CHAPTER 2

AIR TO SUB-SURFACE COMMUNICATION IN AN
ANISOTROPIC CONDUCTING HALF SPACE*

2. A INTRODUCTION

Several theoretical studies on various aspects of the boundary-value problem dealing with the dipole antennas near the interface of a conducting half-space have been dealt at length in text-books (Sommerfeld (1949), King (1961), Jordan (1968), Wait (1962a), Banos (1966), Stratton (1941)). A number of articles have appeared in the special issue of IEEE Trans. on Ant. and wave propagation, on the "Electro-magnetic waves in the earth" (1963). Wait and Campbell (1953), Durani (1962), Wait (1961), Bonnister (1965, 67), Banos (1966) etc. have investigated the field behaviour in the isotropic conducting half-space when the transmitting dipole is placed on or above it and have derived the appropriate approximations for the electro-magnetic fields at various distance-ranges. Wait (1962a) studied the ground-wave

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propagation over inductive surface and presented the numerical results in terms of surface-impedance. However, all these investigations were confined to isotropic structures, but from numerous field observations it is found that under the continents the top layer is sedimentary and inherently anisotropic with respect to the conductivity. The jungle environment may also be more appreciably characterized by a uniaxial conducting medium such that the average conductivity in the horizontal direction differs from that in the vertical direction (Wait (1967)), hence considerable efforts have been made in the past decade to investigate the response of the uniaxial conducting medium. (Tikhonov (1959), Chetaev (1962), Wait (1966), Bergman (1965), Praus (1965), Sinha and Bhattacharyya (1967), Sinha (1968), (1969).

In this chapter, fields of a vertical electric dipole placed at the air-earth interface, have been evaluated and expressed in terms of known functions. The numerical computations have been performed and curves are plotted to examine qualitatively, the field modifications due to anisotropy at different sub-surface locations.
2.1 FORMULATION

We consider a uniaxial model conductor \( Z > 0 \), illustrated in figure 2.1 characterized by the conductivity tensor \( \sigma \) expressed as

\[
\sigma = \begin{bmatrix}
\sigma_l & 0 & 0 \\
0 & \sigma_t & 0 \\
0 & 0 & \sigma_l
\end{bmatrix}
\]  

(2.1)

where \( \sigma_l \) is longitudinal conductivity and \( \sigma_t \) is the transverse conductivity. Conductivity of the upper half space is assumed to be \( \sigma_0 \). Three simplifications have been made i.e.,

(i) Magnetic permeability \( \mu \) is everywhere the same and is that of free space;

(ii) Displacement currents are neglected throughout;

and (iii) Major contribution to the field is the ground wave, as no account is made of the ionosphere.

The source of excitation is taken to be a vertical electric dipole, with a current moment \( "I dl" \) and placed on the surface of an anisotropic conducting medium. Time varies according to \( e^{jωt} \) (hereafter suppressed in the equations).

Maxwell's equations in the lower half-space i.e., \( Z > 0 \) are written as

\[
\nabla \times \vec{H} = (\sigma^t) \vec{E}
\]

(2.2)

\[
\nabla \times \vec{E} = -iω \mu \vec{H}
\]

(2.3)

where \( \vec{E} \) and \( \vec{H} \) are electric and magnetic fields, respectively, \( (\sigma^t) \) is a conductivity tensor given by equation (2.1), \( ω \) = frequency of the wave and \( \mu \) = magnetic permeability.
As \( \text{div} \vec{E} = 0 \), Electric and Magnetic fields can be expressed in terms of vector potential \( \vec{A} \) and scalar potential \( \phi \) by the relations given by

\[
\vec{H} = \text{curl} \vec{A} \quad (2.4)
\]

and

\[
\vec{E} = -i\omega \mu \vec{A} - \gamma \cdot \text{grad} \phi \quad (2.5)
\]

If \( \vec{A} \) and \( \phi \) are chosen such that

\[
\text{div} \vec{A} + \sigma_t \phi = 0 \quad (2.6)
\]

a combination of equations (2.2) to (2.6) leads to

\[
\nabla^2 F_{z1} = r^2 \frac{1}{r_1} F_{z1} + \frac{\partial}{\partial z} \text{div} F(1 - \frac{r_1}{r_t}) \quad (2.7)
\]

where

\[
r^2 = i\omega \mu \sigma_t \cdot \nabla
\]

However in the upper half-space \((z \leq 0)\),

\[
(\nabla^2 - r_0^2) F_{z0} = 0 \quad (2.8)
\]

where

\[
r_0^2 = i\omega \mu \sigma_o \cdot \nabla
\]

Expressions for the vector potentials are given by

\[
F_{z0} = \frac{1}{4\pi} \frac{d}{dl} \left[ \int_{-\infty}^{\infty} \frac{\lambda}{u_0} e^{-u_0(z+h)} J_0(\lambda \sigma) d\lambda \right.
\]

\[
\left. + \int_{0}^{\infty} A_0(\lambda) e^{u_0 z} J_0(\lambda \sigma) d\lambda \right] \quad (2.9)
\]

and

\[
F_{z1} = \frac{1}{4\pi} \frac{d}{dl} \left[ \int_{0}^{\infty} A_1(\lambda) e^{-u_0 z} J_0(\lambda \sigma) d\lambda \right. \quad (2.10)
\]

where \( u_0 = (\lambda^2 + r_0^2)^{\frac{1}{2}}, \quad s = (\lambda^2 \sigma_0^2 + r^2)^{\frac{1}{2}}, \quad J_0(\lambda \sigma) \) is zero order Bessel function and \( A_0(\lambda) \) and \( A_1(\lambda) \) are undetermined functions of \( \lambda \) and can be evaluated from the boundary conditions, which require that
the tangential electric \( E_{\rho_0} = E_{\rho_1} \) and magnetic fields \( H_{\rho_0} = H_{\rho_1} \) are continuous across the interface \( Z = 0 \).

Tangential field components \( E_{\rho} \) and \( H_{\rho} \) are connected to the vector potential by the following relations:

\[
\begin{align*}
E_{\rho_0} &= \frac{1}{c_0} \frac{\partial^2 F_{\omega_0}}{\partial \rho \partial z}, \quad H_{\rho_0} = -\frac{\partial F_{\omega_0}}{\partial \rho} \\
E_{\rho_1} &= \frac{1}{c_1} \frac{\partial F_{\omega_1}}{\partial \rho \partial z}, \quad H_{\rho_1} = -\frac{\partial F_{\omega_1}}{\partial \rho}.
\end{align*}
\]  

(2.11)

After some algebraic calculations, we get

\[
A_0(\lambda) = \frac{r^2 u_0 - r_0^2 s}{r^2 u_0 + r_0^2 s} \quad \text{and} \quad A_1(\lambda) = \frac{2r^2 \lambda}{r^2 u_0 + r_0^2 s} e^{-u \lambda} \]  

(2.12)

considering conductivity of air negligibly small is, \( r_0^2 = 0 \), (this situation is quite common for low radio-frequency communication and is termed as quasi static approximation valid for observation distances very small to air-wave lengths). (The vector potential in the lower half-space is given by from equations (2.10) and (2.12)),

\[
E_{\omega_1} = \frac{1}{c_1} \frac{d}{dz} \int_{-\infty}^{\infty} \exp (-sz) J_0 (\lambda \rho) d\lambda
\]  

(2.13)

Equation (2.13) can be expressed in terms of the known functions with the help of Foster's integral given by

\[
F = \int_{-\infty}^{\infty} \frac{e^{-u \lambda}}{u} J_0 (\lambda \rho) d\lambda = I_0 \left[ \frac{\pi}{2} (R - z) \right] K_0 \left[ \frac{\pi}{2} (R + z) \right]
\]  

(2.14)

where \( I_0 \) and \( K_0 \) are modified Bessel functions of first and second kind of order zero and \( R = (\rho^2 + z^2)^{1/2} \).
Using equation (2.14) and equation (2.13) can be re-written as:

\[ f_{z1} = \frac{i}{2} \frac{d}{dx} \left[ \exp (-2z_1^2) \right] \frac{d}{dx} \left[ J_0 (2 \pi x) \right] dx \]

\[ = \frac{i}{2} \frac{d}{dx} \left[ \frac{1}{2 \pi z_1} \right] \left[ F_0 (\alpha_1) + k_0 (\beta_1) \right] \]

\[ f_{z1} = \frac{i}{2} \frac{d}{dx} \left[ \frac{1}{2 \pi z_1} \right] \left[ F_0 (\alpha_1) + k_0 (\beta_1) \right] \]

\[ (2.15) \]

Where

1. \( \alpha_1 = \frac{r_1 - z_1}{\pi} \)
2. \( \beta_1 = \frac{r_1 + z_1}{\pi} \)
3. \( \delta_1 = \frac{\alpha}{2m} \)
4. \( \delta_2 = \frac{\alpha}{2m} (1 + \cos \beta) \)
5. \( R_1 = (\delta_1^2 + \delta_2^2)^{1/2} \)
6. \( z_1 = m \delta_1 \)
7. \( s_1 = \frac{r_1}{m} \)

and \( \alpha = (\mu \sigma_0)^{1/2} R_1 \), \( \beta = \tan^{-1} \frac{\delta_1}{z_1} \)

\( m = (\frac{\sigma_0}{\delta_1})^{1/2} \) (coefficient of anisotropy) \( (2.17) \)

The existing electric and magnetic fields in the medium \( \Omega > 0 \) may be obtained from the following relations.

(derived from equations (2.2) to (2.4),)

\[ H_{\psi 1} = - \frac{\delta F_{z1}}{\delta \psi} \]
\[ \psi_{\psi 1} = \frac{i}{\sigma_0} \frac{\delta^2 F_{z1}}{\delta \psi \delta z_1} \] and \( \psi_{\psi 1} = \frac{i}{\sigma_0} \frac{\delta^2 F_{z1}}{\delta \psi \delta z_1} \)

\[ E_{z1} = - \omega \mu \epsilon F_{z1} + \frac{1}{\epsilon_0} \frac{\epsilon^2 F_{z1}}{\epsilon_0} \]

\( (2.18) \)
NUMERICAL EVALUATION

Using equations (2.15) to (2.18) the electromagnetic fields, inside the conducting medium, can be derived and expressed in terms of known parameters \( \beta \) and \( m \) (2.17) and are given by:

\[
H_{\theta} = \frac{1}{2 \pi} \frac{d i}{d \rho_1} \frac{\alpha}{2 \sqrt{2 m}} \sin \beta \left[ \left( 1 + \cos \beta \right) I_0 K_1 + \frac{1}{2} \left( 1 - \cos \beta \right) I_1 K_0 \right] + \frac{\alpha^2}{m} \cos \beta \left( I_0 K_0 - I_1 K_1 \right)
\]  

(2.19)

\[
E_{\phi} = \frac{1}{2 \pi} \frac{d i}{d \rho_1} \frac{\sin \beta \cos \beta}{2 \sqrt{2 m}} \left[ \frac{1}{2} m^2 + \cos \beta \left( 3 m^2 - \alpha^2 \right) I_0 K_1 + \frac{1}{2} m^2 - \cos \beta \left( 3 m^2 + \alpha^2 \right) I_1 K_0 \right]
\]

\[
+ \frac{\alpha}{2 \sqrt{2 m}} \sin \beta \left( 3 m^2 + \cos \beta \left( 3 m^2 + \alpha^2 \right) I_0 K_1 \right)
\]

\[
+ \frac{\alpha}{2 \sqrt{2 m}} \sin \beta \cos \beta \left[ \left( 3 m^2 + \cos \beta \left( 3 m^2 + \alpha^2 \right) I_0 K_1 \right) + \left( 3 m^2 - \cos \beta \left( 3 m^2 - \alpha^2 \right) I_1 K_0 \right) \right]
\]

(2.20)

\[
E_z = \frac{1}{2 \pi} \frac{d i}{d \rho_1} \frac{\alpha}{2 \sqrt{2 m}} \left[ \frac{1}{2} m^2 (1 + \cos \beta) (2 - 3 \sin^2 \beta) \right.
\]

\[
+ \frac{\alpha^2}{2 \sqrt{2 m}} \left. \sin \beta \cos \beta \right] \left( 2 - 3 \sin^2 \beta \right)
\]

\[
+ \left[ \frac{1}{2} m^2 (1 - \cos \beta) (2 - 3 \sin^2 \beta) \right.
\]

\[
- \alpha^2 \sin \beta \cos \beta \left] \right. \left. I_1 K_0 \right)
\]

Continued...
\[ + \frac{i \frac{\alpha}{2 \sqrt{2}}}{\sqrt{2}} \{ \frac{1}{2} \alpha \cdot \cos (3 \cos^2 \beta - 1) \} I_0 \cdot K_0 \\
+ 3 \frac{\alpha}{\sqrt{2}} \cdot \sin^2 \beta \cdot \cos \beta I_1 \cdot K_1 \\
+ \{ \frac{1 + \cos \beta}{m^2} \cdot (2 - 3 \sin^2 \beta) \} I_2 \cdot K_2 \\
+ \frac{\alpha^2}{m^2} \sin^2 \beta \cdot \cos \beta \} I_0 \cdot K_0 \} \]

In equations (2.19), (2.20) and (2.21) the modified Bessel functions \( I_0, K_0, I_1 \) and \( K_1 \) are defined as

\[
I_0 \left( \sqrt{\frac{\alpha}{2 g_1}} \right) = \text{ber} \left( g_1 \right) + i \text{bei} \left( g_1 \right), \\
K_0 \left( \sqrt{\frac{\alpha}{2 g_2}} \right) = \text{ker} \left( g_2 \right) + i \text{kei} \left( g_2 \right), \\
I_1 \left( \sqrt{\frac{\alpha}{2 g_1}} \right) = -i \left[ \text{ber} \left( g_1 \right) + i \text{bei} \left( g_1 \right) \right] g_1, \quad (2.22) \\
K_1 \left( \sqrt{\frac{\alpha}{2 g_2}} \right) = i \left[ \text{ker} \left( g_2 \right) + i \text{kei} \left( g_2 \right) \right] g_2.
\]

2.4 NUMERICAL RESULTS AND DISCUSSIONS

For plotting purposes electro-magnetic fields are obtained in the dimensionless form by normalising the moduli of \( E_z \) and \( E_z \) components by \( (1 \, d/2 \pi \sigma L R_1^3) \) and that of magnetic component \( H_\phi \) by \( (1 \, d/2 \pi R_1^2) \). (For normalising the fields and distance longitudinal conductivity is preferred as it is less subject to errors in the actual field problems, (Keller (1966)). Variations of the
normalised values of the electromagnetic field components, with the numerical distance \( r \) are illustrated graphically, in figures (2.2) to (2.8).

Electromagnetic energy reaching at point P \((R, B)\) in the conducting medium (Figure 2.1) in two ways. One is along 'R' and the other through air along the interface, which leaks down in the conducting medium and has significant contribution in the range \( r/R > 1 \) and \( \beta > mz \) (ie., \( \beta > 45^\circ \)). Resultant of these two causes peaks and kinks in the \( \mathcal{E}_\phi - \alpha \) and \( \mathcal{E}_z - \alpha \) curves (Figures 2.4 - 2.7).

Position of these kinks and peaks, shifts towards greater values of \( \alpha \), for increasing anisotropic coefficient. The physical explanation for this may be that with increase in the value of coefficient of anisotropy, \( m \), the damping of the wave propagating through the conducting medium decreases and therefore the wave-zone increases to the greater distances from the dipole.

Significant effect of anisotropy is observable in \( \mathcal{E}_\phi \) component, only, at the larger values of the numerical distance. At the surface of the anisotropic medium, ie., \( \beta = 90^\circ \), (i) the magnetic field component is not affected by anisotropy, and, (ii) the amplitude of \( \mathcal{E}_z \) component is proportional to the square of the coefficient anisotropy. (This agrees with Sinha (1969) \( ^a \)).
Near the source dipole ($\zeta = 0.1 \rightarrow 0.7$) the variation of $|E_N|$ component with $\zeta$ is quite different in the following two cases: (Figures 2.5 and 2.6).

(A) When the source and observer both are on the surface of the earth i.e., $\beta = 90^\circ$

(B) When the source is at the interface but the observer is inside the conducting medium i.e., $\beta = 60^\circ, 80^\circ, 82^\circ, 89^\circ$ or $89.9^\circ$.

In the first case with the increase in the numerical distance, the amplitude value of $|E_N|$ component increases sharply and for one particular value of $\zeta$, increase in the value of $\alpha'$, decreases the amplitude value, appreciably, however, in the second case amplitude of the $|E_N|$ component has almost the same value near the source dipole (significantly for smaller values of $\beta$) and then at larger distances, after reaching a maximum value, attenuation is incurred.

Another observation in the second case, which is contrary to the first case ($\beta = 90^\circ$), is that with increase in the value of $\alpha'$ amplitude value increases, proportionately.