CHAPTER 4

NORM – BASED JOINT AND DECOUPLED MIMO ANTENNA SELECTION ALGORITHMS

4.1 INTRODUCTION

There are situations, where the channel may not be sufficiently time-varying. In such cases, time available may be less between channel changes and very low-complexity algorithms will be required for antenna selection. A small performance compromise will have to be made in these cases. With this in mind, there are three norm - based transmit/receive algorithms stated in this chapter. These algorithms will be useful especially in i.i.d. conditions and may perform little less in correlation conditions. Section 4.2 states the significant contribution. Section 4.3 gives the signal model. Sections 4.4, 4.5 and 4.6 propose three norm-based algorithms. Section 4.7 is on complexity analysis. Section 4.8 is on simulation considerations and results. Section 4.9 gives the summary of the chapter.

4.2 SIGNIFICANT CONTRIBUTION

A relation assuming low-SNR conditions is derived for capacity. Based on this expression, three algorithms are proposed. Though the algorithms may be suitable especially in low-SNR conditions, for high SNR conditions also they may be applied with a small loss on capacity. The proposed algorithms are joint norm-based algorithm (JNBA), decoupled absolute-value based algorithm (DAVBA) and decoupled norm-based
algorithm (DNBA). Complexity analysis is made for the algorithms. Simulations have been done to show their performance and complexity.

4.3 SIGNAL MODEL AND THE PROPOSED NORM-BASED ALGORITHMS

The system model is described by equations (3.1) and (3.2). The capacity of a MIMO system with equal power distribution at the transmit side is given by equation (2.1) here repeated as (4.1).

\[ C = \log_2 \det \left[ I_{M_R} + \frac{\rho}{L_T} HH^H \right] \]  

(4.1)

On doing singular value decomposition (SVD) on $H$, (4.1) changes to

\[ C = \log_2 \det \left[ I_{M_R} + \frac{\rho}{L_T} \Lambda \right] \]  

(4.2)

where $\Lambda$ is a diagonal matrix made up of eigen values of $HH^H$. Since $\left[ I_{M_R} + \frac{\rho}{L_T} \Lambda \right]$ is a diagonal matrix, its determinant can be written as

\[ C = \left( \frac{\rho}{L_T} \lambda_1 + 1 \right) \left( \frac{\rho}{L_T} \lambda_2 + 1 \right) \ldots \left( \frac{\rho}{L_T} \lambda_r + 1 \right) \]  

(4.3)

$r$ being the rank of $H$ and $\lambda$ being the eigen values of $HH^H$. For low-SNR conditions, second and higher degree $\rho$ terms can be neglected compared to the first-degree terms, and the above equation becomes

\[ C = \log_2 \left( 1 + \frac{\rho}{L_T} \lambda_1 + \frac{\rho}{L_T} \lambda_2 + \ldots + \frac{\rho}{L_T} \lambda_r \right) \]

\[ = \log_2 \left( 1 + \frac{\rho}{L_T} (\lambda_1 + \lambda_2 + \ldots + \lambda_r) \right) \]

\[ = \log_2 \left( 1 + \frac{\rho}{L_T} \text{tr}(\Lambda) \right) \]  

(4.4)
where $\text{tr}(A)$ refers to the summation of diagonal elements.

Equation (4.4) may be written as

$$C = \log_2 \left( 1 + \frac{\rho}{l_T} \text{tr}(HH^H) \right)$$

(4.5)

Further equation (4.5) may be written as

$$C = \log_2 \left( 1 + \frac{\rho}{l_T} \|H\|_F^2 \right)$$

(4.6)

where $F$ refers to Frobenius norm. Equation (4.6) can also be written as

$$C = \log_2 \left( 1 + \frac{\rho}{l_T} \sum_{i=1,j=1}^{M_T} |h_{ij}^2| \right)$$

(4.7)

where the summation is in two dimensions.

Equation (4.7) says that under low-SNR conditions, capacity is maximised if $\sum_i |h_{ij}^2|$ is maximised. Hence, in the antenna selection problem, we have to select the maximum-$|h_{ij}^2|$ terms. Since simply selecting the required number of maximum-$|h_{ij}^2|$ will not culminate in selecting the required number of transmit and receive antennas, separate selections have to be applied. Hence, decoupling concept can be applied here. Equation (4.7) can be written as (4.8) and (4.9).

$$C = \left[ |h_{11}^2| + |h_{12}^2| + \ldots + |h_{1M_T}^2| \right] + \left[ |h_{21}^2| + |h_{22}^2| + \ldots + |h_{2M_T}^2| \right] + \ldots + \left[ |h_{M_T 1}^2| + |h_{M_T 2}^2| + \ldots + |h_{M_T M_T}^2| \right]$$

(4.8)

$$C = \left[ |h_{11}^2| + |h_{21}^2| + \ldots + |h_{M_T 1}^2| \right] + \left[ |h_{12}^2| + |h_{22}^2| + \ldots + |h_{M_T 2}^2| \right] + \ldots + \left[ |h_{1M_T}^2| + |h_{2M_T}^2| + \ldots + |h_{M_T M_T}^2| \right]$$

(4.9)
Equation (4.8) is in terms of row-wise summations and (4.9) is in terms of column-wise summations. In this perspective, three computationally very efficient algorithms are proposed for transmit/receive antenna selection. As an approximation process, these algorithms can be used also in high-SNR conditions if complexity is an issue in the application and a small loss in capacity performance is not an issue.

### 4.4 JOINT NORM – BASED ALGORITHM

In this, the best link is found among all the links. The transmit and receive antennas corresponding to the best link are selected. Then, from all the transmit antennas linking to this particular receive antenna, the best transmit antennas are selected, where the best ones are in terms of the absolute values of the channel coefficient. This can be justified based on the expectation that the links to the best receive antenna may be better than the links to the other receive antennas. Similarly, from all the receive antennas linking to the best transmit antenna, the best receive antennas are selected, where the best ones are in terms of the absolute values of the channel coefficient. This can be justified based on the expectation that the links to the best transmit antenna may be better than the links to the other transmit antennas. The algorithm is as follows.

1. Define \( S_R = \{1, 2, \ldots, M_R\} \), and \( S_T = \{1, 2, \ldots, M_T\} \).

2. Find \((P_1, Q_1)\) such that \((P_1, Q_1) = \arg \max_{i \in S_R; j \in S_T} |h_{ij}|\), where \( |h_{ij}| \) is a channel coefficient of channel matrix of \( M_R \times M_T \).

3. Delete \( P_1 \) from \( S_R \) and redefine \( S_R \) devoid of \( P_1 \). Delete \( Q_1 \) from \( S_T \) and redefine \( S_T \) devoid of \( Q_1 \).

4. Loop
a. Choose $P_2$ such that $P_2 = \arg \max_{i \in S_R} |h_{iQ_2}|$

b. Delete $P_2$ from $S_R$ and redefine $S_R$ devoid of $P_2$.

c. Go to Loop until $M_R - L_R$ indices are in $S_R$.

5. Remove set $S_R$ from the original set $\{1, 2, 3, \ldots, M_R\}$ and assign it to $\mathcal{L}_R$. $\mathcal{L}_R$ is the set of receive antennas selected.

6. $H = H_{\mathcal{L}_R}$

7. Loop

a. Choose $Q_2$ such that $Q_2 = \arg \max_{j \in S_T} |h_{P_2J}|$

b. Delete $Q_2$ from $S_T$ and redefine $S_T$ devoid of $Q_2$.

c. Go to Loop until $M_T - L_T$ indices are in $S_T$.

8. Remove set $S_T$ from the original set $\{1, 2, 3, \ldots, M_T\}$ and assign it to $\mathcal{L}_T$. $\mathcal{L}_T$ is the set of transmit-antennas selected.

4.5 **DECOUPLED NORM-BASED ALGORITHM**

This algorithm almost implements (4.7). The norms are calculated for the rows. The rows of maximum norms are selected and the row index will be the antenna index. The $H$ matrix is modified by excluding the unselected antennas or the corresponding rows. Then, the same procedure as done for rows is followed for columns. This algorithm is decoupled because it excludes the unselected receive antennas from the problem of finding the best transmit antennas or the corresponding rows are excluded from participating in transmit antenna selection process.

1. Define $S_R = \{1, 2, \ldots, M_R\}$, $S_T = \{1, 2, \ldots, M_T\}$
2. For all $j \in S_R$, calculate the vector norm of each row

$$a_j = \|H_{j,:}\|^2$$

3. Loop

   a. Choose $P$ such that $P = \arg \max_{j \in S_R} a_j$

   b. Delete $P$ from $S_R$ and redefine $S_R$ devoid of $P$.

   c. Go to Loop until $M_R - L_R$ indices are in $S_R$

4. Remove set $S_R$ from the original set $\{1, 2, 3, \ldots, M_R\}$ and assign it to $\mathcal{E}_R$. $\mathcal{E}_R$ is the set of receive-antennas selected.

5. $H = H_{\mathcal{E}_R}$

6. For all $k \in S_T$, calculate the vector norm $b_k = \|H_{k,:}\|^2$

7. Loop

   a. Choose $Q$ such that $Q = \arg \max_{k \in S_T} b_k$

   b. Delete $Q$ from $S_T$ and redefine $S_T$ devoid of $Q$.

   c. Go to Loop until $M_T - L_T$ indices are in $S_T$

8. Remove set $S_T$ from the original set $\{1, 2, 3, \ldots, M_T\}$ and assign it to $\mathcal{E}_T$. $\mathcal{E}_T$ is the set of transmit-antennas selected.

### 4.6 DECOUPLED ABSOLUTE VALUE-BASED ALGORITHM

In DNBA, vector norm of each row is carried out and then comparison is done among these norms. This is for receive antenna selection. Similar process is done for transmit antenna selection through the processing
on columns. It is worthwhile at this point to note that vector norm involves multiplication of complex elements. Each complex element is multiplied by its conjugate. Each complex-complex multiplication involves 4 real-real multiplications and 2 real-real summations. Hence, by avoiding complex-complex multiplications we can save 6 real-real flops for each complex-complex multiplication. Addition of M complex numbers involves 2M-2 real-real additions. Ultimately, the number of real-real flops matters. In this perspective, if simple summations are used and the squared absolute values of the simple summations are used for comparison, the complexity can be significantly reduced. This is what is done in DAVBA. DAVBA consumes significantly less number of real-real flops than DNBA does. The algorithm is as follows:

1. Define $S_R = \{1, 2, \ldots, M_R\}$, $S_T = \{1, 2, \ldots, M_T\}$ and channel matrix $H$ of $M_R \times M_T$ dimension.

2. For all $i \in S_R$, calculate the squared absolute values of sum of elements along each row, $a_i = \left| \sum_j h_{ij} \right|^2$.

3. Loop
   a. Choose $P$ such that $P = \arg \max_{i \in S_R} a_i$
   b. Delete $P$ from $S_R$ and redefine $S_R$ devoid of $P$
   c. Go to Loop until $M_R - |I_R|$ indices are in $S_R$

4. Remove set $S_R$ from the original set $\{1, 2, 3, \ldots, M_R\}$ and assign it to $I_R$. $I_R$ is the set of receive antennas selected.

5. $H = H_{|I_R|}$. Now, $H$ will contain only the rows corresponding to the selected receive antennas.
6. For all \( j \in S_T \), calculate the squared absolute values of sum of elements along each column, \( \beta_j = \left| \sum_i h_{ij} \right|^2 \)

7. Loop

   a. Choose \( Q \) such that \( Q = \arg \max_{j \in S_T} \beta_j \)

   b. Delete \( Q \) from \( S_T \) and redefine \( S_T \) devoid of \( Q \)

   c. Go to Loop until \( M_T - I_T \) indices are in \( S_T \)

8. Remove set \( S_T \) from the original set \( \{1, 2, 3, \ldots, M_T\} \) and assign it to \( \xi_T \). \( \xi_T \) is the set of transmit antennas selected.

### 4.7 COMPLEXITY ANALYSIS

JNBA needs to find the absolute values of all the elements of the channel matrix. The number of elements are \( M_R M_T \). For each absolute value of a complex number, it is necessary to first carry out multiplication between the complex number and its conjugate and then carry out a square root. Hence, each absolute value demands one complex-complex multiplication and one square root operation. Based on these facts, we can conclude that the algorithm has to carry out \( M_R M_T \) number of complex-complex multiplications. The DAVBA needs to carry out only the addition of the channel coefficients along a row or a column and then the squared norm values of the summed values. In this perspective, it has to carry out \( M_T - 1 \) complex-complex summations for each row and a total of \( (M_T - 1) M_R \) for all the rows. Additional \( M_R \) flops are required for finding the squared absolute values of all the summations. Similar discussion is applicable for the transmit antenna selection. The DNBA carries out vector multiplication. Hence, this algorithm is the most complicated of the three proposed norm-based algorithms. It must be noted that decoupling effect is involved in both
DAVBA and DNBA. Lines numbered 5 of both the algorithms, given in sections 4.5 and 4.6 convey the decoupling effect. Table 4.1 gives the complexity expressions of algorithms in complex-complex flops and real-real flops. Table 4.3 gives the outage capacity and the real-real flop complexity of various algorithms for selected antennas up to 7. The complexities have been given separately for $M_R = M_T = 8$ and $L_R = L_T = 4$ case in Table 4.2. In this, in addition to absolute flops, normalised flops have been given to compare easily.

**Table 4.1** Expressions for complex-complex flops and real-real flops of various algorithms

<table>
<thead>
<tr>
<th>Name of Algorithm</th>
<th>Complexity in complex-complex flops</th>
<th>Complexity in real-real flops</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed JNBA</td>
<td>$M_T M_R$</td>
<td>$6M_T M_R$</td>
</tr>
<tr>
<td>Proposed DAVBA</td>
<td>$(M_T-1)M_R + M_R + (L_R-1)M_T + M_T$</td>
<td>$\frac{1}{2}(2(M_T-1)-2) \cdot 6M_R + {\frac{1}{2}(L_R-1)-2} \cdot 6M_T$</td>
</tr>
<tr>
<td>Proposed DNBA</td>
<td>$2M_T M_R - M_R + 2L_R M_T - M_T$</td>
<td>$6M_T M_R + {\frac{1}{2}(M_T-1)-2} M_R + 6M_T \cdot {\frac{1}{2}(L_R-1)-2} M_T$</td>
</tr>
</tbody>
</table>

**Table 4.2** Complexity of various algorithms in real-real flops for $M_R = M_T = 8$ and $L_R = L_T = 4$

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Absolute Complexity</th>
<th>Complexity Normalised with respect to DAVBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed JNBA</td>
<td>384</td>
<td>1.71</td>
</tr>
<tr>
<td>Proposed DAVBA</td>
<td>224</td>
<td>1</td>
</tr>
<tr>
<td>Proposed DNBA</td>
<td>704</td>
<td>3.14</td>
</tr>
</tbody>
</table>
Table 4.3 Complexity of and outage capacity by various algorithms

<table>
<thead>
<tr>
<th>Number of Antennas</th>
<th>Complexity in Real-Real flops</th>
<th>Outage Capacity in bits/s/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JNBA</td>
<td>DAV BA</td>
</tr>
<tr>
<td>1</td>
<td>384</td>
<td>176</td>
</tr>
<tr>
<td>2</td>
<td>384</td>
<td>192</td>
</tr>
<tr>
<td>3</td>
<td>384</td>
<td>208</td>
</tr>
<tr>
<td>4</td>
<td>384</td>
<td>224</td>
</tr>
<tr>
<td>5</td>
<td>384</td>
<td>240</td>
</tr>
<tr>
<td>6</td>
<td>384</td>
<td>256</td>
</tr>
<tr>
<td>7</td>
<td>384</td>
<td>272</td>
</tr>
</tbody>
</table>

4.8 SIMULATION CONSIDERATIONS AND RESULTS

For all the simulations, a 2000-channel average has been obtained. The type of channel assumed was of Rayleigh flat-fading type, where the elements of $\mathbf{H}$ are i.i.d. complex Gaussian of zero-mean and unit variance. The outage probability assumed was 0.1. The following plots have been obtained for JNBA, DNBA and DAVBA.

i. The outage capacity in bits/s/Hz versus $L_R$ and $L_T$, where $L_R = L_T$

ii. The outage capacity in bits/s/Hz versus SNR in dB for two different $L_R$ and $L_T$, $L_R = L_T = 3$ and 4.

iii. The cumulative probability density versus instantaneous capacity in bits/s/Hz for two different $L_R$ and $L_T$, $L_R = L_T = 3$ and 4.
iv. The number of real-real flops versus $L_R$ and $L_T$, where $L_R = L_T$.

Figure 4.1 shows the variation of outage capacity performance with the number of selected antennas for five different selection schemes including the decoupled random one. The JNBA is the one, which delivers the same performance as the joint optimal one in single antenna selection. The JNBA demands a little more amount of flops than the DAVBA. But, the performance in single antenna selection is significantly superior to the DAVBA. The complexity of JNBA is significantly lower than the DNBA. The performances in single transmit and receive antenna selections of all the proposed algorithms are significantly superior to the decoupled random selection. For other $L_R$ and $L_T$ values, though the performances of all the proposed simple algorithms are less than the joint optimal, their complexities are negligibly low compared to the complexities of the joint optimal selection. The JNBA algorithm delivers the lowest performance among the three proposed algorithms for more than one $L_R$ and $L_T$ values.

![Figure 4.1](image-url)  

**Figure 4.1** Variation of the outage capacity performance in bps/Hz with the number of selected antennas
Figure 4.2  Variation of outage capacity in bps/Hz with SNR in dB for $M_T=M_R=8$ and $L_R=L_T=4$  

Figure 4.3  The variation of cumulative probability density with different instantaneous capacities in bps/Hz. with SNR in dB =20, $M_T=M_R=8$ and $L_R=L_T=3$ and $4$
DAVBA’s performance is close to the performance of DNBA. But, the complexity of DAVBA is significantly lower than that of the DNBA. Hence, for more than one $L_R$ and $L_T$ values, though the performance of DNBA is superior to DAVBA, the performance/complexity tradeoff of DAVBA can be understood to be better than that of the DNBA. Figure 4.2 is on variation of the outage capacity with the SNR in dB. It can be seen that the difference between the joint optimal and the norm-based algorithms is low on the low-SNR side as equation (4.7) suggests. Figure 4.3 has been obtained to see the capacity distribution of different algorithms for two different $L_R$ and $L_T$ values. It can be seen that the highest and the lowest instantaneous capacities achieved are almost the same for DNBA and DAVBA. The capacity distributions are also not much different for these two schemes. The distributions of the instantaneous capacities of all the three proposed algorithms are considerably better than that of the random one. Figure 4.4 is on the real-real flops versus $L_R$ and $L_T$ for the three proposed algorithms.

![Figure 4.4 Variation of real-real flops with the number of antennas selected with $L_R = L_T$.](image-url)
4.9 SUMMARY

In this chapter, three norm-based computationally simple, yet, significantly well-performing transmit/receive algorithms were proposed. The proposed norm-based joint and decoupled algorithms, joint optimal selection, and decoupled random selection were simulated under i.i.d. conditions for instantaneous capacity distribution and average-based outage capacity. The exact complexities were calculated in terms of expressions. An example calculation was made for all these algorithms. The algorithms proposed in this chapter as well as all other algorithms discussed so far are all for the instantaneous antenna selection. In instantaneous antenna selection concept, the channel has to be identified within the training sequence time, and selection must be carried out fast. The proposed norm-based algorithms are very fast and can be used even in channels that may be somewhat fast varying. They can also be used, where low-cost or low-speed processors need to be used. The norm-based algorithms discussed in this chapter are all for transmit/receive antenna selection. These algorithms have not been compared with other transmit/receive algorithms in terms of performance and complexity. It is necessary to have a unified study or investigation of the transmit/receive algorithms and understand their performances and complexities relatively. This exercise is carried out in the next chapter.