Abstract

The main results of this thesis are combinatorial in nature. We will be mainly working with the continuous automorphisms on the torus $\mathbb{T}^2$ and with the continuous self maps on $\mathbb{R}$. The thesis is conveniently divided into five chapters.

The general mathematical setting is that of an abstract dynamical system with discrete time parameter, that is, a pair (X, f) where X is a topological space and f a continuous mapping of X into itself. We are interested in the action of the iterates of f on X.

Chapter-1 is introductory in nature. We explain the basic notions of discrete dynamical systems and some important results emphasizing the role of the set of periodic points and the set of periods in chaos. We discuss briefly about the definitions of chaos due to Devaney and Li-Yorke.

It is already known [21] that for a hyperbolic (having no eigen values on the unit circle) continuous toral automorphism, the periodic points are precisely the rational points. In chapter-2, we calculate the set of periodic points for other continuous toral automorphisms; this happens to be the subgroup generated by $\mathbb{Q} \times \mathbb{Q} \cup$ (a line with rational slope). In fact, for all non-hyperbolic continuous toral automorphisms, there are uncountably many periodic points. Also, we prove that: every such subgroup is the set of periodic points for some continuous toral automorphism.

In Chapter-3, we first discuss some well known results about $Per(f)$ due to Sharkovski [17], Baker [5] and many others; our main result of this chapter is similar in spirit to these. We prove that there are exactly 8 subsets of $\mathbb{N}$ which can occur as $Per(T)$ for some continuous toral automorphism T. We solve the problem separately for hyperbolic and nonhyperbolic automorphisms. It is interesting that, for nonhyperbolic toral automorphisms, we are able to list the set $Per(T)$ in terms of the minimal polynomial

of T.

In chapter-4, we introduce the notion of *special points* and *nonordinary points* of a dynamical system. These notions are new to the literature, though they arise very naturally. By a special point we mean a point in the system which is unique by possessing some dynamical property. We call a point to be *ordinary* if it is "like" points near it. The points which are not ordinary are called *nonordinary.* It is observed that for systems with finitely many nonordinary points, the idea of nonordinary points and the idea of special points coincide.

We prove in [25] that the special points are actually inside the closure of full orbits of periodic points, critical points and possibly the limits at infinity. We call a system $(\mathbb{R}, f)$ to be *simple* if there are only finitely many kinds of orbits (upto order conjugacy). We describe completely, a class of simple systems namely, homeomorphisms with finitely many nonordinary points and give a general formula for counting. Also we prove that there are exactly 26 continuous self maps on $\mathbb{R}$ with a unique nonordinary point.

The main result of chapter-5 is obtained while making an attempt to answer the question: Exactly which maps on $\mathbb{R}$ are topologically conjugate to a polynomial?. The main theorem of this chapter, gives a necessary condition for a continuous self map on $\mathbb{R}$ to be conjugate to a polynomial.