CHAPTER 1

INTRODUCTION

1.1 OBSERVATIONAL BACKGROUND:

Recently a photograph of an unexpectedly large ($2 \times 10^{20}$ cm radius) disk of cool dust and gas surrounding a bright unresolved nucleus in the active galaxy NGC 4261 has been obtained by Jaffe et al., (1993) by using the planetary camera on the Hubble space telescope (Fig. 1-1). This is a direct evidence of the presence of gaseous accretion disk around a compact object.

Prior to this discovery the host of other observations which require the mechanism of accretion for their satisfactory explanations, can be classified into two categories. The radiations from the x-ray binaries such as Cen X-3, Her X-1, Cyg X-1 and the like form the first category of observations. These are characterised by high luminosity ($10^{37}$ to $10^{38}$ ergs/sec), strong x-ray emission with $\hbar \nu \sim 1$-10 kev and rapid time variability (Shakura and Sunyaev 1973, Apparao and Chitre 1976).

Emission from Quasars and active galactic nucleii (AGN) form the other class of observations where the luminosities $10^{46}$ to $10^{48}$ erg/sec (Novikov and Throne 1973, Lyden Bell 1969) are invoked.

Accretion of gas onto a compact object forms the most widely investigated model for both the above classes of objects.
1.2 A GENERAL SCENARIO OF THE ACCRETION

A compact object which may be a black hole, a neutron star or a white dwarf and which accretes gas from the surrounding can emit radiation at the expense of the gravitational potential energy of the gas particles. While the total luminosity is decided by the amount of gas falling on to the compact object per-sec, the peak of the spectrum is determined by the gravitational binding energy of each gas particle. Gravitational potential will be higher on the surface of a neutron star than on a white dwarf and therefore accretion onto a neutron star can produce hard X-rays as compared to white dwarf. Further the amount of gas available increases in case the compact object forms a close binary with an early type normal star (Shkura and Sunyaev 1972, Pringle and Rees 1973, and Novikov and Thorne 1973). While a compact object in a close binaries forms a model for X-ray binaries, Lyden-Bell (1969) was first to propose an accreting super massive black hole \((M \sim 10^8 M_\odot)\) for the model of quasars and active galactic nuclei. Different accretion models have been investigated and several review articles have been written on the dynamics of accretion, such as Lightman et al., (1978), Pringle (1981), Begelman et al., (1984) and Frank et al., (1992).
1.3 SPHERICALLY SYMMETRIC ACCRETION:

The accretion onto a compact object is radially or spherically symmetric, if either the compact object is at rest or the incoming matter does not possess angular momentum. Studies of such spherical symmetric accretion were made by Bondi (1952), Shvartsman (1971), Shapiro (1973) and Cox and Smith (1976). Bondi (1952) worked out rigoursly the dynamics of accretion for the stationary state. They had shown that the accretion rate is proportional to the square of the mass of the stars and to the density of the gas at infinity while it varies inversely as the sound velocity in the gas at infinity, provided the accreting gas obeys a polytropic equation of state. The role of the self-gravitation for spherically symmetric accretion has been discussed by Wandel (1984).

1.4 THIN DISK MODEL:

The importance of angular momentum of the gas in binary accretion was first discussed by Prendergast and Burbidge (1968). It was realised that if the infalling gas has angular momentum, then it will form a rotating disk around the central object. The thin disk model was first studied by Pringle and Rees (1972), and Shakura and Sunyaev (1973) and it's subsequent modification by Novikov and Thorne (1973) was termed as the standard accretion disk model ($\alpha$ - model). It was further developed by many workers. Such as Shakura and Sunyaev (1976), Lightman, Shapiro
and Rees (1978), Pringle (1981), Petterson (1983), Frank et al., (1992). The steady state structure of \( \alpha \) - model is governed by the laws of conservation of mass, angular momentum and energy and by the nature of viscosity. The dominant source of viscosity is probably the chaotic magnetic field and the turbulence in the gas flow.

The earlier models of accretion disk were based on the Newtonian theory of gravitation. Novikov and Throne (1973) and Throne (1974) have also studied the general relativistic effects specially on the inner region of the disk which is very close to the event horizon. The study of \( \alpha \) - model disk is generally divided in three parts. (I) Radial structure (II) Vertical structure (III) The propagation of radiation from the interior of the disk to it's surface. The main assumptions are that (a) central plane of the disk coincide with equatorial plane (b) The companion star in the binary system has a negligible influence. (c) The disk is thin. The main problems studied are the luminosity and the spectrum of the radiation and are found to depend on the accretion rate.

1.5 THICK DISK:

The study of thick disks were started by Fishbone and Moncrief (1976). Accretion disk becomes thick when the accretion rate exceeds the critical value (super critical accretion) or when the pressure force becomes large. Thick disk remains in hydrodynamical equilibrium but its rotational velocity is not
Keplarian as the pressure gradient force is important. Fishbone and Moncrief (1976) have considered stationary and pure azimuthal flow of perfect fluid disk around a Kerr black hole for the special case of isentropic flow. They have solved the relativistic Euler's equation, as obtained from the conservation of the energy momentum tensor of perfect fluid. An analytical theory of the hydrodynamical structure of accreting disk (without self gravitation but with pressure) orbitting around a axially symmetric stationary compact object, was also discussed by Abramowicz et al., (1978). They have indicated five different possibilities regarding the disk structure in terms of angular momentum distribution.

(i) $l_0 < l_{ms}$: disk will not form.

(ii) $l_0 = l_{ms}$: disk exist as an infinitesimally thin unstable ring located on the circle $r = r_{ms}$.

(iii) $l_{ms} < l_0 < l_{mb}$ : disk can form without cusp but with only one cusp.

(iv) $l_0 = l_{mb}$ a cusp is formed and is located at the marginally closed equipotential surface.

(v) $l_0 > l_{mb}$: disk has no cusp.

In the above $l_{ms}$ and $l_{mb}$ are the absolute values of the angular momentum of the disk with Keplarian circular velocities at marginally stable and marginally bound values of the radii respectively and $l_0$ is the constant angular momentum.
A full-fledged theory of the thick accretion disk around a black hole was first discussed by Paczynski and Witta (1980). The accretion flow was considered to be confined to a thin layer at the disk surface. The shape of the disk with steady state accretion rate was obtained by assuming hydrostatic equilibrium, local heat balance and the standard conservation laws of mass, angular momentum and energy. The viscosity does not effect the results as long as it is small. Paczynski and Witta (1980) have used a pseudo-Newtonian potential to describe gravitational field.

The angular momentum distribution is no longer Keplerian and the vertical structure of the disk becomes a torous with two deep and narrow funnels along the rotation axis and having cusp very close to compact object (Rees, 1984) (fig.1.2). Most of the accreting energy is emitted from these funnels. The radiative flux becomes very high and it drives mass lost from the funnel in the forms of two relativistic jets pointing in opposite directions (Lynden-Bell 1978).

1.6. STABILITY:

The stability of the disk has been discussed by several authors such as Lightman and Eardley (1974), Pringle, and Rees (1973), Shakura and Sunyaev (1976), Papalizou and Pringle (1984, 1985, 1987), but no definite conclusion exist. The only
criterion available is that a necessary (but not sufficient) criterion for the stability is that the specific angular momentum should not decrease outwards. Two kinds of stability analysis can be made (i) Local and (ii) Global stability analysis. In the local stability analysis the wavelength of the perturbation are supposed to be smaller than the characteristic scale length over which the unperturbed dynamical variables change significantly. In the global analysis one constructs an eigenvalue equation and forms a variational principle. The solution of such an analysis is valid for all wavelengths of the perturbations.

1.7 STABILITY OF THIN DISKS:

Earlier studies of instabilities in thin disks were due to Lightman and Eardly (1974 a,b), Pringle et al., (1973), Shakura and Sunyaev (1976). There are two types of instabilities - the viscous instability, discovered by Lightman and Eardly (1974) and the thermal instability, discussed initially by Pringle et al., (1973). Lightman and Eardly (1974) have solved the time dependent equations for the accretion disks and discussed the stability of thin accretion disk for an axially symmetric perturbation. In original $\alpha$ - model, they have found that the inner region of the disk is unstable, Pringle, et al., (1973), Lightman, et al., (1974), Eardly and Lightman (1974) have attempted to improve the original model only to find another model whose inner region is thermally unstable.
Piran (1978) has shown that the stability of disk depends very sensitively on the viscosity law and a relatively small variation in viscosity law stabilizes the disk but the difficulty is that one does not know how to evaluate a modified turbulent or a magnetic viscosity. Okuda (1980) has claimed that inner region of the disk is thermally unstable unless the accretion rate is low. He has discussed the stability of accretion disk around a neutron star by considering the role of pressure gradient forces in the angular momentum and in the conservation equations, while keeping the same viscosity law as in the original $\alpha$-model.

All the above mentioned stability analysis are based on the local analysis, using the Newtonian gravitational field. Using local analysis Kato and Fuko (1980) have discussed the radial oscillations of a thin gaseous disk around a Schwarzschild black hole. A very interesting phenomena have observed by Treves et al., (1989), that the thin disk stability has a limit cyclic behaviour. In fact, Bathe and Pringle (1983) were first to draw one's attention to the limit cyclic behaviour while discussing the evolution of viscous disk for modelling the eruption of dwarf novae.

The local stability of accretion disk is studied rigoursly by Dubrulle and Knobloch (1992) with the help of Reynolds number ranging from $10^{14}$ for a disk around a white dwarf to $10^{26}$ for an active galactic nuclei. They have claimed that an unbounded, viscous, compressible, rotating shear flow is stable with respect to the infinitesimal three dimensional disturbances.
The global instabilities have intensively been studied by Narayan et al., (1987), Lindzen and Barker (1985), and Glatzel (1989), Dubrulle (1990), Zahna (1990), Dubrulle and Valdettaro (1991) have claimed that most unstable modes have low - cross stream wave number while large wave number disturbances are damped. Such global instability generate structure of the order of the size of the system.

1.8 STABILITY OF THICK ACCRETION DISK:

The methods for the study of the stability of thick disk are primarily due to Chandrasekhar (1964), Chandrasekhar and Friedman (1972 a,b) and have been extensively used by Chakraborty and Prasanna (1981,1982) and Chakraborty and Mishra(1991,1993). The dynamical stability of differentially rotating fluid having uniform entropy and uniform specific angular momentum for the non-axisymmetric perturbation has been studied by Papaloizu and Pringle (1984,1985,1987). They undertake the global stability analysis and find that disks are generally unstable. The instability reported by Papaloizu and Pringle has given rise to lot of activities amongst the theoraticians to find ways to stabilise the disk.

Self gravitation has been introduced to stabilize the disk and was found to have an important role in thick accretion disk towards its stability (Abramowicz et al., 1984). Self gravitation has also been exploited to study the structure by Bodo and Curir (1992). They have computed the equilibrium structure of self
gravitating thick accretion disk by an iterative procedure which produces a final density distribution in equilibrium with the potential coming from it. They have also discussed the main difference in their properties with models computed without self gravitation. Abramowicz, et al., (1984) in a preliminary study of the global effect of gravity have shown that its role can be important even in the case of small masses of disk because it perturbs the delicate balance between gravitational and centrifugal forces and the structure of the disk is very sensitive to such balance. The non axisymmetric instabilities of a self gravitating thick accretion disk have been discussed by Goodman and Narayan (1988), and Papaloizu and Suvonije (1991), which showed that some mode of instability could be damped where as some others could be amplified when self gravity is induced. The effect of self gravity around a Kerr black hole has been discussed by Will (1974,75), by using the perturbative technique in the framework of general relativity for a slowly rotating black hole.

1.9 GENERAL RELATIVISTIC EFFECTS:

As the inner edge of the disk can go very near to the event horizon the general relativistic effects become quite significant. This was particularly emphasized by Prasanna and Coworkers, Prasanna (1991). Earlier many works such as Novikov and Thorne (1973) have included the effect of general relativity
as correction terms over the Newtonian gravity. Witta (1980) has used a Pseudo Newtonian potential which has been claimed to reproduce some of the features of the Schwarzschild geometry. The Pseudo Newtonian potential has a singularity at $\frac{r}{m} = 2$ and so can not be used very near to the event horizon.

A truly general relativistic calculation of the disk structure was performed by Fishbone and Moncrief (1978). They derive the general solution without self gravitation of the appropriate relativistic Euler equation for the special case of isentropic, stationary and purely azimuthal flow. Prasanna and Chakraborty (1981) have given a detailed general relativistic equations for the study of structure and stability of charged fluid disk around compact object, neglecting the self gravitation of disk itself. These equations have been used by Chakraborty and Prasanna (1982) to develop the complete set of dynamical equations governing the structure and axi-symmetric perturbations for a perfect fluid disk rotating around Schwarzschild black hole. For the stability analysis they have used normal mode analysis to set up a characteristic value equation and study the stability criteria by trial function method as developed by Chandrasekhar and Friedman (1972,a,b).

Chakraborty and Prasanna (1982) have discussed the general relativistic effects by comparing the profiles of the steady state parameters such as velocity and pressure along the equatorial plane for Schwarzschild and Newtonian gravity calculations. They have shown that for the same values of the
inner edge \( f_{in} \) and outer edge \( f_{out} \) Newtonian disk occupy more volume than the relativistic disk. In the Newtonian disk the pressure at any point is higher while the velocity is lower, than in the case of relativistic disk (Fig. 1-3).

There are some other possible general relativistic effects: as Hanawa (1989) and Bhaskaran and Prasanna (1989) have claimed from the collections of flux luminosity, for a single black body temperature, fit for the emission from the inner edge of the disk.

We have extended the work of Chakraborty and Prasanna (1981, 1982). We have developed the general set of hydrodynamical equations for non-self gravitating perfect fluid in the Kerr background geometry and have obtained solutions for thin disks with accretion, as well as thick disks, without accretion, rotating around a linearised Kerr black hole. We have also studied the solutions of thick disks rotating around a Schwarzschild black hole with adiabatic as well as a polytropic equation of state (Chakraborty and Mishra 1993).

We have observed several features such as the formation of the cusp at the inner edge of disk, and the restriction on the possible value of some of the parameters which determine the steady state structures of the disk. The formation of the cusp at inner edge has been reported earlier by Abramowicz, et al., (1978) and Chakraborty and Prasanna (1982). We investigated it further and examined a mathematical criteria for the existence of the cusp.
We have also investigated the frequency of oscillation of axisymmetric perturbations. We find that the frequency depends on (i) The choice of trial function (ii) The size of the disk and (iii) Underlying gravitation field Chakraborty and Mishra (1991,1993). We find that the frequency of the axisymmetric oscillation of the disk is higher in general theory of relativity than in Newtonian gravitation; by taking those steady state solutions in which the boundary of the disk is same both for the general relativity and Newtonian gravitational fields and by taking the same trial functions in both the cases.

We present the basic mathematical equations for the dynamics of the disk a Kerr geometry in chapter 2. Chapter 3 is devoted for the structure and stability of the rotating fluid disk around a compact object using the Newtonian description of the gravitational field. This forms the kind of ground work for finding out the differences when the Newtonian field is replaced by Schwarzschild gravitational field. This is presented in chapter 4. The thin accreting and thick (non-accreting) disks around a line arised Kerr geometry are discussed in chapter 5.

As the temperature of the disk may be quite high, it is expected that the gas is in a plasma state. It is, therefore, more appropriate to treat the matter of the disks as plasma and to use the equations of magneto-hydrodynamics for their study. This point was particularly emphasized by Prasanna and Bhaskaran (1989) Prasanna, Tripathi and Das (1989).
In plasma dynamics, particle approach also gives same understanding, especially when the density is low. Dynamics of charged particle in an external magnetic field, superimposed over a curved background geometry, can be regarded as a first step towards a more rigorous study of plasma dynamics on a curved geometry.

Charged particle dynamics in an external electromagnetic field, superimposed or a background geometry has been studied by Prasanna & Verma (1977), Prasanna and Vishveshwara (1978), Chakraborty and Prasanna (1982), Prasanna and Chakraborty (1981), Dadhich and Witta (1982). They have considered uniform and dipolar magnetic fields, both in Schwarzschild and Kerr background geometries. The most important results they found is the existence of stable orbits, even very near to the event horizon. This was the result which encouraged Prasanna and Coworkers, to undertake the studies of disk dynamics, using the full general theory of relativity as the inner edge of the disk could go very near to event horizon. Prasanna & Vishveshwara (1978) have studied the charged particle dynamics in an uniform magnetic field in Kerr geometry. The rotation axis of the black hole is considered to be parallel (or anti parallel) to the asymptotic magnetic field. We have considered a more general situation in which the rotation axis and asymptotic field are not parallel, in general. Some aspects of this part of the study, are presented in chapter 6.
CAPTIONS FOR FIGURES

Fig. (1.1) - Left: A composite image of NGC 4261 in V band taken by the Palomar mountain 48" telescope.

Right: A Hubble Space Telescope image taken with the planetary camera of the nuclear region of NGC 4261 (Jaff et al., 1993).

Fig. (1.2) - Meridional section of a thick disk showing deep and narrow funnels along the rotation axis and the cusps.

Fig. (1.3) - Boundary of the disk in relativistic formulation (Solid line) and in Newtonian formulation (dashed lines).
THICK ACCRETION DISKS

Fig 1.2