Chapter 2

Theoretical Models

2.1 Introduction

Relativistic nucleus - nucleus collisions have been studied very intensively in recent years. Increasing the projectile mass number and energies increases the possibility of formation of Quark-Gluon Plasma (QGP) [16]. In relativistic nucleus-nucleus collisions, it is highly important to analyze the nuclear geometry. The emission of projectile fragments has been studied in several experiments and large number of models have been introduced for the better understanding of physics behind projectile fragmentation in the nucleus - nucleus collision i.e. the projectile fragmentation mechanism. Various models have been developed in support of the ultra-relativistic heavy ion experimental results [17] but unfortunately few models are available for the explanation of relativistic heavy ion experimental results, especially in the MeV region [18].

Physics model always helps experiments to proceed in right direction to achieve their physics goals but in reality experiments drive the models. The ultimate test of model is the successful explanation of experimental results. Therefore, before going for experiment we must understand related physics models in
depth. In this connection, I have summarised some of the models used in the present study of projectile fragmentation.

2.2 Participant Spectator Model (PS Model)

PS Model was proposed by Knoll et al [19] in the year 1977. PS Model based on the assumption that, all the nucleons during collision act incoherently. In this model, a straight line motion of the projectile at high energy defines a simple picture of collision geometry [20, 21]. The interacting system, in this model, in relativistic nucleus - nucleus collisions can be divided into three parts or regions namely (1) Participants (Target & Projectile) region (2) Projectile Spectators region and (3) Target Spectators region.

The overlapping region of colliding nuclei is called participant region, from where newly created particles occur and the remaining parts of nuclei which do not participate in collision are called target spectator region and projectile spectator region, respectively. The velocity of participant has a wide distribution from zero to projectile velocity. The velocity of target spectator in laboratory frame of reference should be zero but in practice, due to small momentum transfer from projectile, it’s velocity is non-zero. Projectile spectator has almost the same velocity as that of projectile. It is known that violent collisions happen in the participant region, and weak excitations and cascade collisions happen in the spectator regions. In the participant region mesons, kaons, photons and lepton pairs, etc., are produced while from spectator regions related fragments are only emitted.

The participant and spectator parts are correlated to each other. It is expected that quark-gluon plasma (QGP) will be formed in participant region at
very high incident energies and a liquid - gas phase transition will happen in spectator regions. The participant and spectator regions are relevant for study of nuclear reaction mechanism, which requires a thorough investigation of fragmentation mechanism of spectator region.

Nucleons in the overlapping region could be scattered via hard N - N collision, where as nucleons in the non - overlapping region ( i.e. spectator region ) would just pass through with minimal changes. The fragments from the participant region are mainly particles like protons, neutrons or pions because energy transfer is much higher than nuclear binding energy. The final state particles from participant region are emitted in all center of mass angles having momentum range allowed by kinematics in $P_E^*/mc$ and $P_T/mc$ plane in center of mass (CM) frame. There is involvement of strong interaction between nucleons of pro-
jectile and target in this region. Both the spectators are more likely to be emitted at \([(P^*/mc, P_T/mc) = (P^*_{\text{frag.}} / A, 0)]\) and \((P^*_{\text{target}} / A, 0)\), respectively. Since very little momentum transfer is needed to form these fragments, the beam and target fragments are often nuclear clusters.

The concept of PS Model are applicable when the de-Broglie wavelength \((\lambda)\) of incident projectile nucleons is small compared to the distance between nucleons inside the nucleus, i.e.,

\[
\lambda = \frac{\hbar}{p} \ll d,
\]

where \(\hbar =\) Reduced planks constant, \(d =\) Internucleon distance and \(p =\) Projectile momentum.

The above equation is satisfied only when projectile energy in CM frame is more than \(10-20\) MeV/A. At CM energy below \(5\) MeV/A, the individuality of each nucleon disappears and both participants and spectators are combined to form well-known compound nucleus.

Experimentally, there are two ways to verify PS Model. One way is to observe multiplicity distributions (Investigation of the participants) and the other way is to measure properties of spectator pieces in fragmentation reaction. PS Model predicts a clear cut separation of participants and spectators regions because of the assumption of clean cut geometry and suggestion that the nucleons in overlap volume participate in reaction while spectators remain unaffected.

If the projectile \((P)\) with radius \(R_P\), charge \(Z_P\) and mass number \(A_P\) collides with target \((T)\) with radius \(R_T\), charge \(Z_T\) and mass number \(A_T\) and both have
uniform densities for a given impact parameter \( b \) then the number of projectile and target participants can be written \([22]\) as

\[
N_T(b) = \int dz d^2s \Theta(R_T^2 - S^2 - Z^2) \Theta(R_T^2 - (b - S)^2),
\]

(2.2)

and

\[
N_P(b) = \int dz d^2s \Theta(R_P^2 - S^2 - Z^2) \Theta(R_P^2 - (b - S)^2),
\]

(2.3)

where \( N_T(b) \) and \( N_P(b) \) are the number of target and projectile participants for a given \( b \). \( \Theta(\times) \) is the step function for which \( \times > 0 \), \( \Theta(\times) = 1 \) and \( \times \leq 0 \), \( \Theta(\times) = 0 \). \( S \) is a component in the parameter \( b \) plane and \( z \) is beam direction. The above equation helps us to calculate overlap volume for target and projectile separately.

The study of projectile fragmentation \([23]\) in high energy heavy-ion collision involves an understanding of peripheral processes as describe in section 1.1. In most of the models, the type of high energy reactions which involves small amount of energy in nuclear reactions are usually explained through a two-step mechanism, although the details of these models differ considerably. In first stage, the projectile undergoes a peripheral collision with target and may lead to knock out of one or more nucleons, followed by De-excitation on a time-scale characteristic of internal nuclear motion. At higher energies (\( \sim 1 \) TeV), it is expected that the projectile nucleus practically becomes transparent and projectile and target nuclei pass through each other as shown in figure 2.1. As a result, a hot baryon free region called the central region is likely to be created along their trajectories. It is believed that during the collision, the energy density and temperature of this baryon-free central region can become too high, so that this region can again melt into quarks and gluons.
2.3 Two Source Projectile Fragment Emission Model

In collisions, due to the existence of relative motion between participant and spectator, friction is caused on the contact layer. The participant and spectator gets heat due to friction [24]. It takes time for heat to be transmitted to the rest parts of spectator and give raise to temperature gradient in spectator region of projectile. The contact layer and rest part are separated from each other because of heat and are considered as two sources to emit nuclear fragments at two different temperatures and is demonstrated in figure 2.2.

The contact layer portion has highest temperature after participant region. The temperature is almost constant in a layer and the layers thickness increases with distance from contact layer as shown in figure 2.2. The fall in temperature is rapid towards the farther side of projectile spectator region. This could lead whole spectator to a non-equilibrium state but the contact layer and rest part are in local equilibrium state. The change in temperature follows power law decay nature and it could be explained from measured charge spectrum of projectile fragments [25].

Let \( n_c \) and \( n_o \) be multiplicities of alpha projectile fragments emitted from contact layers closer to participant region and rest part of the projectile spectator, respectively. It is also possible for heavy projectile clusters getting excited and decay into few alpha projectile fragments in a very short time and the distance travelled before decay is close to the vertex of the event. The multiplicity of alpha projectile fragments measured in final state of interaction is denoted by \( n_a \). The relationship between the mean multiplicities are given by
\[ < n_c > = k < n_a >, \quad (2.4) \]

and

\[ < n_o > = (1 - k) < n_a >, \quad (2.5) \]

where \( k \) is an extra parameter.

Let \( P_c(n_c) \), \( P_o(n_o) \) be the probability for contact layer and rest part of spectator to emit \( n_c \), \( n_o \) fragments, respectively. \( P_c(n_c) \), \( P_o(n_o) \) from [25] are given by

\[
P_c(n_c) = \frac{1}{< n_c > [1 - \exp(-n_{cm} / < n_c >)]},
\]

and

\[
P_o(n_o) = \frac{1}{< n_o > [1 - \exp(-n_{om} / < n_o >)]},
\]

where \( n_{cm} \) and \( n_{om} \) are the maximum values of \( n_c \) and \( n_o \), respectively.

The probability \( P(n_a) \) for spectator to emit \( n_a \) fragments depends only on the sum of \( n_c \) and \( n_o \) [24]. That is

\[
P(n_a) = \int_0^{n_a} P_c(n_c)P_o(n_a - n_c)dn_c. \quad (2.8)
\]

On substituting equation (2.6) and (2.7) in (2.8), we get

\[
P(n_a) = \frac{1}{(< n_c > < n_o >)} \int_0^{n_a} \exp(-n_c / < n_c >) \left[ 1 - \exp(-n_{cm} / < n_c >) \right] \left[ 1 - \exp(-n_{om} / < n_o >) \right] dn_c.
\]
\[ P(n_a) \approx \frac{4n_a}{\langle n_a \rangle^2} \exp \left( \frac{-2n_a}{\langle n_a \rangle} \right). \]  

(2.9)

In the case of \( k = 0.5 \) and maximum multiplicity of alpha projectile fragments \( n_{a\text{max}} \gg 2n_{cm} \gg 2n_{om} \), then

\[ P(n_a) \approx \frac{4n_a}{\langle n_a \rangle^2} \exp \left( \frac{-2n_a}{\langle n_a \rangle} \right). \]  

(2.10)

Especially for a maximum number of alpha projectile fragments \( (n_{a\text{max}}) \) values, eq. (2.10) becomes

\[ P(n_a) \approx \frac{4n_a}{n_a^2} \exp \left( \frac{-2n_a}{n_a} \right). \]  

(2.11)

Alpha projectile fragments emission from each source is assumed to be isotropic [24] in the source rest frame. As \( z \) denote the incident direction of projectile, \( xoz \) plane will be the reaction plane. The three components of alpha projectile fragments momentum \( (p_{x,y,z}) \) in the source rest frame are assumed to have Gaussian distributions [21] with same width \( \sigma_p \). Therefore, we have

\[ P_{p_{x,y,z}}(p_{x,y,z}) = \frac{1}{(2\pi\sigma_p^2)} \exp \left( -\frac{p_{x,y,z}^2}{2\sigma_p^2} \right). \]  

(2.12)

Then, the transverse momentum

\[ p_T = \sqrt{(p_x^2 + p_y^2)}, \]  

(2.13)

has Rayleigh scattering distribution [25] as

\[ p_{p_T}(p_T) = \frac{p_T}{\sigma_p^2} \exp \left( -\frac{p_T^2}{2\sigma_p^2} \right). \]  

(2.14)
Figure 2.2: Schematic overview of contact layer and fine layers of temperatures at rest parts of projectile and target spectators across the incoming direction of projectile. Darker area represent lower temperature regions. The projectile is approaching toward the target and target is at rest. We are considering only two basic regions of projectile spectator i.e. contact layer very close to participant region and rest of the spectator part just for convenience in calculation.
By considering two-source emission of alpha projectile fragments, final state transverse momentum ($p_T$) distribution should be the sum of two Rayleigh scattering distributions, i.e. active sources are represented by each distribution [25]

$$p_{pT}(p_T) = \frac{A_H p_T}{\sigma_H^2} \exp\left(-\frac{p_T^2}{2\sigma_H^2}\right) + \frac{A_L p_T}{\sigma_L^2} \exp\left(-\frac{p_T^2}{2\sigma_L^2}\right), \quad (2.15)$$

where $\sigma_H$ and $\sigma_L$ are widths of $p_T$ distributions for alpha projectile fragments emitted from sources of high and low temperatures, respectively. $A_H$ and $A_L$ denote the number of alpha projectile fragments contribution from two sources where temperature of the sources are considered as high and low, respectively.

2.4 Compound Nucleus Model

In this model, it is assumed that after projectile enters the nucleus, it interacts very strongly with the nucleons and projectile loses its initial energy and parameters and also adopts a new nuclear configuration, i.e. a combination of projectile and target nucleus, and there is a possibility that target nucleus might be completely merged into the projectile nucleus volume and a compound nucleus is formed, but it is not a necessary condition. The conservation of energy is apparent that compound nucleus will be formed at a highly excited state from which it has to decay to its own ground state by emitting gamma-rays or to another neighboring residual nucleus in ground or excited state by emitting a particle. The schematic diagram for compound nucleus model is shown in figure 2.3.

The behavior of projectile fragmentation and their cross-section has been explained in many cases with the help of this model. These exists between entry of projectile and formation of complete compound nucleus. In this period, projectile may travel back and forth in between surface points or may break up or pick up nucleons and go through a complicated energy mixing with the nucleons
before settling down to $t_c$ equilibrium state of compound nucleus. The experimental life-time of such a compound state is of the order of $t_c \approx 10^{-12} - 10^{-16}$ s, compared to small time required for single traversal of the nucleus that is of the order of $10^{-19} - 10^{-22}$ s. The experimental observed values of $t_c$ for compound state will provide the number of traversals in a typical case of $10^3$ to $10^7 \cdot 10^{10}$, before a compound nucleus is formed [26].

The theory of compound nucleus of nuclear cross-section is based on two assumptions: one of it is the incident particle after entering nucleus forms a compound nucleus, in such a way that the final equilibrium state of compound nucleus does not remember how it was formed. In other words, the compound state will be independent of methods of formation of compound state as long as the compound nuclear state is same. It also means that the final state products are independent from the initial state conditions. The second one is that the decay of compound state depends on the properties of compound state and not on how it was formed. This was experimentally demonstrated by the famous experiment of S. N. Ghoshal [26], where he produced same compound nucleus $^{\ast 64}Zn_{30}$ with bombardment of $^{60}Ni_{28}$ by alpha particles and of $^{63}Cu_{29}$ by protons with appropriate energies to reach same excitation energy. The reactions that were observed

(a) $^{60}Ni_{28}$ (α, n) $^{63}Zn_{30}$
(b) $^{60}Ni_{28}$ (α, 2n) $^{62}Zn_{30}$
(c) $^{60}Ni_{28}$ (α, pn) $^{62}Zn_{30}$
(d) $^{60}Ni_{28}$ (p, n) $^{63}Zn_{30}$
(e) $^{60}Ni_{28}$ (p, 2n) $^{62}Zn_{30}$
(f) $^{60}Ni_{28}$ (p, pn) $^{62}Zn_{30}$.

In all the above cases on formation of compound nucleus, they will decay in such a way that
Figure 2.3: Schematic diagram of light nucleus interaction with heavier nucleus and heavy nucleus-nucleus interactions are respectively shown in (a) and (b).
\[ x + X \rightarrow C \rightarrow y + Y \]  \hspace{1cm} (2.16)

and

\[ \sigma(x, y) = \sigma_c(x)G_c(y), \]  \hspace{1cm} (2.17)

where \( x \) is incident particle, \( X \) is target, \( y \) is produced and emitted particles, \( C \) is compound nucleus and \( Y \) is projectile or target fragments. Then the cross-section for a given reaction \( (x, y) \) is a two-step process. That is formation of compound nucleus for which cross-section is given by \( \sigma_c(x) \) and \( G_c(y) \) the probability that compound state for \( C \) decays with emission and production of \( y \) with a projectile fragment \( Y \). In Equation (2.17) assume one, the decay probability is dependent only on property of compound nucleus state and is independent of the way it was formed. Hence, if there is a reaction

\[ x + A \rightarrow C \rightarrow z + Z. \]  \hspace{1cm} (2.18)

Then we can write

\[ \sigma(x, z) = \sigma_c(x, A)G_c(z). \]  \hspace{1cm} (2.19)

Similarly, for a reaction

\[ y + B \rightarrow C \rightarrow z + Z, \]  \hspace{1cm} (2.20)

one can then write

\[ (y, z) = \sigma_c(y, A)G_c(z). \]  \hspace{1cm} (2.21)

The above equations (2.16, 2.17, 2.18, 2.19, 2.20 and 2.21) and the hypothesis of compound nucleus we get

\[ (p, n) : (p, 2n) : (p, pn) = (\alpha, n) : (\alpha, 2n) : (\alpha, pn). \]  \hspace{1cm} (2.22)

\( G_c(z) \) is assumed to be the same for proton induced or \( \alpha \)-induced reaction for the
same produced or emitted particles \( z \), if compound nucleus state \( C \) is the same. Ghoshal’s experiment proved this relation empirically [26]. The subsequently performed similar experiments confirms the above assumption. From uncertainty principle, it is well-known that

\[
\Delta E \Delta t \approx \hbar \rightarrow \Delta E \approx \frac{\hbar}{\Delta t}. \tag{2.23}
\]

On substituting \( \Delta t = t_C \), \( \Delta E \) provides the energy width of level and is referred to as \( t_C \). We can then write \( t_C \) as

\[
t_C = \frac{\hbar}{t_C} = \hbar \lambda_C. \tag{2.24}
\]

\( t_C \) is proportional to \( \lambda_C \), the probability per unit time of decay from the level. If the widths are less than energy difference between successive states (\( \Delta \)), then cross-section for excitation as a function of incident energy will have discrete structure. On the other hand, if \( t_C >> \Delta \) then cross-section become smoothly varying function of energy. \( \Delta \) may also consists of many partial widths corresponding to different types of emitted particles and different energy channels can be scaled in the case of resonances (\( t_C << \Delta \)). In the case of interaction of neutrons with a nucleus major modes of interactions will be (i) elastic scattering \((n, n)\), inelastic scattering \((n, n')\) and \((n, R)\); where \((n, n)\) correspond to inelastic scattering and \((n, R)\) corresponds to proper reaction. According to above equations the total width \( (t_C) \) is then given by

\[
t_C = t_n + t_R, \tag{2.25}
\]

where \( t_n \) represents width for elastic scattering and \( R \) represents reaction including inelastic scattering. Now we are in a situation to define a quantity \( (t'_C) \), the reduced width, in such a way that the overall rate of decay of compound nucleus
state C to a state I, can be written as

$$\frac{\lambda_I}{\lambda_C} = \tau_i'$$  \hspace{1cm} (2.26)

where $\lambda_C$ is total probability per unit time of compound nuclear state decay and $\lambda_I$ is partial probability for decay in a given channel (i). The excitation energy $U$ of the compound nucleus is

$$U = (\frac{M_T}{M_T + M_P})T_P + S_P$$  \hspace{1cm} (2.27)
where $M_T$ and $M_P$ are atomic masses of target and incident projectile, respectively. $T_P$ is the laboratory kinetic energy of incident projectile and $S_P$ is the binding energy of particle P in the compound nucleus. As projectile have a finite energy spread the “quasi-stationary state” of compound nucleus includes many excited states. The lack of detailed knowledge about this composition of states causes most of the difficulties in analyzing compound nucleus model. However this problem is not serious for thermal neutrons because only a single excited state is involved [26].