ABSTRACT

The present investigation deals with the performance prediction of machining system with mixed standbys having permanent as well as additional repairmen. The individual along with common cause failure have been taken account while developing Markov model based on finite population queueing theory. The concepts of correlated queue and dependence are also incorporated. If any unit fails, it is sent immediately for repair if server is available, otherwise waits in the queue. When all spares are used, the system begins to work in degraded mode. The queue size distribution in equilibrium state for the system is established using recursive method. Some performance measures viz. expected number of failed units, the expected number of operative units in the system etc. have been established to help in expediting the decision making process.

3.1 Introduction

The study of fascinating area of machine repair problems via queuing theory approach can play a crucial role in predicting system descriptors of manufacturing and production systems, computer and communication systems, transportation and distribution systems, etc.. The vital needs of modern science and technology insistently put forward the task of developing a systematic approach to the study of any phenomena related to blocking and delay of resources in the machining environment of day-to-day activities. In industrial world, the machine repair problems arise in many areas wherein a machine may fail and thus requires a corrective action by the repairmen after which it again starts working properly. If at any time more than one machine needs the repairman’s attention, an interference queue occurs. The provision of repair facility in addition to standbys helps in the smooth running of the machining system with least interference due to machine failure which is an unavoidable phenomenon.

Machine failure has a highly influence on the performance of any machining system. The machining failure may be balanced by providing spare part support and
better repair facility by the system designers and decision makers of any organization so that the production may not suffer. The important recent past research works done in the area of machine repair problem with spares are reported in a recent survey article by Haque and Armstrong (2007). Zang et al. (2006) obtained availability and reliability of k-out-of-(M+N): G warm standbys systems. Wang et al. (2007b) discussed a profit analysis of the M/M/R machine repair problem. Jain et al. (2008c) discussed the performance analysis of double ended Markovian queue for machining system with spares. Mishra and Shukla (2010) did optimality analysis of machine interference model with spares with the help of the N-R method to optimize the total cost function. Yuan and Meng (2011) discussed reliability analysis of a warm standby repairable system with priority in use. Ke and Wu (2012) investigated a machine repair problem with homogeneous machines and standbys available, in which multiple technicians are responsible for supervising these machines.

It is worthwhile to report some important contributions in the area of machine repair problem in which the failure and repair rates depend on the state of the system (i.e. state dependent rates). Jain and Singh (2005) studied state dependent bulk service queue of a machining system with delayed vacation. Improved generic indices for machine repair system with state dependent rates, reneging, spares and additional repairmen were obtained by Jain and Kumari (2007). Jain et al. (2008b) analyzed state dependent system with mixed standbys and two modes of failure. Jain and Pandey (2009) explored a $M^X/G^{a,b}/1$ of state dependent queue with balking, multiple vacations and setup times. Ahmad and Onno (2010) studied the transient behavior of a state dependent M/M/1/K queue during the busy period. Wang (2011c) discusses an inspection model that is established under an assumption that the system's state can be classified into four states corresponding to a three-stage failure process. The failed state is always observed immediately, but the other three states, namely normal, minor defective and severe defective, can only be identified by an inspection. Eryilmaz and Tank (2012) studied a series system with two active components and a single cold standby unit. The two simultaneously working states are assumed to be dependent.

The provision of additional removable repairmen in the case of long queue may be helpful in reducing the waiting time as well as balking behavior of the caretaker of the failed machines in many real time systems working in machining environment. The concepts of correlated queue and controllable rates have been
studied by a few researchers. The machining systems with additional repairmen have been studied by many researchers (cf. Jain et al., 2000b; Jain and Maheshwari, 2003). Jain et al. (2004c) analyzed M/M/C interdependent machining system with mixed spares and controllable rates. Jain and Sharma (2005) developed controllable multi server queue with balking. The M/M/\(a,b\)/C interdependent queueing model with controllable arrival rates has been discussed by Sitrarasu et al. (2007). Markovian queueing model with reneging, no passing and additional servers has been developed by Jain et al. (2007e). Jain et al. (2008d) studied time sharing queueing system with nopassing and removable additional servers. Jain and Aggrwal (2009) gave optimal policy for bulk queue with multiple types of server breakdown. Allen and Ming (2010) obtained the queue length distribution of a multiple-server queueing system with time varying arrival and service rates when these rates are high. Jain and Upadhyaya (2011) presented modeling and analysis of unreliable Markovian multiserver finite-buffer queue with discouragement and synchronous working vacation policy. Sakuma and Inoie (2012) analyzed a multi-server queueing system with multiple vacations in which the vacation duration of each server is exponentially distributed.

A common cause failure may be defined as any instance where multiple units or elements fail due to a single cause. Machine repair problems with common cause failures are studied by several research workers in different frame-works (cf. Zhang et al., 2006; Wang et al., 2007a). Exponential asymptotic property of a parallel repairable system with common cause failure was discussed by Shen et al. (2008). A system of n-identical component with human and common-cause failure was modeled by El-Damcese (2009b). Levitin and Xing (2010) presented an algorithm for evaluating performance distribution of complex series-parallel multi state systems with common cause failures caused by propagation of failures in the system elements. The reliability and availability are considered for four series configurations with both warm and cold standby with the existence of common cause failure of the system at all states by Hajeeh (2011). Kancev and Cepin (2012b) presented a new method for explicit modeling of single component failure event within multiple common cause failure groups simultaneously.

In this investigation, machine repair problem with mixed standbys, additional repairmen, state dependent rates and common cause failure is studied. The rest of the
chapter is arranged as follows. The description of the model and notations used are
given in section 3.2. In section 3.3, the transition flow rates and equations governing
the model are constructed. The queue size distribution of the system for steady state is
obtained in section 3.4. Some performance measures such as mean queue length,
expected waiting time, etc. are established in Section 3.5. Finally the conclusion is
drawn in section 3.6.

3.2 Model Description

Consider machine repair system with mixed standbys, additional repairmen,
state dependent rates and common cause failures. For formulating the model
mathematically, the following assumptions are made:

- The arrival and repair processes of the permanent and additional server finite
capacity queueing system are correlated and governed by a bivariate Poisson
process having the joint probability mass function of the form:

\[
P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_n + \mu_n \varepsilon) t} \frac{\min(x_1, x_2)}{x_1! x_2!} \left(\frac{\lambda_n - \varepsilon}{\mu_n \varepsilon}\right)^{x_1} \left(\frac{\mu_n - \varepsilon}{\lambda_n \varepsilon}\right)^{x_2}
\]

\[x_1, x_2 = 0, 1, 2, \ldots; \quad 0 < \lambda_n, \mu_n; \quad 0 < \varepsilon < \min(\lambda_n, \mu_n)\]

with parameters \(\lambda_n(n = 0, 1, 2, \ldots, L), \mu_n(n = 1, 2, \ldots, L)\) and \(\varepsilon\) as mean
dependence rate between failure rate and repair rate.

- There are M operating and k types of mixed standbys units in the system.

- The system will work with at least m operating units whereas for normal
functioning M operating units are required.

- The repair facility consists of R permanent repairmen and k additional removable
repairmen for the smooth running of the system and to maintain the amount of
production up to a desired goal.

- If the number of failed units is more than the permanent repairmen then we
employ the additional removable repairmen one by one depending upon work
load.

- In case when all standbys are exhausted, the remaining online units fail in a
degraded fashion with faster rate.

- The switching time from standby to operating state and repaired to standby state
are assumed to be negligible.

- After repair the failed unit is as good as before failure.

- The system will fail when there are \(L = M + S^{(k)} - m + 1\) or more failed units in
the system; \(S^{(k)} = \sum_{i=1}^{k} S_i\).
We develop the mathematical model by using some notations which are given below:

- $M$: The number of operating units in the machining system
- $S_i$: The number of $i^{th}$ ($1 \leq i \leq k$) type spare units
- $R$: The number of permanent repairmen
- $k$: The number of additional removable repairmen
- $\lambda$: Failure rate of operating units
- $\lambda_d$: Degraded failure rate of operative units when all spare units exhausted
- $\alpha_i$: Failure rate of $i^{th}$ ($1 \leq i \leq k$) type spare unit
- $\mu$: Repair rate of permanent repairmen
- $\mu_j$: Repair rate of $j^{th}$ ($j=1, 2, \ldots, k$) additional repairmen
- $\mu_f$: Faster repair rate of permanent repairmen
- $\varepsilon$: Mean dependence rate i.e. covariance between failure and repair processes
- $\lambda_c$: Common cause failure rate

### 3.3 The Transition Flow Rates and Governing State Equations

When there are $n$ failed units in the system, the state dependent transition rates $\lambda_n$ and $\mu_n$ corresponding to failure and repair processes respectively are given by

\[
\lambda_n = \begin{cases} 
M(\lambda - \varepsilon) + (S_i - n)(\alpha_i - \varepsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \varepsilon), & 0 \leq n < S_i \\
M(\lambda - \varepsilon) + (S^{(i)} - n)(\alpha_i - \varepsilon) + \sum_{j=i+1}^{k} S_j(\alpha_j - \varepsilon), & S^{(i)} \leq n < S^{(i+1)} \\
(M + S^{(k)} - n)(\lambda_d - \varepsilon), & S^{(k)} \leq n < L \\
0, & \text{Otherwise}
\end{cases}
\]

where,

\[
S^{(i)} = S_1 + S_2 + S_3 + \ldots + S_i,
\]

**For 0th level:** At this level repair is done by the permanent repairmen. Here

\[
\mu_n = \begin{cases} 
n(\mu - \varepsilon), & 0 \leq n < R \\
R(\mu - \varepsilon), & R \leq n < S_i - 1
\end{cases}
\]
For 1st level: At this level additional repairmen start repair. Here

\[
\mu_n = \begin{cases} 
R(\mu_1 - \varepsilon) + (\mu_1 - \varepsilon), & R + 1 < n < S_1 \\
R(\mu_f - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon), & S_1 + 1 < n \leq S^{(2)} \\
R(\mu_f - \varepsilon) + \sum_{j=1}^{i} (\mu_j - \varepsilon), & S^{(i-1)} + 1 < n \leq S^{(i+1)} \\
R(\mu_f - \varepsilon) + \sum_{j=1}^{k} (\mu_j - \varepsilon), & S^{(k)} < n \leq L
\end{cases}
\]

Using appropriate state dependent rates given as above we construct the governing steady state equations as:

- \[ [M(\lambda - \varepsilon) + S_1 (\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon)] P_n(0) + \mu P_1(0) = 0 \] (3.1)

- \[ [M(\lambda - \varepsilon) + (S_1 - n)(\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon) + n(\mu - \varepsilon)] P_n(0) + [M(\lambda - \varepsilon) + (S_1 - n + 1)](\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon)] P_{n+1}(0) = 0, \quad 1 \leq n < R \] (3.2)

- \[ [M(\lambda - \varepsilon) + (S_1 - R)(\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon) + R(\mu - \varepsilon)] P_R(0) + [M(\lambda - \varepsilon) + (S_1 - R + 1)](\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon)] P_{R+1}(0) = 0 \] (3.3)

- \[ [M(\lambda - \varepsilon) + (S_1 - n)(\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon) + R(\mu - \varepsilon)] P_n(0) + [M(\lambda - \varepsilon) + (S_1 - n + 1)](\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon)] P_{n+1}(0) = 0, \quad R \leq n < S_1 - 1 \] (3.4)

- \[ [M(\lambda - \varepsilon) + (\alpha_1 - \varepsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon) + R(\mu - \varepsilon)] P_{S_1}(0) + [M(\lambda - \varepsilon) + 2(\alpha_1 - \varepsilon)] + \sum_{j=2}^{k} S_j (\alpha_j - \varepsilon)] P_{S_1+1}(0) = 0 \] (3.5)
\[-[M(\lambda - \varepsilon) + (S^{(2)} - \frac{R + 1}{\mu}) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_{n+1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_{n+2} (l) = 0 \]  
\( (3.6) \)

\[-[M(\lambda - \varepsilon) + (S^{(2)} - n) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_n (l) + [M(\lambda - \varepsilon) + (S^{(2)} - n+1) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{n-1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_{n+1} (l) = 0, \]  
\( R+1 < n < S_1 \)  
\( (3.7) \)

\[-[M(\lambda - \varepsilon) + (S^{(2)} - S_1) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_{S_1} (l) + [M(\lambda - \varepsilon) + (S^{(2)} - S_1+1) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{S_1-1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 - \varepsilon)] P_{S_1+1} (l) + [M(\lambda - \varepsilon) + (\alpha_1 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{S_1-1} (0) = 0 \]  
\( (3.8) \)

\[-[M(\lambda - \varepsilon) + (S^{(2)} - n) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_n (l) + [M(\lambda - \varepsilon) + (S^{(2)} - n+1) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{n-1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_{n+1} (l) = 0, \]  
\( S_1 < n < S^{(2)} \)  
\( (3.9) \)

\[-[M(\lambda - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_{S^{(2)}} (l) + [M(\lambda - \varepsilon) + (\alpha_2 - \varepsilon)] + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{S^{(2)}-1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_{S^{(2)}+1} (l) = 0 \]  
\( (3.10) \)

\[-[M(\lambda - \varepsilon) + (S^{(2)} - n) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_n (l) + [M(\lambda - \varepsilon) + (S^{(2)} - n+1) (\alpha_2 - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{n-1} (l) + [R(\mu_i - \varepsilon) + (\mu_1 + \mu_2 - 2\varepsilon)] P_{n+1} (l) = 0, \]  
\( S^{(2)} < n < S^{(3)} \)  
\( (3.11) \)

\[-[M(\lambda - \varepsilon) + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon) + R(\mu_i - \varepsilon) + \sum_{j=3}^{i} (\mu_j - \varepsilon)] P_{S^{(3)}} (l) + [M(\lambda - \varepsilon) + (\alpha_i - \varepsilon)] + \sum_{j=3}^{k} S_j (\alpha_j - \varepsilon)] P_{S^{(3)}-1} (l) + [R(\mu_i - \varepsilon) + \sum_{j=3}^{i} (\mu_j - \varepsilon)] P_{S^{(3)}+1} (l) = 0 \]  
\( (3.12) \)
\[-[M(\lambda - \epsilon) + (S^{(i)} - n) (\alpha_j - \epsilon) + \sum_{j=1}^{k} S_j (\alpha_j - \epsilon) + R(\mu_t - \epsilon) + \sum_{j=1}^{i} (\mu_j - \epsilon) P_n(1) + [M(\lambda - \epsilon)] \]

\[+ (S^{(i)} - n+1)(\alpha_j - \epsilon) + \sum_{j=1}^{k} S_j (\alpha_j - \epsilon) ] P_{n-1}(1) + [R(\mu_t - \epsilon) + \sum_{j=1}^{i} (\mu_j - \epsilon) ] P_{n+1}(1) = 0, \]

\[S^{(i)} < n < S^{(i+1)} \quad (3.13)\]

\[- [M(\lambda_d - \epsilon) + R(\mu_t - \epsilon) + \sum_{j=1}^{k} (\mu_j - \epsilon) ] P_n(1) + [(M+1)(\lambda_d - \epsilon)] P_{n-1}(1) + [R(\mu_t - \epsilon)] \]

\[+ \sum_{j=1}^{k} (\mu_j - \epsilon) ] P_{n+1}(1) = 0 \quad (3.14)\]

\[- [(M+S^{(k)} - n)(\lambda_d - \epsilon) + R(\mu_t - \epsilon) + \sum_{j=1}^{k} (\mu_j - \epsilon) ] P_n(1) + [(M+S^{(k)} - n+1)(\lambda_d - \epsilon)] P_{n-1}(1) \]

\[+ [R(\mu_t - \epsilon) + \sum_{j=1}^{k} (\mu_j - \epsilon) ] P_{n+1}(1) = 0, \quad S^{(k)} < n < L \quad (3.15)\]

\[- [R(\mu_t - \epsilon) + \sum_{j=1}^{k} (\mu_j - \epsilon) ] P_L(1) + [(M+S^{(k)} - L+1)(\lambda_d - \epsilon)] P_{L-1}(1) = 0 \quad (3.16)\]

### 3.4 Queue Size Distribution

The steady state solution of equations (3.1)-(3.16) is obtained using the recursive technique obtained as follows:

\[
P_n(0) = \begin{cases} 
A^n P_0(0), & 0 \leq n \leq R \\
\frac{n!}{\prod_{i=1}^{n-R-1} K_{R+1}} \left(1 + \sum_{j=1}^{n-R-1} \prod_{i=j}^{n-R-1} K_{R+1} \right) \beta P_0(0), & R < n \leq S_1 - 1 
\end{cases}
\]

(3.17)

where

\[A^n = \frac{\prod_{i=0}^{n-1} M(\lambda - \epsilon) + (S_i - i)(\alpha_j - \epsilon) + \sum_{j=2}^{k} S_j (\alpha_j - \epsilon)}{\mu^n}, \quad \beta = \frac{A^{R+1}}{(R+1)!}, \]

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\[
K_{R+i} = \frac{M(\lambda - \epsilon) + (S_i - i)(\alpha_i - \epsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \epsilon)}{R\mu},
\]

\[
P_n(l) = \left\{ \begin{array}{ll}
1 + \sum_{i=1}^{n-R-1} \prod_{j=i}^{n-R-1} \delta_{R+j} & R + 1 \leq n \leq S_i - 1 \\
\sum_{i=1}^{S_i - R} \prod_{j=i}^{S_i - R} \delta_{R+j} & n = S_i + 1 \\
1 + \sum_{i=1}^{n-S_i} \prod_{j=i}^{S_i - 1} \delta_{R+j} - C_0 \left[ 1 + \sum_{i=2}^{n-S_i} \prod_{j=i}^{S_i - 1} \delta_{R+j} \right] & S_i + 1 < n \leq S^{(2)} \\
1 + \sum_{i=1}^{n-S_i} \prod_{j=i}^{S_i - 1} \delta_{R+j} & S^{(i)} + 1 < n \leq S^{(i+1)} \\
1 + \sum_{i=1}^{n-S_i} \prod_{j=i}^{S_i - 1} \delta_{R+j} - C_2 \left[ 1 + \sum_{i=2}^{n-S_i} \prod_{j=i}^{S_i - 1} \delta_{R+j} \right] & S^{(k)} + 1 < n \leq L 
\end{array} \right.
\]

where

\[
\delta_{R+i} = \frac{M(\lambda - \epsilon) + (S_i - i)(\alpha_i - \epsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \epsilon)}{R(\mu_f - \epsilon) + (\mu_i - \epsilon)},
\]

\[
\gamma_{S_i+1} = \frac{M(\lambda - \epsilon) + (S^{(2)} - i)(\alpha_2 - \epsilon) + \sum_{j=3}^{k} S_j(\alpha_j - \epsilon)}{R(\mu_f - \epsilon) + (\mu_i + \mu_2 - 2\epsilon)}
\]

\[
U_{S^{(i)}+j} = \frac{M(\lambda - \epsilon) + (S^{(i)} - j)(\alpha_i - \epsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \epsilon)}{R(\mu_f - \epsilon) + \sum_{j=1}^{i} (\mu_j - \epsilon)},
\]

\[
V_{S^{(i)+1}} = \frac{M + S^{(k)} - i(\lambda_d - \epsilon)}{R(\mu_f - \epsilon) + \sum_{i=1}^{k} (\mu_i - \epsilon)}
\]

\[
C_0 = \frac{M(\lambda - \epsilon) + (S_i - i)(\alpha_i - \epsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \epsilon)}{R(\mu_f - \epsilon) + (\mu_i + \mu_2 - 2\epsilon)},
\]

\[
C_2 = \frac{M(\lambda - \epsilon) + (S^{(k-1)} - j)(\alpha_{k-1} - \epsilon) + S_k(\alpha_k - \epsilon)}{R(\mu_f - \epsilon) + \sum_{i=1}^{k} (\mu_i - \epsilon)}
\]

\[
C_1 = \frac{M(\lambda - \epsilon) + (S^{(k-1)} - j)(\alpha_{k-1} - \epsilon) + \sum_{j=2}^{k} S_j(\alpha_j - \epsilon)}{R(\mu_f - \epsilon) + \sum_{i=1}^{k} (\mu_i - \epsilon)}
\]
$P_0$ is obtained by using normalizing condition, i.e.

$$
\sum_{n=0}^{S-1} P_n(0) + \sum_{n=R+1}^{L} P_n(1) = 1
$$

(3.19)

### 3.5 Performance Measures

Some performance indices of interest in terms of probabilities are obtained as given below:

- The expected number of failed units in the system is

$$
E(n) = \sum_{n=0}^{S-1} nP_n(0) + \sum_{n=R+1}^{L} nP_n(1)
$$

(3.20)

- The expected number of operative units in the system is

$$
E(o) = M - \sum_{n=S^{(k)}+1}^{L} (n - S^{(k)})P_n(1)
$$

(3.21)

- The expected number of idle permanent repairmen in the system is

$$
E(i) = \sum_{n=0}^{R-1} (R - n)P_n(0)
$$

(3.22)

- The expected number of busy permanent repairmen in the system is

$$
E(b) = R - E(i)
$$

(3.23)

- The expected number of busy additional repairmen in the system is

$$
E(A) = \sum_{n=1}^{k} \sum_{n=S^{(k)}+1}^{L} P_n(1) + k \sum_{n=S^{(k)}+1}^{L} P_n(1)
$$

(3.24)

- The expected number of mixed standbys repairmen in the system is

$$
E(S) = \sum_{n=0}^{g^{(k)}} (S^{(k)} - n)P_n(1)
$$

(3.25)

- The throughput is obtained using the following

$$
T = \sum_{n=0}^{R} n(\mu - \varepsilon)P_n(0) + \sum_{n=R+1}^{S-1} R(\mu - \varepsilon)P_n(0) + \sum_{n=R+1}^{L} \left[ R(\mu_f - \varepsilon) + \sum_{j=1}^{L} (\mu_j - \varepsilon) \right] P_n(1)
$$

(3.26)

- The production rate of system is given by

$$
\eta = 1 - \frac{E(n)}{L}
$$

(3.27)
3.6 Cost Analysis

In this section, our main aim is to provide a cost function in order to find optimal number of repairmen and standbys. The different cost factors used to construct cost function are given below:

\[ C_M \] Cost per unit time of the operating units when the system works in normal mode.

\[ C_S \] Cost per unit time for providing a standby unit.

\[ C_I \] Cost per unit time of a permanent repairman when he is idle.

\[ C_B \] Cost per unit time of a permanent repairman when he is providing repair.

\[ C_A \] Cost per unit time of an additional repairman when he is providing repair.

The expected total cost per unit time is given by:

\[
E(C) = C_M \left( \sum_{n=0}^{S_i} P_n(0) + \sum_{n=R+1}^{S_{(k)}} P_n(1) \right) + C_S E(S) + C_I E(I) + C_B E(B) + C_A E(A) \tag{3.28}
\]

3.7 Conclusion

In this chapter, we have analyzed a machining system consisting of mixed spares and a repair facility having both permanent and additional repairmen. The provision of mixed spares and additional repairmen may be helpful to facilitate the uninterrupted service as well as to provide regular magnitude of production up to a desired grade of demand. The assumptions of common cause failure, controllable rate and mean dependence rates between the failure and repair processes make our model more realistic and robust to tackle real time system working in machining environment. The expressions for several system characteristics derived explicitly can be further employed to find out the optimal combination of spares and repairmen to facilitate the insights to the system designers to achieve the pre-specified target of production.