CHAPTER 7

ULTRASONIC WAVE’S INTERACTION AT FLUID-POROUS PIEZOELECTRIC LAYERED INTERFACE

7.1 Introduction

Numerous ultrasonic devices, based on either SAW or BAW, have been developed by making use of piezoelectric materials for a variety of applications. These devices are usually fabricated with a layered geometry, because of its high sensitivity, great band width and enhanced reception characteristics (Goldberg et al., 1997; Powell et al., 1998). Analysis of BAW in multilayered structures, composed of stacked piezoelectric, dielectric and metallic materials, has once again attracted the attention of engineers and researchers working on radio frequency components such as stacked filter, coupled resonator filters, and multiplexers built on solidly mounted resonators and thin film bulk acoustic resonators. Research into ultrasonic non destructive evaluation techniques for the inspection of multilayered structures relies strongly on the use of modeling tools which calculate dispersion curves and reflection and transmission spectra.

Many methods have been developed for studying the acoustic waves and ultrasonic waves spectra in layered media and majority of them are based on the matrix formalism. Fahmy and Adler (1973) proposed a model, based on a transfer matrix approach, to solve the plane wave propagation problems in piezoelectric multilayer.

Keeping the importance of the reflection-transmission phenomenon through layered media to a wide range of technical application in view, particularly in surface acoustic wave devices and characterization of composite materials and smart structures for the analysis of acoustic wave interaction with anisotropic and piezoelectric multilayers, the reflection and transmission of ultrasonic waves from a
fluid loaded porous piezoelectric layered structure is investigated in this Chapter. The layered structure is considered to be consisting of ‘n’ number of layers of PPM sandwiched between FHS and PPHS. The formal solution for the mechanical displacements, electric potentials, mechanical stresses and electrical displacements are obtained in the Second section. In the Section 3, the transfer matrix technique is used to study the layered materials. The analytical expressions of the elements of transfer matrix are derived. In the next section, amplitude ratios of reflected and transmitted waves are derived with the help of boundary conditions. The energy ratios of reflected and transmitted waves along with the expressions of interaction energy are also obtained in the Section 4. The analytical expression of surface impedance is derived in the Section 5. The effects of frequency, porosity, angle of incidence, layer thickness, number of layers and piezoelectricity on the energy ratios and surface impedance are studied for different configurations of the layered materials with the help of numerical computations in the next section. The results of Nayfeh and Taylor (1988) and Vashisht and Khurana (2002) are obtained as special cases from the studied model, which are in agreement with the earlier established results. The results obtained in the chapter 6 are also obtained as special case from the present model. A validation of the numerical technique (transfer matrix method) is also established by computing and comparing the results with an alternative numerical technique.

7.2 Formulation of the problem

We consider a laminated plate of thickness \( h \), consisting of \( n \) porous piezoelectric layers each having 6mm symmetry, bonded at their interfaces (Fig. 7.1). Let \( h_j (j = 1, 2, ..., n) \) be the thickness of the \( j^{th} \) layer. The laminated plate is assumed to be overlying a PPHS and underlying a FHS. The FHS occupies the region \(-\infty < x_3 < 0\) and the PPHS occupies the region \( h < x_3 < \infty\). A plane elastic wave in the FHS, making an angle \( \theta \) with the \( x_3 \) axis, is assumed to strike the interface \( x_3 = 0 \). This elastic wave proceeds through the ‘\( n\)’ intervening porous piezoelectric layers to emerge into the PPHS. In addition to one mode of reflected wave in the FHS, five waves are transmitted into the PPHS. The propagating transmitted modes are
represented as \( q_{P_1}, q_{S_1} \) and \( q_{P_2} \) waves and transmitted evanescent electric potential modes are represented by \( PE_1 \) and \( PE_2 \).

![Geometry of the problem](image)

**Figure 7.1** Geometry of the problem

### 7.3 Transfer Matrix

It has been proved in the Chapter 6 that, 10 waves can propagate in the bounded PPM, out of which 5 waves are upgoing waves and 5 waves are downgoing waves. Let \( q_i^{(j)} (i=1,2,...,10; j=1,2,...,n) \) correspond to the roots of the equation (6.5) in the \( j^{th} \) layer. Let \( q_i^{(j)} (i=1,2,...,5; j=1,2,...,n) \) represent vertical slowness of downward going waves and these values correspond to those roots of the equation
(6.5) whose imaginary parts are positive. Similarly, $q_i^{(j)}$ ($i = 6, 7, ..., 10; j = 1, 2, ..., n$) correspond to those roots of the equation (6.5) whose imaginary parts are negative and represent upward travelling waves. $q_1^{(j)}$, $q_2^{(j)}$ and $q_3^{(j)}$ correspond to propagating $qP_1$ wave, $qS_1$ wave and $qP_2$ wave and $q_4^{(j)}$ and $q_5^{(j)}$ correspond to evanescent electric potential wave modes in the $j^{th}$ layer.

These 10 roots $q_i^{(j)}$ can be arranged as

$$q_{10}^{(j)} = -q_1^{(j)}, q_9^{(j)} = -q_2^{(j)}, q_8^{(j)} = -q_3^{(j)}, q_7^{(j)} = -q_4^{(j)}, q_6^{(j)} = -q_5^{(j)}.$$  \hspace{1cm} (7.1)

Repeating the same steps, as detailed out in the Chapter 6, the formal solutions for the mechanical displacements, electric potentials, mechanical stresses and electrical displacements in the $j^{th}$ layer can be written as

$$(u_1^{(j)}, u_3^{(j)}, u_1^{*^{(j)}}, u_3^{*^{(j)}}, \Phi^{(j)}, \Phi^{*^{(j)}}) = \sum_{i=1}^{10} (l, r_{ij}^{(j)}, s_{ij}^{(j)}, r_{ij}^{(j)} + s_{ij}^{(j)}) B_{ii}^{(j)} \exp \left( t \omega \left( \frac{1}{c} x_i + q_i^{(j)} x_3 - t \right) \right), \hspace{1cm} (7.2)$$

$$(\sigma_{31}^{(j)}, \sigma_{33}^{(j)}, \sigma_{35}^{(j)}, D_3^{(j)}, D_3^{*^{(j)}}) = \sum_{i=1}^{10} (g_{1i}^{(j)}, g_{2i}^{(j)}, g_{3i}^{(j)}, g_{4i}^{(j)}, g_{5i}^{(j)}) B_{ii}^{(j)} \exp \left( t \omega \left( \frac{1}{c} x_i + q_i^{(j)} x_3 - t \right) \right), \hspace{1cm} (7.3)$$

where, the coefficients $r_k^{(j)}$ ($i = 1, 2, ..., 10; \kappa = 1, 2, ..., 5$) can be obtained from the equation (6.7) by making use the material parameters of the $j^{th}$ layer.

Further,

$$g_{1i}^{(j)} = t \omega \left[ c_{55} q_i^{(j)} + (c_{55} r_i^{(j)} + e_{15} r_i^{(j)} + \zeta_{15} r_i^{(j)}) / c \right],$$

$$g_{2i}^{(j)} = t \omega \left[ c_{33} r_i^{(j)} + m_{33} r_i^{(j)} + e_{33} r_i^{(j)} + \zeta_{33} r_i^{(j)} q_i^{(j)} + (c_{31} + m_{33} r_i^{(j)}) / c \right],$$

$$g_{3i}^{(j)} = t \omega \left[ (m_{33} r_i^{(j)} + R r_i^{(j)} + \zeta_{3} r_i^{(j)} + e_{3} r_i^{(j)}) q_i^{(j)} + (m_{11} + R r_i^{(j)}) / c \right],$$

$$g_{4i}^{(j)} = t \omega \left[ (e_{33} r_i^{(j)} + \zeta_{3} r_i^{(j)} - \bar{\zeta}_{33} r_i^{(j)} - A_{33} r_i^{(j)}) q_i^{(j)} + (e_{13} + \zeta_{3} r_i^{(j)}) / c \right],$$

$$g_{5i}^{(j)} = t \omega \left[ (\zeta_{33} r_i^{(j)} + e_{3} r_i^{(j)} - A_{33} r_i^{(j)} - \bar{\zeta}_{33} r_i^{(j)}) q_i^{(j)} + (\zeta_{31} + e_{3} r_i^{(j)}) / c \right]. \hspace{1cm} (7.4)$$

The equations (7.2) and (7.3) can be written as

$$V^{(j)} = X^{(j)} W^{(j)} S^{(j)}, \hspace{1cm} (7.5)$$
where

$$\mathbf{V}^{(j)} = [u_1^{(j)} u_3^{(j)} u_3^{* (j)} \Phi^{(j)} D_3^{(j)} \sigma^{(j)} \sigma_{13}^{(j)} \Phi^{(j)} D_3^{(j)}]^T,$$

$$\mathbf{X}^{(j)} =
\begin{bmatrix}
\eta_{11} & -\eta_{12} & -\eta_{13} & -\eta_{14} & -\eta_{15} & -\eta_{16} \\
-\eta_{21} & \eta_{22} & -\eta_{23} & -\eta_{24} & -\eta_{25} & -\eta_{26} \\
-\eta_{31} & -\eta_{32} & \eta_{33} & -\eta_{34} & -\eta_{35} & -\eta_{36} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} & \sigma_{26} \\
\sigma_{31} & \sigma_{32} & -\sigma_{33} & -\sigma_{34} & -\sigma_{35} & -\sigma_{36} \\
\sigma_{41} & -\sigma_{42} & -\sigma_{43} & \sigma_{44} & -\sigma_{45} & -\sigma_{46} \\
\sigma_{51} & -\sigma_{52} & \sigma_{53} & -\sigma_{54} & -\sigma_{55} & -\sigma_{56} \\
\sigma_{61} & -\sigma_{62} & -\sigma_{63} & \sigma_{64} & -\sigma_{65} & -\sigma_{66} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\end{bmatrix}
\quad \text{and}
$$

$$\mathbf{S}^{(j)} = [B_{11}^{(j)} B_{10}^{(j)} B_{12}^{(j)} B_{19}^{(j)} B_{13}^{(j)} B_{18}^{(j)} B_{14}^{(j)} B_{117}^{(j)} B_{15}^{(j)} B_{18}^{(j)}]^T,$$

$$\mathbf{W}^{(j)} =
\begin{bmatrix}
E_1^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E_{10}^{(j)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E_2^{(j)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E_3^{(j)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & E_8^{(j)} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E_4^{(j)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E_7^{(j)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E_5^{(j)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad \text{and}
$$

$$E_i^{(j)} = \exp(\im \omega q_i^{(j)} x), \quad (i = 1, 2, ..., 10). \quad (7.6)$$

Here, superscript $^T$ denotes the transpose of matrix, $\mathbf{V}^{(j)}$ is the field vector corresponding to the $j$th layer and $\mathbf{S}^{(j)}$ is the amplitudes vector corresponding to $j$th layer.

The relation between mechanical displacements, electric potentials, mechanical stresses and electrical displacements, at the top of the $j$th layer, and the amplitudes corresponding to the $j$th layer can be written as

$$\mathbf{V}_j = \mathbf{X}^{(j)} \mathbf{W}^{(j)} \mathbf{S}^{(j)}, \quad (7.7)$$
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where

\[
V_j^- = [V^{(j)}]_{x_i = h_i + h_{i+1} + \ldots + h_j}, \quad (2 \leq j \leq n) \quad \text{and} \quad V_i^- = [V^{(i)}]_{x_i = 0},
\]

\[
W_j^- = [W^{(j)}]_{x_i = h_i + h_{i+1} + \ldots + h_j}, \quad (2 \leq j \leq n) \quad \text{and} \quad W_i^- = [W^{(i)}]_{x_i = 0},
\]  

(7.8)

Similarly, the relation between mechanical displacements, electric potentials, mechanical stresses and electrical displacements at the bottom of the \( j \)th layer with the amplitudes corresponding to \( j \)th layer can be written as

\[
V_j^+ = X^{(j)} W_j^+ S^{(j)}.
\]  

(7.9)

where

\[
V_j^+ = [V^{(j)}]_{x_i = h_i + h_{i+1} + \ldots + h_j}, \quad (1 \leq j \leq n),
\]

\[
W_j^+ = [W^{(j)}]_{x_i = h_i + h_{i+1} + \ldots + h_j}, \quad (1 \leq j \leq n).
\]  

(7.10)

Eliminating the common amplitude vector \( S^{(j)} \) from the equations (7.7) and (7.9), we obtain

\[
V_j^- = X^{(j)} W_j^- (W_j^+)^{-1} (X^{(j)})^{-1} V_j^+.
\]  

(7.11)

The matrix \((X^{(j)})^{-1}\) is obtained as

\[
(X^{(j)})^{-1} = \begin{bmatrix}
    y_{41} & z_{41} & z_{42} & y_{42} & z_{43} & y_{43} & y_{21} & z_{21} & z_{22} & y_{22} \\
y_{41} & -z_{41} & -z_{42} & y_{42} & -z_{43} & y_{43} & y_{21} & -z_{21} & -z_{22} & y_{22} \\
y_{44} & z_{44} & z_{45} & y_{45} & z_{46} & y_{46} & y_{23} & z_{23} & z_{24} & y_{24} \\
y_{44} & -z_{44} & -z_{45} & y_{45} & -z_{46} & y_{46} & y_{23} & -z_{23} & -z_{24} & y_{24} \\
y_{47} & z_{47} & z_{48} & y_{48} & z_{49} & y_{49} & y_{25} & z_{25} & z_{26} & y_{26} \\
y_{47} & -z_{47} & -z_{48} & y_{48} & -z_{49} & y_{49} & y_{25} & -z_{25} & -z_{26} & y_{26} \\
y_{31} & z_{31} & z_{32} & y_{32} & z_{33} & y_{33} & y_{11} & z_{11} & z_{12} & y_{12} \\
y_{31} & -z_{31} & -z_{32} & y_{32} & -z_{33} & y_{33} & y_{11} & -z_{11} & -z_{12} & y_{12} \\
y_{34} & z_{34} & z_{35} & y_{35} & z_{36} & y_{36} & y_{13} & z_{13} & z_{14} & y_{14} \\
y_{34} & -z_{34} & -z_{35} & y_{35} & -z_{36} & y_{36} & y_{13} & -z_{13} & -z_{14} & y_{14}
\end{bmatrix},
\]  

(7.12)

where
\[ z_1 = s_{14} l(s_{14} s_{14} - s_{12} s_{13}) , \]
\[ z_2 = -s_{12} l(s_{14} s_{14} - s_{12} s_{13}) , \]
\[ z_3 = -s_{13} l(s_{14} s_{14} - s_{12} s_{13}) , \]
\[ z_4 = s_{11} l(s_{14} s_{14} - s_{12} s_{13}) , \]
\[ z_{21} = -(\alpha_{21}(z_{11} t^{(j)} + z_{14} r^{(j)}) + \alpha_{12}(z_{14} t^{(j)} + z_{13} r^{(j)}) + \alpha_{13}(z_{14} t^{(j)} + z_{13} r^{(j)})), \]
\[ z_{22} = -(\alpha_{21}(z_{12} t^{(j)} + z_{14} r^{(j)}) + \alpha_{12}(z_{12} t^{(j)} + z_{14} r^{(j)}) + \alpha_{13}(z_{12} t^{(j)} + z_{14} r^{(j)})), \]
\[ z_{23} = -(\alpha_{11}(z_{11} t^{(j)} + z_{13} r^{(j)}) + \alpha_{22}(z_{11} t^{(j)} + z_{13} r^{(j)}) + \alpha_{33}(z_{11} t^{(j)} + z_{13} r^{(j)})), \]
\[ z_{24} = -(\alpha_{31}(z_{11} t^{(j)} + z_{14} r^{(j)}) + \alpha_{22}(z_{12} t^{(j)} + z_{14} r^{(j)}) + \alpha_{33}(z_{12} t^{(j)} + z_{14} r^{(j)})), \]
\[ z_{25} = -(\alpha_{41}(z_{11} t^{(j)} + z_{13} r^{(j)}) + \alpha_{42}(z_{14} t^{(j)} + z_{13} r^{(j)}) + \alpha_{43}(z_{14} t^{(j)} + z_{13} r^{(j)})), \]
\[ z_{26} = -(\alpha_{41}(z_{12} t^{(j)} + z_{14} r^{(j)}) + \alpha_{42}(z_{12} t^{(j)} + z_{14} r^{(j)}) + \alpha_{43}(z_{12} t^{(j)} + z_{14} r^{(j)})), \]
\[ z_{31} = -(\alpha_{21}(z_{11} s^{(j)} + z_{31} g^{(j)}) + \alpha_{31}(z_{14} s^{(j)} + z_{31} g^{(j)}) + z_{12}(\alpha_{21} r^{(j)} + \alpha_{31} r^{(j)} + \alpha_{41} r^{(j)})), \]
\[ z_{32} = -(\alpha_{12}(z_{12} g^{(j)} + z_{14} s^{(j)}) + \alpha_{22}(z_{12} g^{(j)} + z_{14} s^{(j)}) + z_{12}(\alpha_{12} r^{(j)} + \alpha_{22} r^{(j)} + \alpha_{42} r^{(j)})), \]
\[ z_{33} = -(\alpha_{13}(z_{11} g^{(j)} + z_{33} s^{(j)}) + \alpha_{33}(z_{14} g^{(j)} + z_{33} s^{(j)}) + z_{12}(\alpha_{13} r^{(j)} + \alpha_{33} r^{(j)} + \alpha_{43} r^{(j)})), \]
\[ z_{34} = -(\alpha_{31}(z_{12} g^{(j)} + z_{31} s^{(j)}) + \alpha_{41}(z_{14} g^{(j)} + z_{31} s^{(j)}) + z_{14}(\alpha_{31} r^{(j)} + \alpha_{41} r^{(j)} + \alpha_{43} r^{(j)})), \]
\[ z_{35} = -(\alpha_{12}(z_{11} g^{(j)} + z_{14} s^{(j)}) + \alpha_{22}(z_{12} g^{(j)} + z_{14} s^{(j)}) + z_{14}(\alpha_{12} r^{(j)} + \alpha_{22} r^{(j)} + \alpha_{42} r^{(j)})), \]
\[ z_{36} = -(\alpha_{13}(z_{11} s^{(j)} + z_{33} s^{(j)}) + \alpha_{33}(z_{14} s^{(j)} + z_{33} s^{(j)}) + z_{14}(\alpha_{33} r^{(j)} + \alpha_{33} r^{(j)} + \alpha_{43} r^{(j)})), \]
\[ z_{41} = \alpha_{21} - 2(\alpha_{21}(z_{11} t^{(j)} + z_{34} r^{(j)}) + \alpha_{12}(z_{14} t^{(j)} + z_{34} r^{(j)}) + \alpha_{13}(z_{14} t^{(j)} + z_{34} r^{(j)})), \]
\[ z_{42} = \alpha_{12} - 2(\alpha_{21}(z_{32} t^{(j)} + z_{35} r^{(j)}) + \alpha_{12}(z_{32} r^{(j)} + z_{35} r^{(j)}) + \alpha_{13}(z_{32} r^{(j)} + z_{35} r^{(j)})), \]
\[ z_{43} = \alpha_{13} - 2(\alpha_{21}(z_{33} t^{(j)} + z_{36} r^{(j)}) + \alpha_{12}(z_{33} r^{(j)} + z_{36} r^{(j)}) + \alpha_{13}(z_{33} r^{(j)} + z_{36} r^{(j)})), \]
\[ z_{44} = \alpha_{31} - 2(\alpha_{21}(z_{31} t^{(j)} + z_{34} r^{(j)}) + \alpha_{22}(z_{31} r^{(j)} + z_{34} r^{(j)}) + \alpha_{33}(z_{31} r^{(j)} + z_{34} r^{(j)})), \]
\[ z_{45} = \alpha_{32} - 2(\alpha_{21}(z_{32} t^{(j)} + z_{35} r^{(j)}) + \alpha_{22}(z_{32} r^{(j)} + z_{35} r^{(j)}) + \alpha_{33}(z_{32} r^{(j)} + z_{35} r^{(j)})), \]
\[ z_{46} = \alpha_{33} - 2(\alpha_{21}(z_{33} t^{(j)} + z_{36} r^{(j)}) + \alpha_{22}(z_{33} r^{(j)} + z_{36} r^{(j)}) + \alpha_{33}(z_{33} r^{(j)} + z_{36} r^{(j)})), \]
\[ y_{11} = t_{14}'(t_{14}' - t_{12}' t_{13}'), \]
\[ y_{12} = -t_{13}'(t_{14}' - t_{12}' t_{13}'), \]
\[ y_{13} = -t_{12}'(t_{14}' - t_{12}' t_{13}'), \]
\[ y_{14} = t_{11}'(t_{14}' - t_{12}' t_{13}'), \]
\[ y_{21} = -(\beta_{21}(y_{11} + y_{13}) + \beta_{12}(y_{11} s^{(j)} + y_{13} s^{(j)}) + \beta_{13}(y_{11} s^{(j)} + y_{13} s^{(j)})), \]
\[ y_{22} = -(\beta_{21}(y_{14} + y_{12}) + \beta_{12}(y_{14} s^{(j)} + y_{14} s^{(j)}) + \beta_{13}(y_{14} s^{(j)} + y_{14} s^{(j)})), \]
\[ y_{23} = -(\beta_{31}(y_{11} + y_{13}) + \beta_{22}(y_{11} s^{(j)} + y_{13} s^{(j)}) + \beta_{33}(y_{11} s^{(j)} + y_{13} s^{(j)})), \]
\[ y_{24} = -(\beta_{31}(y_{14} + y_{12}) + \beta_{22}(y_{14} s^{(j)} + y_{14} s^{(j)}) + \beta_{33}(y_{14} s^{(j)} + y_{14} s^{(j)})). \]
\[ y_{24} = - (\beta_{31} (y_{14} + y_{12}) + \beta_{22} (y_{12} s_2^{(j)} + y_{14} s_2^{(j)}) + \beta_{33} (y_{12} s_4^{(j)} + y_{14} s_4^{(j)})), \]
\[ y_{25} = - (\beta_{41} (y_{11} + y_{13}) + \beta_{42} (y_{11} s_2^{(j)} + y_{13} s_2^{(j)}) + \beta_{43} (y_{11} s_4^{(j)} + y_{13} s_4^{(j)})), \]
\[ y_{26} = - (\beta_{41} (y_{14} + y_{12}) + \beta_{42} (y_{12} s_2^{(j)} + y_{14} s_2^{(j)}) + \beta_{43} (y_{12} s_4^{(j)} + y_{14} s_4^{(j)})), \]
\[ y_{31} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{32} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{33} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{34} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{35} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{36} = - (y_{11} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)}) + y_{12} (\beta_{21} s_1^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)})), \]
\[ y_{41} = \beta_{31} - 2 (\beta_{21} (y_{13} + y_{14}) + \beta_{12} (y_{31} s_2^{(j)} + y_{34} s_2^{(j)}) + \beta_{33} (y_{31} s_4^{(j)} + y_{34} s_4^{(j)})), \]
\[ y_{42} = \beta_{31} - 2 (\beta_{21} (y_{32} + y_{33}) + \beta_{12} (y_{32} s_2^{(j)} + y_{35} s_2^{(j)}) + \beta_{33} (y_{32} s_4^{(j)} + y_{35} s_4^{(j)})), \]
\[ y_{43} = \beta_{31} - 2 (\beta_{21} (y_{33} + y_{36}) + \beta_{12} (y_{33} s_2^{(j)} + y_{36} s_2^{(j)}) + \beta_{33} (y_{33} s_4^{(j)} + y_{36} s_4^{(j)})), \]
\[ y_{44} = \beta_{31} - 2 (\beta_{21} (y_{34} + y_{36}) + \beta_{12} (y_{34} s_2^{(j)} + y_{34} s_2^{(j)}) + \beta_{33} (y_{34} s_4^{(j)} + y_{34} s_4^{(j)})), \]
\[ y_{45} = \beta_{31} - 2 (\beta_{21} (y_{35} + y_{36}) + \beta_{12} (y_{35} s_2^{(j)} + y_{35} s_2^{(j)}) + \beta_{33} (y_{35} s_4^{(j)} + y_{35} s_4^{(j)})), \]
\[ y_{46} = \beta_{31} - 2 (\beta_{21} (y_{36} + y_{36}) + \beta_{12} (y_{36} s_2^{(j)} + y_{36} s_2^{(j)}) + \beta_{33} (y_{36} s_4^{(j)} + y_{36} s_4^{(j)})), \]
\[ y_{47} = \beta_{41} - 2 (\beta_{21} (y_{31} + y_{34}) + \beta_{22} (y_{31} s_2^{(j)} + y_{34} s_2^{(j)}) + \beta_{23} (y_{31} s_4^{(j)} + y_{34} s_4^{(j)})), \]
\[ y_{48} = \beta_{41} - 2 (\beta_{21} (y_{32} + y_{35}) + \beta_{22} (y_{32} s_2^{(j)} + y_{35} s_2^{(j)}) + \beta_{23} (y_{32} s_4^{(j)} + y_{35} s_4^{(j)})), \]
\[ y_{49} = \beta_{41} - 2 (\beta_{21} (y_{33} + y_{36}) + \beta_{22} (y_{33} s_2^{(j)} + y_{36} s_2^{(j)}) + \beta_{23} (y_{33} s_4^{(j)} + y_{36} s_4^{(j)})), \]
\[ t_{11} = s_{34}^{(j)} - 2 (\beta_{21} s_3^{(j)} + \beta_{31} s_3^{(j)} + \beta_{41} s_3^{(j)} + (\beta_{12} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{42} s_3^{(j)})) s_{24}^{(j)}, \]
\[ t_{12} = s_{35}^{(j)} - 2 (\beta_{21} s_3^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)} + (\beta_{12} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{42} s_3^{(j)})) s_{25}^{(j)}, \]
\[ t_{13} = s_{34}^{(j)} - 2 (\beta_{21} s_5^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)} + (\beta_{12} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{42} s_3^{(j)})) s_{24}^{(j)}, \]
\[ t_{14} = s_{35}^{(j)} - 2 (\beta_{21} s_5^{(j)} + \beta_{31} s_2^{(j)} + \beta_{41} s_3^{(j)} + (\beta_{12} s_1^{(j)} + \beta_{22} s_2^{(j)} + \beta_{42} s_3^{(j)})) s_{25}^{(j)}, \]
\[ s_{11} = g_{14}^{(j)} - 2 \left( (\alpha_{12} g_{11}^{(j)} + \alpha_{31} g_{12}^{(j)} + \alpha_{41} g_{13}^{(j)}) r_{14}^{(j)} + (\alpha_{12} g_{11}^{(j)} + \alpha_{22} g_{12}^{(j)} + \alpha_{42} g_{13}^{(j)}) r_{14}^{(j)} + (\alpha_{13} g_{11}^{(j)} + \alpha_{33} g_{12}^{(j)} + \alpha_{43} g_{13}^{(j)}) r_{14}^{(j)} \right), \]
\[ s_{12} = g_{15}^{(j)} - 2 \left( (\alpha_{12} g_{11}^{(j)} + \alpha_{31} g_{12}^{(j)} + \alpha_{41} g_{13}^{(j)}) r_{15}^{(j)} + (\alpha_{12} g_{11}^{(j)} + \alpha_{22} g_{12}^{(j)} + \alpha_{42} g_{13}^{(j)}) r_{15}^{(j)} + (\alpha_{13} g_{11}^{(j)} + \alpha_{33} g_{12}^{(j)} + \alpha_{43} g_{13}^{(j)}) r_{15}^{(j)} \right), \]
\[ s_{13} = r_{44}^{(j)} - 2((\alpha_{21} r_{51}^{(j)} + \alpha_{31} r_{52}^{(j)} + \alpha_{41} r_{53}^{(j)}) r_{14}^{(j)} + \\
(\alpha_{12} r_{51}^{(j)} + \alpha_{22} r_{52}^{(j)} + \alpha_{32} r_{53}^{(j)}) r_{34}^{(j)} + (\alpha_{13} r_{51}^{(j)} + \alpha_{33} r_{52}^{(j)} + \alpha_{43} r_{53}^{(j)}) r_{44}^{(j)}), \]
\[ s_{14} = r_{55}^{(j)} - 2((\alpha_{21} r_{51}^{(j)} + \alpha_{31} r_{52}^{(j)} + \alpha_{41} r_{53}^{(j)}) r_{15}^{(j)} + \\
(\alpha_{12} r_{51}^{(j)} + \alpha_{22} r_{52}^{(j)} + \alpha_{32} r_{53}^{(j)}) r_{35}^{(j)} + (\alpha_{13} r_{51}^{(j)} + \alpha_{33} r_{52}^{(j)} + \alpha_{43} r_{53}^{(j)}) r_{45}^{(j)}), \]
\[ \alpha_{11} = (r_{11}^{(j)} + 2 r_{12}^{(j)} \alpha_{32} r_{31}^{(j)}) / 2(r_{11}^{(j)})^2, \]
\[ \alpha_{12} = (2 \alpha_{42} \alpha_{32} r_{12}^{(j)} r_{33}^{(j)} - \alpha_{32} r_{12}^{(j)} - 2 \alpha_{11} \alpha_{42} r_{13}^{(j)} r_{11}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{13} = 2(\alpha_{32} \alpha_{43} r_{12}^{(j)} r_{33}^{(j)} - \alpha_{11} \alpha_{43} r_{13}^{(j)} r_{11}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{21} = (\alpha_{11} r_{11}^{(j)} - 2 \alpha_{11} \alpha_{41} r_{13}^{(j)} r_{11}^{(j)} + 2 \alpha_{32} \alpha_{41} r_{12}^{(j)} r_{33}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{22} = (\alpha_{32} r_{11}^{(j)} + 2 \alpha_{32} \alpha_{42} r_{13}^{(j)} r_{13}^{(j)} - 2 \alpha_{32} \alpha_{42} r_{33}^{(j)} r_{11}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{31} = (2 \alpha_{41} \alpha_{32} r_{13}^{(j)} r_{31}^{(j)} - \alpha_{32} r_{31}^{(j)} - 2 \alpha_{32} \alpha_{41} r_{13}^{(j)} r_{11}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{32} = (r_{31}^{(j)} / 2(r_{32}^{(j)} r_{41}^{(j)} - r_{12}^{(j)} r_{31}^{(j)}), \]
\[ \alpha_{33} = (2 \alpha_{32} \alpha_{43} r_{13}^{(j)} r_{13}^{(j)} - \alpha_{32} \alpha_{43} r_{11}^{(j)} r_{53}^{(j)}) / r_{11}^{(j)}, \]
\[ \alpha_{41} = -(2 \alpha_{43} / r_{11}^{(j)})(\alpha_{11} r_{11}^{(j)} r_{11}^{(j)} - \alpha_{32} r_{42} r_{31}^{(j)}) , \]
\[ \alpha_{42} = -(2 \alpha_{43} / r_{11}^{(j)})(\alpha_{32} r_{42} r_{11}^{(j)} - \alpha_{32} r_{41} r_{12}^{(j)}), \]
\[ \alpha_{43} = (r_{11}^{(j)} / 2(r_{13}^{(j)} r_{11}^{(j)} - 2 r_{13}^{(j)} (\alpha_{11} r_{11}^{(j)} r_{11}^{(j)} - \alpha_{32} r_{42} r_{31}^{(j)}) - \\
2 r_{33}^{(j)} (\alpha_{32} r_{42} r_{11}^{(j)} - \alpha_{32} r_{41} r_{12}^{(j)})), \]
\[ \beta_{11} = (1 + 2 \beta_{32} g_{21}^{(j)}) / 2, \]
\[ \beta_{12} = 2 \beta_{32} \beta_{42} g_{23}^{(j)} - \beta_{32} - 2 \beta_{11} \beta_{42}, \]
\[ \beta_{13} = 2(-\beta_{11} \beta_{43} + \beta_{32} \beta_{43} g_{23}^{(j)}), \]
\[ \beta_{21} = \beta_{11} - 2 \beta_{11} \beta_{41} + 2 \beta_{32} \beta_{41} g_{23}^{(j)}, \]
\[ \beta_{22} = \beta_{32} + 2 \beta_{32} \beta_{42} g_{21}^{(j)} - 2 \beta_{32} \beta_{42} g_{23}^{(j)} , \]
\[ \beta_{31} = 2 \beta_{32} \beta_{41} g_{21}^{(j)} - \beta_{32} g_{21}^{(j)} - 2 \beta_{32} \beta_{41} g_{23}^{(j)}, \]
\[ \beta_{32} = 1/2(g_{22}^{(j)} - g_{21}^{(j)}), \]
\[ \beta_{33} = 2(\beta_{43} g_{23}^{(j)} - \beta_{32} \beta_{43} g_{23}^{(j)}), \]
\[ \beta_{41} = -2 \beta_{43}(\beta_{11} g_{41}^{(j)} - \beta_{32} g_{21}^{(j)} g_{42}^{(j)}), \]
\[ \beta_{42} = -2 \beta_{43}(\beta_{32} g_{24}^{(j)} - \beta_{32} g_{14}^{(j)}), \]
\[ \beta_{43} = 1/2(g_{43}^{(j)} - 2 \beta_{11} g_{41}^{(j)} + 2 \beta_{32} g_{21}^{(j)} g_{42}^{(j)} - 2 g_{23}^{(j)} (\beta_{32} g_{42}^{(j)} - \beta_{32} g_{41}^{(j)})). \]

The equation (7.11) can be written as

\[ V_j = T_j V_j^+, \]  

\[ (7.14) \]

where

\[ T_j = X^{(j)} W_j^+ (W_j^+)^{-1} (X^{(j)})^{-1}. \]  

\[ (7.15) \]
\( T_j \) is the transfer matrix relating the mechanical displacements, electric potentials, mechanical stresses and electric displacements at the top of the \( j^{th} \) layer to the corresponding quantities at the bottom of the \( j^{th} \) layer.

By successive repetition of the above process for each layer and by invoking the continuity conditions of mechanical displacements, electric potentials, mechanical stresses and electric displacements at the common interfaces in the layered structure, the relation between the field vectors at the top and bottom of the laminated plate is given by
\[
\mathbf{V}_1^- = T^* \mathbf{V}_n^+ ,
\]
where
\[
\mathbf{V}_1^- = [V^{(l)}]_{x_i=0} ,
\]
\[
\mathbf{V}_n^+ = [V^{(n)}]_{x_i=h_1+h_2+...+h_n} ,
\]
\[
T^* = T_1 T_2 ... T_n .
\]

\( T^* \) is the global transfer matrix relating mechanical displacements, electric potentials, mechanical stresses and electric displacements at the top and the bottom of the laminated plate.

7.4 Reflection and Transmission Coefficients

7.4.1 Amplitude ratios of reflected and transmitted

The mechanical displacements, electric potentials, mechanical stresses and electric displacements in the PPHS can be written as
\[
(u^{(hs)}_1, u^{(hs)}_3, u^{(hs)}_4, u^{(hs)}_5, \Phi^{(hs)}, \Phi^{(hs)}_{II}) = \\
\sum_{i=1}^5 (l^{(hs)}_i, r^{(hs)}_{2i}, r^{(hs)}_{3i}, r^{(hs)}_{4i}, r^{(hs)}_{5i}) B^{(hs)}_{li} \exp \left( i \omega \left( \frac{1}{c} x_i + q^{(hs)}_i x_3 - t \right) \right) ,
\]
\[
(\sigma^{(hs)}_{31}, \sigma^{(hs)}_{33}, \sigma^{(hs)}_{43}, D^{(hs)}_3, D^{(hs)}_{53}) = \\
\sum_{i=1}^5 (g^{(hs)}_{li}, g^{(hs)}_{2i}, g^{(hs)}_{3i}, g^{(hs)}_{4i}, g^{(hs)}_{5i}) B^{(hs)}_{li} \exp \left( i \omega \left( \frac{1}{c} x_i + q^{(hs)}_i x_3 - t \right) \right) ,
\]
where \( q^{(hs)}_i (i=1,2,...,5) \) correspond to the roots of the equation (6.5) in the PPHS.
whose imaginary parts are positive. The coefficients \( r_{ik}^{(hs)} (i = 1, 2, \ldots, 5; k = 1, 2, \ldots, 5) \) can be obtained from the equation (6.7) by using the material parameters of the PPHS and 

\[
g_{1i}^{(hs)} = i \omega \left[ c_{35} q_{i}^{(hs)} + (c_{55} r_{ik}^{(hs)} + e_{15} r_{4i}^{(hs)} + \zeta_{15} r_{5i}^{(hs)}) / c \right],
\]

\[
g_{2i}^{(hs)} = i \omega \left[ (c_{33} r_{ik}^{(hs)} + m_{33} r_{5i}^{(hs)} + e_{33} r_{4i}^{(hs)} + \zeta_{33} r_{5i}^{(hs)}) q_{i}^{(hs)} + (c_{31} + m_{33} r_{2i}^{(hs)}) / c \right],
\]

\[
g_{3i}^{(hs)} = i \omega \left[ (m_{33} r_{ik}^{(hs)} + R r_{5i}^{(hs)} + \zeta_{33} r_{4i}^{(hs)} + e_{3} r_{5i}^{(hs)} + n_{33} r_{2i}^{(hs)}) q_{i}^{(hs)} + (m_{1} + R r_{2i}^{(hs)}) / c \right],
\]

\[
g_{4i}^{(hs)} = i \omega \left[ (e_{33} r_{ik}^{(hs)} + \zeta_{33} r_{5i}^{(hs)} - \zeta_{33} r_{4i}^{(hs)} - A_{33} r_{5i}^{(hs)}) q_{i}^{(hs)} + (e_{31} + \zeta_{33} r_{2i}^{(hs)}) / c \right],
\]

\[
g_{5i}^{(hs)} = i \omega \left[ (\zeta_{33} r_{ik}^{(hs)} + e_{3} r_{5i}^{(hs)} - A_{33} r_{4i}^{(hs)} - \zeta_{33} r_{5i}^{(hs)}) q_{i}^{(hs)} + (\zeta_{31} + e_{3} r_{2i}^{(hs)}) / c \right].
\]

Making use of the equations (7.18) and (7.19), the relation between the mechanical displacements, electrical potentials, mechanical stresses and electric displacements in the PPHS and the wave amplitudes corresponding to PPHS can be written as

\[
\mathbf{V}^{(hs)} = \mathbf{T}_{hs} \mathbf{S}^{(hs)},
\]

where

\[
\begin{align*}
\mathbf{V}^{(hs)} & = [u_{1}^{(hs)} \ u_{2}^{(hs)} \ u_{3}^{(hs)} \ \sigma_{33}^{(hs)} \ \Phi_{3}^{(hs)} \ D_{3}^{(hs)} \ \sigma_{13}^{(hs)} \ \Phi_{13}^{(hs)} \ D_{13}^{(hs)}]^{T} \bigg|_{x = h}, \\
\mathbf{S}^{(hs)} & = [E_{1}^{(hs)} \ E_{2}^{(hs)} \ E_{3}^{(hs)} \ E_{4}^{(hs)} \ E_{5}^{(hs)}]^{T}.
\end{align*}
\]

\[
\begin{bmatrix}
E_{1}^{(hs)} & E_{2}^{(hs)} & E_{3}^{(hs)} & E_{4}^{(hs)} & E_{5}^{(hs)} \\
E_{1}^{(hs)} & r_{12}^{(hs)} & E_{2}^{(hs)} & r_{13}^{(hs)} & E_{3}^{(hs)} & r_{14}^{(hs)} & E_{4}^{(hs)} & r_{15}^{(hs)} & E_{5}^{(hs)} \\
r_{31}^{(hs)} & E_{1}^{(hs)} & r_{32}^{(hs)} & E_{2}^{(hs)} & r_{33}^{(hs)} & E_{3}^{(hs)} & r_{34}^{(hs)} & E_{4}^{(hs)} & r_{35}^{(hs)} & E_{5}^{(hs)} \\
g_{21}^{(hs)} & E_{2}^{(hs)} & g_{22}^{(hs)} & E_{2}^{(hs)} & g_{23}^{(hs)} & E_{3}^{(hs)} & g_{24}^{(hs)} & E_{4}^{(hs)} & g_{25}^{(hs)} & E_{5}^{(hs)} \\
r_{41}^{(hs)} & E_{1}^{(hs)} & r_{42}^{(hs)} & E_{2}^{(hs)} & r_{43}^{(hs)} & E_{3}^{(hs)} & r_{44}^{(hs)} & E_{4}^{(hs)} & r_{45}^{(hs)} & E_{5}^{(hs)} \\
g_{41}^{(hs)} & E_{1}^{(hs)} & g_{42}^{(hs)} & E_{2}^{(hs)} & g_{43}^{(hs)} & E_{3}^{(hs)} & g_{44}^{(hs)} & E_{4}^{(hs)} & g_{45}^{(hs)} & E_{5}^{(hs)} \\
g_{51}^{(hs)} & E_{1}^{(hs)} & g_{52}^{(hs)} & E_{2}^{(hs)} & g_{53}^{(hs)} & E_{3}^{(hs)} & g_{54}^{(hs)} & E_{4}^{(hs)} & g_{55}^{(hs)} & E_{5}^{(hs)} \\
g_{51}^{(hs)} & E_{1}^{(hs)} & g_{52}^{(hs)} & E_{2}^{(hs)} & g_{53}^{(hs)} & E_{3}^{(hs)} & g_{54}^{(hs)} & E_{4}^{(hs)} & g_{55}^{(hs)} & E_{5}^{(hs)} \\
g_{51}^{(hs)} & E_{1}^{(hs)} & g_{52}^{(hs)} & E_{2}^{(hs)} & g_{53}^{(hs)} & E_{3}^{(hs)} & g_{54}^{(hs)} & E_{4}^{(hs)} & g_{55}^{(hs)} & E_{5}^{(hs)}
\end{bmatrix},
\]

\[
E_{i}^{(hs)} = \exp(i \omega q_{i}^{(hs)} h), \quad (i = 1, 2, \ldots, 5).
\]

By invoking the conditions of continuity of mechanical displacements, electric potentials, mechanical stresses and electric displacements at the interface of the \( n \)th layer and the PPHS and also using the equations (7.15) and (7.20), we obtain
\[ \mathbf{V}_1^- = \mathbf{T} \mathbf{S}^{(h_s)}, \quad \text{where} \quad \mathbf{T} = \mathbf{T}^* \mathbf{T}_{h_s}. \]  

(7.23)

The displacements and normal stress in the FHS can be written as

\[
(u_{1}^{fs}, u_{2}^{fs}) = \sum_{k=1}^{2} (1, W_k^{fs}) U_k^{fs} \exp \left( \imath \omega \left( \frac{x_1}{c} + (-1)^{k+1} q^{fs} x_3 - t \right) \right),
\]

(7.24)

\[
\sigma_{33}^{fs} = \imath \omega \sum_{k=1}^{2} \rho^{fs} c U_k^{fs} \exp \left( \imath \omega \left( \frac{x_1}{c} + (-1)^{k+1} q^{fs} x_3 - t \right) \right),
\]

(7.25)

where

\[
W_1^{fs} = q^{fs} c, \quad W_2^{fs} = -q^{fs} c, \quad q^{fs} = \frac{1}{c} \sqrt{\frac{c^2}{(v^{(f)})^2} - 1},
\]

(7.26)

where \( v^{(f)} \) is the longitudinal incident wave velocity in the fluid medium. \( \rho^{fs} \) is the fluid density. \( k = 1 \) for the incident wave and \( k = 2 \) for the reflected wave.

7.4.2 Boundary conditions

The boundary conditions at the interface \( x_3 = 0 \) can be written as

(a) Mechanical Boundary Conditions: The boundary conditions on stresses and mechanical displacements at the interface between a fluid medium and a poroelastic medium are

(i) \( \sigma_{33}^{(i)} + \sigma^{(i)} = \sigma_{33}^{fs}, \)

(7.27)

(ii) \( \sigma^{(i)} = f \sigma_{33}^{fs}, \)

(7.28)

(iii) \( \sigma_{13}^{(i)} = 0, \)

(7.29)

(iv) \( (1 - f) \dot{u}_3^{(i)} + f \dot{u}_3^{(i)} = \dot{u}_3^{fs}. \)

(7.30)

Here, dot represents the differentiation with respect to time.

(b) Electrical (open case) Boundary Conditions: The FHS is considered to be non-piezoelectric, so the continuity of normal component of the electric displacements implies that

(v) \( D_3^{(i)} = 0, \)

(7.31)

(vi) \( D_3^{(i)} = 0. \)

(7.32)
Substituting the equations (7.17) and (7.23)-(7.25) into the equations (7.27)-(7.32), a non-homogeneous linear system is obtained and that can be written as

\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
  a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
  a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\
  a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\
  a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
  a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \\
\end{bmatrix}
\begin{bmatrix}
  B_{11}^{(fs)} \\
  B_{12}^{(fs)} \\
  B_{13}^{(fs)} \\
  B_{14}^{(fs)} \\
  B_{15}^{(fs)} \\
  U_2^{fs} \\
\end{bmatrix}
= \begin{bmatrix}
  \rho^{fs} c \\
  \rho^{fs} c \\
  0 \\
  q^{fs} c \\
  0 \\
  0 \\
\end{bmatrix}
U_1^{fs},
\]  

(7.33)

where

\[
\begin{align*}
a_{11} &= T_{41} + T_{71}, a_{12} = T_{42} + T_{72}, a_{13} = T_{43} + T_{73}, a_{14} = T_{44} + T_{74}, a_{15} = T_{45} + T_{75}, \\
a_{16} &= -\rho^{fs} c, a_{21} = T_{71}, a_{22} = T_{72}, a_{23} = T_{73}, a_{24} = T_{74}, a_{25} = T_{75}, a_{26} = -f \rho^{fs} c, \\
a_{31} &= T_{81}, a_{32} = T_{82}, a_{33} = T_{83}, a_{34} = T_{84}, a_{35} = T_{85}, a_{36} = 0, \\
a_{41} &= (1 - f)T_{21} + fT_{31}, a_{42} = (1 - f)T_{22} + fT_{32}, a_{43} = (1 - f)T_{23} + fT_{33}, \\
a_{44} &= (1 - f)T_{24} + fT_{34}, a_{45} = (1 - f)T_{25} + fT_{35}, a_{46} = q^{fs} c, \\
a_{51} &= T_{61}, a_{52} = T_{62}, a_{53} = T_{63}, a_{54} = T_{64}, a_{55} = T_{65}, a_{56} = 0, \\
a_{61} &= T_{10,1}, a_{62} = T_{10,2}, a_{63} = T_{10,3}, a_{64} = T_{10,4}, a_{65} = T_{10,5}, a_{66} = 0.
\end{align*}
\]  

(7.34)

Solving the non-homogeneous system of equations (7.33), the reflected and transmitted amplitude ratios are obtained as

\[
\frac{U_2^{fs}}{U_1^{fs}} = \frac{A' - Y - Y_1}{A' + Y + Y_1},
\]  

(7.35)

and

\[
\frac{B_i^{(fs)}}{U_1^{fs}} = \frac{2 \rho^{fs} c \left( dt_{k-1} - f dt_k \right)}{A' + Y + Y_1}, \quad (i = 1, 2, ..., 5; \ k = 2) .
\]  

(7.36)

where expressions for \( A' \), \( Y \), \( Y_1 \) and \( dt_k (k = 1, 2, ..., 10) \) are given as follows:

\[
A' = (T_{41} + T_{71}) dt_1 - (T_{42} + T_{72}) dt_3 + (T_{43} + T_{73}) dt_5 - (T_{44} + T_{74}) dt_7 + (T_{45} + T_{75}) dt_9 ,
\]

\[
Y = \left[ ((1 - f)T_{21} + fT_{31}) dt_1 - ((1 - f)T_{22} + fT_{32}) dt_3 - ((1 - f)T_{23} + fT_{33}) dt_5 - ((1 - f)T_{24} + fT_{34}) dt_7 + ((1 - f)T_{25} + fT_{35}) dt_9 \right] \rho^{fs} / q^{fs} ,
\]
\[ Y_i = - \left[ (1 - f) T_{21} \frac{1}{dt_2} - (1 - f) T_{22} \frac{1}{dt_4} + (1 - f) T_{23} \frac{1}{dt_6} - (1 - f) T_{24} \frac{1}{dt_8} + (1 - f) T_{25} \frac{1}{dt_{10}} \right] f \frac{q_s}{\rho_s}, \]

\[
\begin{array}{cccc}
T_{72} & T_{73} & T_{74} & T_{75} \\
T_{82} & T_{83} & T_{84} & T_{85} \\
T_{62} & T_{63} & T_{64} & T_{65} \\
T_{10,2} & T_{10,3} & T_{10,4} & T_{10,5} \\
\end{array}
\begin{array}{cccc}
T_{42} & T_{72} & T_{43} & T_{73} \\
T_{82} & T_{83} & T_{84} & T_{85} \\
T_{62} & T_{63} & T_{64} & T_{65} \\
T_{10,2} & T_{10,3} & T_{10,4} & T_{10,5} \\
\end{array}
\]

\[
\begin{array}{cccc}
T_{71} & T_{73} & T_{74} & T_{75} \\
T_{81} & T_{83} & T_{84} & T_{85} \\
T_{61} & T_{63} & T_{64} & T_{65} \\
T_{10,1} & T_{10,3} & T_{10,4} & T_{10,5} \\
\end{array}
\begin{array}{cccc}
T_{41} & T_{71} & T_{43} & T_{73} \\
T_{81} & T_{83} & T_{84} & T_{85} \\
T_{61} & T_{63} & T_{64} & T_{65} \\
T_{10,1} & T_{10,3} & T_{10,4} & T_{10,5} \\
\end{array}
\]

\[
\begin{array}{cccc}
T_{71} & T_{72} & T_{74} & T_{75} \\
T_{81} & T_{82} & T_{84} & T_{85} \\
T_{61} & T_{62} & T_{64} & T_{65} \\
T_{10,1} & T_{10,2} & T_{10,4} & T_{10,5} \\
\end{array}
\begin{array}{cccc}
T_{41} & T_{71} & T_{42} & T_{72} \\
T_{81} & T_{82} & T_{84} & T_{85} \\
T_{61} & T_{62} & T_{64} & T_{65} \\
T_{10,1} & T_{10,2} & T_{10,4} & T_{10,5} \\
\end{array}
\]

\[
\begin{array}{cccc}
T_{71} & T_{72} & T_{73} & T_{75} \\
T_{81} & T_{82} & T_{83} & T_{85} \\
T_{61} & T_{62} & T_{63} & T_{65} \\
T_{10,1} & T_{10,2} & T_{10,3} & T_{10,5} \\
\end{array}
\begin{array}{cccc}
T_{41} & T_{71} & T_{42} & T_{72} \\
T_{81} & T_{82} & T_{83} & T_{85} \\
T_{61} & T_{62} & T_{63} & T_{65} \\
T_{10,1} & T_{10,2} & T_{10,3} & T_{10,5} \\
\end{array}
\]

\[
\begin{array}{cccc}
T_{71} & T_{72} & T_{73} & T_{74} \\
T_{81} & T_{82} & T_{83} & T_{84} \\
T_{61} & T_{62} & T_{63} & T_{64} \\
T_{10,1} & T_{10,2} & T_{10,3} & T_{10,4} \\
\end{array}
\begin{array}{cccc}
T_{41} & T_{71} & T_{42} & T_{72} \\
T_{81} & T_{82} & T_{83} & T_{84} \\
T_{61} & T_{62} & T_{63} & T_{64} \\
T_{10,1} & T_{10,2} & T_{10,3} & T_{10,4} \\
\end{array}
\]

(7.37)

7.4.3 Energy ratios

Energy partitioning between different reflected and transmitted waves is calculated across a surface element of unit area at the interface. The time average of normal acoustic flux \( P \) over a period, denoted by \( \langle P \rangle \), represents the average energy transmission per unit surface area per unit time.

As described in the Section 6.3.3, the normal acoustic flux in FHS and PPHS can be expressed as
\[ P_{fs} = -\sigma_{33}^{(f)} \overline{u}_3^{(f)}, \]  
\[ P = -\left( \sigma_{31}^{(l)} \overline{u}_1^{(l)} + \sigma_{33}^{(l)} \overline{u}_3^{(l)} + \sigma^{*(l)} \overline{u}_3^{(l)} - \overline{D}_2^{(l)} \Phi^{(l)} - \overline{D}_3^{(l)} \Phi^{(l)} \right). \]  

(7.38) 

(7.39)

Using the equations (7.24), (7.25) and (7.38), the average energy flux of incident and reflected waves are obtained as

\[ \langle P \rangle_I = \frac{1}{2} \omega^2 q^{fs} \rho^{fs} c^2 \left| U_{1}^{fs} \right|^2, \]  

(7.40)

and

\[ \langle P \rangle_R = - \frac{1}{2} \omega^2 q^{fs} \rho^{fs} c^2 \left| U_{2}^{fs} \right|^2. \]  

(7.41)

Similarly, making use of the equations (7.17), (7.23) and (7.39), the average energy flux of transmitted waves are obtained as

\[ \langle P_i \rangle = \frac{1}{2} (\omega) \text{Re} \left( g_{li}^{(i)} + g_{2i}^{(i)} r_{li}^{(i)} + g_{3i}^{(i)} r_{3i}^{(i)} - g_{4i}^{(i)} r_{4i}^{(i)} - g_{5i}^{(i)} r_{5i}^{(i)} \right) B_{li}^{(h s)} B_{lj}^{(h s)}, \quad (i = 1, 2, ..., 5). \]  

(7.42)

The energy ratios of the reflected and transmitted waves are denoted as

\[ E_R = \frac{\langle P_R \rangle}{\langle P_I \rangle}, \quad E_i = \frac{\langle P_i \rangle}{\langle P_I \rangle}, \quad (i = 1, 2, ..., 5). \]  

(7.43)

The interaction energy ratios, which account for interaction between stress/electric potential field and mechanical/ electric displacement fields of different transmitted waves, are described as

\[ E_{ij} = \frac{\langle P_{ij} \rangle}{\langle P_I \rangle}, \quad (i, j = 1, 2, ..., 5 \text{ and } i \neq j). \]  

(7.44)

where

\[ \langle P_{ij} \rangle = - \frac{(\omega)}{2} \text{Re} \left( g_{li}^{(i)} + g_{2i}^{(i)} r_{li}^{(i)} + g_{3i}^{(i)} r_{3i}^{(i)} - g_{4i}^{(i)} r_{4i}^{(i)} - g_{5i}^{(i)} r_{5i}^{(i)} \right) B_{li}^{(h s)} B_{lj}^{(h s)}. \]  

(7.45)

Making use of the equations (7.40)-(7.45), the expressions for the reflected, transmitted and interaction energy ratios are obtained as

\[ E_R = \frac{\left| U_{2}^{fs} \right|^2}{\left| U_{1}^{fs} \right|^2}, \]  

(7.46)

\[ E_i = \text{Re} \left( g_{li}^{(i)} + g_{2i}^{(i)} r_{li}^{(i)} + g_{3i}^{(i)} r_{3i}^{(i)} - g_{4i}^{(i)} r_{4i}^{(i)} - g_{5i}^{(i)} r_{5i}^{(i)} \right) / (\omega) B_{li}^{(h s)} B_{lj}^{(h s)} \left| U_{i}^{fs} \right|^2, \quad (i = 1, 2, ..., 5), \]  

(7.47)
\[ E_{ij} = \Re \left( g_{i}^{(1)} + g_{2i}^{(1)} r_{ij}^{(1)} + g_{3i}^{(1)} r_{3j}^{(1)} - g_{4i}^{(1)} r_{4i}^{(1)} - g_{5j}^{(1)} r_{5j}^{(1)} \right) / (i \omega) \frac{B_{i j}^{(h s)}}{U_{i}^{fs}} \times \frac{B_{j i}^{(h s)}}{U_{j}^{fs}}, \]

\((i, j = 1, 2, ..., 5 ; i \neq j). \) \( (7.48) \)

For conservation of energy, it is verified that

\[ \sum_{i=1}^{5} E_{i} + E_{\text{int}} - E_{R} = 1, \] \( (7.49) \)

where \( E_{\text{int}} = \sum_{i,j=1 \atop i \neq j}^{5} E_{ij} \) is the resultant interaction energy between the transmitted waves.

### 7.5 Normal Surface Impedance

The surface impedance \( Z \) is the transfer function between pressure and velocity. It provides the necessary information to calculate the characteristic impedance and wave number of material in closed form. The amount of energy reflected or lost when sound passes from one medium to another of greater impedance is largely determined by impedance difference or mismatches. The normal surface impedance \( Z \) is defined as

\[ Z = \frac{p_{fs}}{v_{fs}}, \] \( (7.50) \)

where \( p_{fs} \) and \( v_{fs} \) are the pressure and normal velocity in the FHS.

The boundary conditions (7.27)-(7.32) at the interface \( x_{3} = 0 \) can be written as

\[ \sigma_{33}^{(1)} + \sigma_{3i}^{x(1)} = -p_{fs}, \] \( (7.51) \)

\[ \sigma_{i3}^{x(1)} = -f \ p_{fs}, \] \( (7.52) \)

\[ \sigma_{13}^{(1)} = 0, \] \( (7.53) \)

\[ (1 - f) \dot{u}_{3}^{(1)} + f \dot{u}_{3}^{x(1)} = v_{fs}. \] \( (7.54) \)

\[ D_{3}^{(1)} = 0, \] \( (7.55) \)

\[ D_{3}^{x(1)} = 0. \] \( (7.56) \)
Using the equations (7.17), (7.23) and (7.50) in the equations (7.51)-(7.56), we obtain

\[
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & \hat{a}_{16} \\
    a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & \hat{a}_{26} \\
    a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & \hat{a}_{36} \\
    a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & \hat{a}_{46} \\
    a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
    a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}
\end{bmatrix}
\begin{bmatrix}
P_{11}^{(fr)} \\
P_{12}^{(fr)} \\
P_{13}^{(fr)} \\
P_{14}^{(fr)} \\
P_{15}^{(fr)} \\
U_{2s}^{fs}
\end{bmatrix} = 0,
\]  

(7.57)

where

\[
\hat{a}_{16} = Z, \quad \hat{a}_{36} = f Z, \quad \hat{a}_{36} = 0, \quad \hat{a}_{46} = 1/\iota \omega.
\]  

(7.58)

The condition of existence of the non-trivial solution of the homogeneous system (7.57) leads to

\[
Z = \left( \frac{\det_3}{(\iota \omega)(f \det_2 - \det_1)} \right),
\]  

(7.59)

where

\[
\begin{align*}
\det_1 &= f(T_{31} - T_{21}) + T_{21} f(T_{32} - T_{22}) + T_{22} f(T_{33} - T_{23}) + T_{23} f(T_{34} - T_{24}) + T_{24} f(T_{35} - T_{25}) + T_{25}, \\
\det_2 &= f(T_{41} + T_{71}) + T_{71} f(T_{42} + T_{72}) + T_{72} f(T_{43} + T_{73}) + T_{73} f(T_{44} + T_{74}) + T_{74} f(T_{45} + T_{75}) + T_{75}, \\
\det_3 &= f(T_{41} + T_{81}) + T_{81} f(T_{42} + T_{82}) + T_{82} f(T_{43} + T_{83}) + T_{83} f(T_{44} + T_{84}) + T_{84} f(T_{45} + T_{85}) + T_{85},
\end{align*}
\]  

(7.60)
7.6 Discussion of Numerical Results

The analytical expressions for the reflected energy ratio, transmitted energy ratios, interaction energy ratio and the surface impedance, obtained in the previous sections, are solved numerically for particular models. $E_1, E_2$ and $E_3$ are energy ratios corresponding to the $qP_1, qS_1$ and $qP_2$ waves and $E_4$ and $E_5$ are energy ratios of $PE_1$ and $PE_2$ wave modes.

The PPHS is considered as PZT-7H, unless otherwise specified. The layered structure embedded between FHS and PPHS is assumed to have different configuration for numerical computation purpose. Unless otherwise specified, the configuration chosen is a two layer plate consisting of a Barium Titanate layer of 1mm thickness lined with a PZT-5H layer of 2mm thickness. The materials Barium Titanate, PZT-5H and PZT-7H are coded here as 1, 2 and 3, respectively. Similarly, the FHS is coded as 0. The elastic, piezoelectric and dielectric coefficients for these ceramics, taken from Auld (1973) and Kar-Gupta and Venkatesh (2006), are listed in the Tables 7.1-7.3. The dynamical coefficients and other parameters used in the study are listed in the Table 7.4.

Table 7.1 Elastic, piezoelectric and dielectric constants of Barium Titanate crystal (code 1)

<table>
<thead>
<tr>
<th>Elastic constants ($GPa$)</th>
<th>Piezoelectric constants ($C/m^2$)</th>
<th>Dielectric constants ($nC/Vm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11} = 150.4$</td>
<td>$c_{12} = 65.63$</td>
<td>$\xi_{11} = 10.8$</td>
</tr>
<tr>
<td>$c_{13} = 65.94$</td>
<td>$c_{33} = 145.5$</td>
<td>$\xi_{33} = 13.1$</td>
</tr>
<tr>
<td>$c_{44} = 43.86$</td>
<td>$m_{11} = 8.8$</td>
<td>$e_{3} = -3.6$</td>
</tr>
<tr>
<td>$m_{33} = 5.2$</td>
<td>$R = 20$</td>
<td>$\varepsilon_{31} = -1.728$</td>
</tr>
<tr>
<td>$e_{15} = 11.4$</td>
<td>$e_{33} = 17.4$</td>
<td>$\zeta_{15} = 4.56, \zeta_{33} = 6.96$</td>
</tr>
<tr>
<td>$\zeta_{31} = -1.728$</td>
<td>$e_3^* = -3.6$</td>
<td>$\bar{\zeta}_3 = -7.5$</td>
</tr>
<tr>
<td>$\rho = 5700 Kg/m^3$</td>
<td>$\zeta_{33} = 6.96$</td>
<td>$A_{11} = 12.8, A_{33} = 15.1$</td>
</tr>
</tbody>
</table>
Table 7.2 Elastic, piezoelectric and dielectric constants of PZT-5H crystal (code 2);

<table>
<thead>
<tr>
<th>Elastic constants (GPa)</th>
<th>Piezoelectric constants (C/m²)</th>
<th>Dielectric constants (nC/Vm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11} = 127.2$</td>
<td>$e_{15} = 17$</td>
<td>$\xi_{11} = 27.71$</td>
</tr>
<tr>
<td>$c_{13} = 84.67$</td>
<td>$e_{33} = 23.2$</td>
<td>$\xi^{*}_{11} = 11.8$</td>
</tr>
<tr>
<td>$c_{33} = 117.4$</td>
<td>$\zeta_{31} = -0.8$</td>
<td>$\xi^{*}_{33} = 13.9$</td>
</tr>
<tr>
<td>$e_{44} = 22.99$</td>
<td>$e^{*}_{3} = -3.6$</td>
<td>$A_{11} = 12.8$</td>
</tr>
<tr>
<td>$m_{33} = 5.2$</td>
<td>$c^{*}_{3} = -7.5$</td>
<td>$A_{33} = 15.1$</td>
</tr>
<tr>
<td>$m_{44} = 8.8$</td>
<td>$R = 20$</td>
<td></td>
</tr>
<tr>
<td>$\rho = 7500 Kg / m³$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.3 Elastic, piezoelectric and dielectric constants of PZT-7H crystal (code 3);

<table>
<thead>
<tr>
<th>Elastic constants (GPa)</th>
<th>Piezoelectric constants (C/m²)</th>
<th>Dielectric constants (nC/Vm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11} = 148$</td>
<td>$e_{15} = 9.3$</td>
<td>$\xi_{11} = 3.984$</td>
</tr>
<tr>
<td>$c_{13} = 74.2$</td>
<td>$e_{33} = 10.99$</td>
<td>$\xi^{*}_{11} = 11.8$</td>
</tr>
<tr>
<td>$c_{33} = 131$</td>
<td>$\zeta_{31} = -0.48$</td>
<td>$\xi^{*}_{33} = 13.9$</td>
</tr>
<tr>
<td>$e_{44} = 25.3$</td>
<td>$e^{*}_{3} = -3.6$</td>
<td>$A_{11} = 12.8$</td>
</tr>
<tr>
<td>$m_{33} = 5.2$</td>
<td>$c^{*}_{3} = -7.5$</td>
<td>$A_{33} = 15.1$</td>
</tr>
<tr>
<td>$m_{44} = 8.8$</td>
<td>$R = 20$</td>
<td></td>
</tr>
<tr>
<td>$\rho = 7700 Kg / m³$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.4 Dynamical coefficients, permeability tensor and other parameters

<table>
<thead>
<tr>
<th>$\rho_f = 1000 Kg / m³$</th>
<th>$\rho_f^{s} = 1000 Kg / m³$</th>
<th>$\mu = 1 \times 10^{-3} Ns / m^2$</th>
<th>$v^{(f)} = 1500 m / s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu = 1 MHz$</td>
<td>$f = 0.2$</td>
<td>$\chi_{11} = 1.0 \times 10^{-10} m^2$</td>
<td>$\chi_{33} = 0.8 \times 10^{-10} m^2$</td>
</tr>
<tr>
<td>$\rho_{11} / \rho = 0.66$</td>
<td>$\rho_{12} / \rho = -0.15$</td>
<td>$\rho_{22} / \rho = 0.64.$</td>
<td></td>
</tr>
</tbody>
</table>

The figure 7.2 depicts the variation of energy ratios of reflected & transmitted waves in a FHS overlying a plate consisting of two layers 1 and 2 over PZT-7H (3) half space in the frequency range 1-6 MHz at incidence angle of 5°. When more than two media are involved, interference effects produce frequency dependent transmission and reflection coefficients for waves travelling through media of finite thickness.
Figure 7.2 Variation of Reflected and Transmitted energy ratios with frequency at $\theta = 5^\circ$, $h_1 = 1\text{mm}$, $h_2 = 2\text{mm}$; two layers between FHS & PPHS $\leftarrow 0|1|2|3 \rightarrow$

The frequency dependence of the energy ratios involves a system of resonances. The behavior of variation of energy ratios with frequency is oscillatory. This oscillating behavior confirms that the reflected and transmitted energy ratios for layered structures are sensitive to changes in the frequency. It is inferred that there are numbers of maxima in the transmission coefficients corresponding to respective minima of reflection coefficient at which the transmission is relatively very high. This behavior could be advantageous in the sense that we can always find a particular frequency at which reflection is low and more energy is transmitted into the other medium. Here, when frequency is 2.36 MHz, then more than 50% of energy is scattered to the transmitting medium. The minima correspond to principal Bragg peaks in the reflected energy ratio and they are accompanied by secondary minima. The energy ratios corresponding to electric potential modes are relatively significant in the frequency range 4-6 MHz.

The effects of number of layers of Barium Titanate between FHS and PZT-7H PPHS, on the variation of reflected and transmitted energy ratios with frequency, are
shown in the figure 7.3. The case is studied from one to four layers. The thickness of each layer is assumed to be 1mm.

![Graphs showing variation of energy ratios with frequency](image)

**Figure 7.3** Variation of Reflected and Transmitted energy ratios with frequency at $\theta = 5^\circ$; (i) $0|1|3$  (ii) $0|1|1|3$  (iii) $0|1|1|1|3$  (iv) $0|1|1|1|1|3$

It is observed that the number of maxima and minima increases in number as the layers are added to the array. The pattern of variation of energy ratios with frequency is oscillatory and repeats itself after small frequency packets. As the number of layers over the substrate decreases, the pattern of variation of transmitted energy ratios shifts toward right. An increase in the thickness of the layer modifies the dynamic response of the system thus leading to strong oscillations in all the curves and thus showing a higher numbers of maxima and minima. The transmission maxima are due to constructive interference effects within the layers. It can also be inferred from here that when the thickness of the layers is changed or increase, then large numbers of local maxima are present. The addition of same kind of layers between FHS & PPHS does not affect significantly the amount of energy going to reflected & transmitted waves.
The figure 7.4 illustrates the effects of periodic stacking of layers 1 and 2 over the PZT-7H half space on the energy ratios. The development of the band structures with the increase of stacks can be seen in this figure. The shallow tightly spaced minima correspond roughly to the thickness resonance of the entire array and they increases in number as layers are added to the array. Further, it is also observed that the minima’s in the reflected energy ratio get deepens and increases in number with the addition of periodic stacks of layers over the PPHS. The number of stop bands ($E_R = 1$) also increases as the stacks of layers are added to the array. The existence of these stop bands are of great importance in design and construction of piezo-composite transducers (Otero et al., 2004). The energy carried out by the transmitted $qP_2$ mode increases with increase in the number of periodic stacks over the PPHS. Comparison of the figures 7.3 and 7.4 reveals that addition of same kind of layers over the PPHS only affects the number of maxima and minima while addition of periodic stacks of different layers affects both the number of maxima and minima’s and magnitude of energy ratios.

Figure 7.4. Variation of Reflected and Transmitted energy ratios with frequency at $\theta = 5^\circ$; (i) $\leftarrow 0|1|2|3 \rightarrow$ (ii) $\leftarrow 0|1|2|1|2|3 \rightarrow$ (iii) $\leftarrow 0|1|2|1|2|1|2|3 \rightarrow$
Variation of real and imaginary part of the surface impedance with the angle of incidence at frequency =1MHz for a plate consisting of two layers 1 and 2 between FHS & PPHS is shown in the figure 7.5. The real part of the surface impedance represents surface resistance and imaginary part corresponds to the reactance. Rapid variation before the angle of incidence $\theta = 32^\circ$ is noticed and real part of $Z$ approaches to zero as the angle of incidence is increased further. The angles, at which the surface impedance approach to zero, correspond to the angles at which total internal reflection occurs. It is only the imaginary part i.e. reactance which contribute and the boundary is said to be purely reactive. The study is used to identify mismatches at the interface because a contrast in acoustic impedance causes reflection.

![Figure 7.5](image-url)

Figure 7.5. Variation of Real and imaginary parts of surface impedance with angle of incidence at frequency=1MHz, $h_1 = 1\text{mm}$, $h_2 = 1\text{mm}$; two layers over PPHS $\leftarrow 0|1\parallel 2|3 \rightarrow$
The thickness of a layer over the PPHS affects the surface impedance significantly. The peak value of \( \text{Re}(Z) \) decreases with increase in the thickness of the layer over the substrate. Thus, as the thickness of the Barium Titanate layer between FHS and PPHS increases, less energy get transmitted into the PPHS, resulting more reflected energy and hence less impedance (Fig. 7.6).

![Graph showing variation of real and imaginary parts of surface impedance with angle of incidence for different values of the layer thickness](image)

**Figure 7.6** Variation of real and imaginary parts of surface impedance with angle of incidence for different values of the layer thickness (i) \( h_l = 1 \text{mm} \) (ii) \( h_l = 3 \text{mm} \) (iii) \( h_l = 5 \text{mm} \):

The effects of thickness of a layer on the variation of surface impedance with frequency are shown in the figure 7.7. The surface impedance shows sinusoidal behavior with frequency. The surface impedance is seen to be sensitive to the changes in the thickness of the layer. This may be understood from the fact that the small changes in the thickness of the layer can result in a relatively large change in the gradient of the velocity profile. In particular, by changing the thickness of the layer, we are changing the waveguide like nature of the layer and this affects the surface impedance. An exchange in the positions of maxima and minima in the oscillatory behavior of surface impedance with frequency are observed. This observation may be
due to the influence of mutual relationships between the acoustic impedances of the individual medium within a layered structure. The change in the thickness of the layer does not affect the magnitude of impedance when incident angle is kept constant.

Figure 7.7 Variation of absolute value of surface impedance with frequency for different values of layer thickness (i) $h_1 = 1\, \text{mm}$ (ii) $h_1 = 2\, \text{mm}$ (iii) $h_1 = 3\, \text{mm}$ (iv) $h_1 = 4\, \text{mm}$; $\leftarrow 0|1|3 \rightarrow$

The figure 7.8 shows the plot of surface impedance with porosity at $\theta = 5^\circ$ and frequency =1MHz for a layered configuration consisting of layers 1 and 2 of thickness 1mm and 2mm respectively between FHS and the PZT-7H substrate. The acoustics impedance shows the linear dependence on porosity which could be quantitatively decreasing by considering the gradual introduction of the fluid phase, in which sound wave velocity is far lower than in the solid phase. The lowest value reached 1.2e7, close to that of biological tissue and water, which is beneficial in improving acoustic matching by minimizing energy loss at the interface of the ceramic and medium. The lower acoustic impedance brings about improved hydrostatic figure of merit and coupling with biological tissues and water, respectively.
Figure 7.8 Variation of absolute value of surface impedance with porosity at $\theta = 5^\circ$, frequency =1MHz; $\leftarrow 0|1|2|3 \rightarrow$

The figure 7.9 demonstrates the variation of reflected, transmitted and interaction energy ratios with the angle of incidence at frequency =10000Hz. The plate consists two layers 1 and 2 & lies over the PZT-7H substrate. The first critical angle, encountered, is that of the $qP_1$ mode of the PPHS. Beyond this angle, this wave component becomes evanescent and decays exponentially from the boundary. As the angle of incidence further increases, a second critical angle, corresponding to $qS_2$ wave, is encountered. Beyond this critical angle, only one wave component, namely $qP_2$ wave propagates in the PPHS. Beyond all critical angles of the transmitted medium, no energy will be transmitted and the incident wave suffers total internal reflection. The energy carried out by the electric potential modes is very small in comparison to other three propagating modes. These energy ratios are so small that we can afford to call these modes as non-propagating modes in the sense of energy progress. It is observed that, interaction energy ratio and energy ratios corresponding to electric potential modes increase abruptly at the critical angles corresponding to propagating modes in PPHS. Thus the layered structure is very selective with respect to the angle of incidence of the propagating wave. The reflected and transmitted
energy ratios change significantly with the change in the porosity. The energy carried out by the transmitted $qP_1$ and $qS_1$ wave decreases as the porosity increases while that of $qP_2$ mode and electric potential modes increase with increase in the porosity of the medium. The variation in porosity affects significantly the interaction energy ratio beyond $\theta = 58^\circ$. The change in the porosity does not affect the critical angles of the transmitted modes.

![Figure 7.9 Variation of Reflected and Transmitted energy ratios with angle of incidence for different porosity; \( \theta \neq 58^\circ \).](image)

Next, to observe the effects of piezoelectric interaction on the energy partitions, the parameters for the non-piezoelectric case are taken as

$$
e_{15} = e_{31} = e_{33} = \zeta_{15} = \zeta_{31} = \zeta_{33} = \xi_{11} = \xi_{33} = \xi_{33}^* = A_{11} = A_{33} = e_3^* = \zeta_3 = 0.$$

The reflection and transmission coefficients for one to four layers between FHS & PPHS in the absence of piezoelectric effect are shown in the figure 7.10. Comparison of the figures 7.3 and 7.10 reveals that, in the absence of piezoelectricity, more energy goes to the reflected wave in comparison to transmitted waves.
7.7 Validation of the Model

To validate the considered model of study here, the earlier established results are obtained as special cases of the present model, which are described as follows:

1. If the thickness of the porous piezoelectric layers reduces to zero or if material of all the layers and PPHS is same i.e. Barium Titanate then the model reduces to fluid loaded PPHS. The results obtained for this reduced case (Fig. 7.11) match with the results shown in the figures (6.6) & (6.7) of the previous chapter.
Figure 7.11 Variation of reflected and transmitted and interaction energy ratios with angle of incidence for different porosity; FHS|PPHS

Figure 7.12 Variation of absolute value of surface impedance with angle of incidence
2. If we neglect the effects of piezoelectricity in all layers and PPHS and effects of porosity in PPHS, the model reduces to poro-elastic layered structure embedded between FHS and an elastic half space (EHS). Consideration of the absence of dissipation effects which arise due to viscosity of saturating fluid, low frequency range and the values of elastic constants as mentioned in Vashishth and Khurana (2002), give the results as shown in the figure 7.12. These results agree with the results of Vashishth and Khurana (2002), as it should be.

3. Further, if we also neglect the effects of porosity in the layers embedded between FHS and EHS in the model deduced in the special case 2, the model reduces to fluid-loaded elastic layered structure. The results obtained for this reduced case (Fig. 7.13) are found in agreement with the results of Nayfeh and Taylor (1988) when elastic constants are taken as given in Nayfeh and Taylor (1988).

![Figure 7.13 Variation of real and imaginary parts of reflected amplitude ratio (rfl) with apparent phase velocity (c)](image)

Further, to verify the results computed by numerical techniques used in the present study, the results are also computed using alternative numerical technique by constructing a rigorous program. In transfer matrix approach, we obtain a non-homogeneous system (7.32) of 6 simultaneous equations. Solution of this system analytically by Cramer’s rule and numerically by Gauss elimination method gives the same results. We also calculate the results using direct method (DM). In this method,
we apply the boundary conditions at each interface simultaneously. For a layered structure consisting of six layers between FHS and PPHS, we obtain a non-homogeneous system and a Global matrix of order 66. The figure 7.14 shows the comparison between the TMM and DM simulation for the energy ratios of reflected and transmitted waves propagating in multilayered structure having configuration $\leftarrow 0|1|2|1|2|3 \rightarrow$ at $\theta = 5^\circ$ in the frequency range 1-6 MHz. A very good agreement between the TMM and DM spectra is observed in the broad range of frequency, which validates the TMM approach.

![Figure 7.14 Variation of Reflected and Transmitted energy ratios with frequency at $\theta = 5^\circ$; (i) $\leftarrow 0|1|2|3 \rightarrow$ (ii) $\leftarrow 0|1|2|1|2|3 \rightarrow$ (iii) $\leftarrow 0|1|2|1|2|1|2|3 \rightarrow$](image)

**7.8 Conclusion**

Knowledge and control of the reflection and transmission of ultrasonic waves in layered media is an important basic problem in underwater acoustics. Following the importance of the reflection-transmission phenomenon through layered media to a wide range of technical applications, particularly in surface acoustic wave devices, composite materials characterization and smart structures for the analysis of acoustic
wave interaction with anisotropic and piezoelectric multilayers, the reflection and transmission of ultrasonic waves from a fluid loaded piezoelectric layered structure having porosity is studied using the transfer matrix technique. The study can be used to identify mismatches at the interface because a contrast in acoustic impedance causes reflection.

The pattern of variation of energy ratios with frequency is oscillatory and repeats itself after small frequency packets. The number of maxima and minima increases in number as the layers are added to the array. The minima’s in the reflected energy ratio get deepens and increases in number with the addition of periodic stacks of layers over the PPHS. The energy carried out by the transmitted $qP_2$ mode increases with increase in the number of periodic stacks over the PPHS.

The number of stop bands also increases as the stacks of layers are added to the array. The existence of these stop bands are of great importance in design and construction of piezo-composite transducers.

The surface impedance decreases with increase in the value of the porosity, which is beneficial in improving acoustic matching by minimizing energy loss at the interface of the ceramic and medium.

The energy carried out by the transmitted $qP_1$ and $qS_1$ waves decreases as the porosity increases while that of $qP_2$ mode and electric potential modes increase with increase in the porosity of the medium.

The reflected energy ratio increases while that of other propagating modes decreases in the absence of piezoelectric interaction.

Three special cases are obtained from the studied model, which are in agreement with the earlier established results. A good agreement between the results computed by using different numerical techniques is found.