}{b_3^4}B^{1/2},
$$

$$
B = \left\{ (4\cos^2 \theta' \sin^2 2\phi')^2 + (3/2 \sin^2 2\phi')^2C + (1 + \cos^2 \theta') \sin^2 \theta' \cos^2 2\phi' \right\}^{1/2}
$$
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Figure 5.1: Effects of $g_e$ and $h_e$ on $b/a$ profile. Model with $p = 0.65, q = 0.60, \theta' = 60°, \phi' = 10°$. $a_1 = a_3 = 0, a_2 = a_4 = 0$ for filled circles. $a_1 = a_3 = 0.2b_0, a_2 = a_4 = 0.9b_0$ for crosses.

We find that the position angle $\Theta_a$ and the axial ratio $b/a$ is the same as those of $fgh$ model of CT00 at large radii, and these parameters are different in the present model at small radii. The reason is not difficult to trace. For smaller radii of the model, one has to 'see' through the model from outer to inner, and light from all radii will contribute and the additional radial functions are effective dominate in the intermediate range of $r$.

The axial ratio at very small $R$ is now different from that of the $fgh$ model of CT00. Thus the effect of extra terms is seen in the inner radius of the model. This fact can be seen from the profile of $b/a$ with respect to $R$ (Figure 5.1). Many real galaxies shows this type of small scale variations in their profiles, rather than the smooth monotonically increasing or decreasing profiles as produced by the CT00's model. Thus our model is more realistic model. It shows
Table 5.1: Basic parameters of NGC 661 and details of observations. These data has been taken from Sahu 1999.

<table>
<thead>
<tr>
<th>Basic parameters of 661:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance = 76.9 Mpc</td>
</tr>
<tr>
<td>RA = 01 44 14.6</td>
</tr>
<tr>
<td>Dec. = 28 42 20</td>
</tr>
<tr>
<td>$M_B = 21.73$</td>
</tr>
<tr>
<td>$r_e = 14.35''$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Details of the Observations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope : 1m UPSO reflector</td>
</tr>
<tr>
<td>Wave bands : VRI</td>
</tr>
<tr>
<td>Detector : 384 x 576 CCD</td>
</tr>
<tr>
<td>Pixel size : 23 µm</td>
</tr>
<tr>
<td>Scale : 0.36&quot; pixel^{-1}</td>
</tr>
<tr>
<td>Seeing (FWHM) : 1.84&quot; (for $R$ filter)</td>
</tr>
</tbody>
</table>

ellipticity variation and the small scale variations in it.

Profile of $b/a$, as presented in Figure (5.1) can be compared with the similar profiles of real galaxies. We present the data on galaxy NGC 661. NGC 661 is rather a featureless galaxy (no sign of shells or dust). It is an $E+$ galaxy. The basic parameters and the observation details are presented in Table 5.1. The observation and analysis were performed by Sahu et al. (1995) and is reproduced in Figure (5.2) and Table 5.2. This galaxy shows a small scale variation in the ellipticity profile at $\sim 0.3r_e$ (see Chakraborty et al. 2001). For a quantitative estimate, we note that $r_e$ for the spherical modified Hubble model is $4.48r_e$ (Thakur and Chakraborty, 2001).

5.4 Correlated projected properties

It was shown in ZC96 that the observed ellipticities and the position angles at small and at large radii, can be used to derive the intrinsic axis ratios of the mass model, as a function of the viewing angles. However, the viewing angles of galaxies are largely unknown, and therefore, it will be interesting to get estimate of the intrinsic axis ratios
Table 5.2: Galaxy data of NGC 661. This table has been taken from Sahu 1999.

<table>
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<tr>
<th>a (arcsec)</th>
<th>$R_V$ (mag. arcsec$^{-2}$)</th>
<th>$V - R$</th>
<th>$R - I$</th>
<th>$\epsilon$</th>
<th>$\theta$</th>
<th>$b_t$</th>
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independent of the viewing angles. In case the observed parameters exhibit correlations, when a model with a given set of intrinsic parameters is viewed in all possible orientations, one can then apply the method of Bayesian statistics and obtain a probable estimate of the intrinsic shape, independent of the viewing angles. Further, ensembles of models having different intrinsic parameters, should be used to obtain a model-independent shape estimate.

Thakur & Chakraborty, (2001a,b) studied the triaxial generalisation of both the spherical $\gamma$ - models of Dehnen (1993) and modified Hubble model, and found strong correlations between the observed axis ratios, evaluated at small and at large radii. For both the models, they found that the correlation plots corresponding to different choices of the intrinsic axis ratios, occupy well-separated regions in the parameter space. They exploit these correlations to set constraints on intrinsic shapes of mass models using Bayesian statistics.

The model presented here, has same axial ratio at large radii, as that of CT00, but at inner radii axial ratios are now different from that of models of CT00. Therefore, it will be worthwhile to examine the correlations between axial ratio $(b/a)_o$ and $(b/a)_\infty$, for the present model, as well.

We (Das & Chakraborty 2001) find that the correlation between the observed axis ratios exists for the present model (Figure 5.3) exist. Further, the pattern of the correlation remains nearly same as that studied by TC01a,b. For different choices of the intrinsic axis ratios, the correlation plot occupies well-separated regions in the parameter space (Figure 5.3). Thus, our model may be included as an ensembles of model for the study of intrinsic shapes of triaxial mass models.

5.5 Results and discussion

We have studied the modified Hubble mass model with the extra radial function added to the radial functions with second order spherical harmonics. The model is more realistic model. The projected surface density, produced varieties of ellipticity and position angle profiles. The profile $b/a$ can be compared with the real galaxy NGC 661. Figure 5.3 shows that the axial ratio $b/a$ at small and large $R$ are strongly correlated, when the model is viewed in all possible direction. For different choices of the intrinsic axis ratios, the correlation plot occupies well-separated regions in the parameter space.
5.5. RESULTS AND DISCUSSION

NGC661

$M_{B} = -21.73$  Seeing(R): 1.84"

Figure 5.2: The $R$ band profiles of surface brightness, ellipticity and the major axis position angle, colour and higher order Fourier coefficient as a function of major axis radius in arcseconds. Data in $R$ band are indicated by triangle, in $B$ by filled circle, in $V$ by open square and in $I$ by open circle. This figure has been taken from Sahu 1999.
Figure 5.3: Correlation between \((\frac{b}{a})_o\) and \((\frac{b}{a})_\infty\) when the model of a chosen \((p, q)\) is projected in all possible viewing angles.