CHAPTER V

INVENTORY MODEL FOR ECONOMIC PRODUCTION OF PULETS
CHAPTER V

INVENTORY MODEL FOR ECONOMIC PRODUCTION
OF PULETS

5.1 INTRODUCTION:

This chapter may be considered as an extension of Chapter IV on Poultry Industry. Here we have successfully tackled one of the most important problems in the Poultry industry, that is, the problem of matching supply and demand of point-of-lay hens against seasonal fluctuations at a minimum cost. To solve the above problem, the cost of adjustment model i.e. a modified form of certain inventory model has been developed and solved by Dynamic Programming approach. The optimal production of hens with minimum cost for meeting the varying but known demands for different periods has been worked out. This problem was posed and attempted by C. Mar - Molinero (1979), though without much success.

Infact, in a poultry industry, seasonal egg-consumption introduces a seasonal pattern in demand for point-of-lay hens. Input into the rearing units is related to the expected demand at the end of rearing
period, while output is related to the demand now. Fluctuations in the demand pattern, together with long rearing time would cause big fluctuations in the pullets being reared and therefore in the cash-flow of the firm, manpower requirements etc. and would therefore seem to be undesirable. If firm tries to smooth out the fluctuations by not carrying out the required change to the full extent, they will end up with younger than average hens in period of low demand, required to be sold at lower price or to be fed for a longer period. Due to above type of fluctuations, there is a cost which has to be adjusted, that is, cost of holding actual population minus cost of holding desired population for the given period.

Mar-Molinero (1979) applied the time series analysis and also used the method given by Larson (1964) to adjust the above cost and pointed out limitations and shortcomings of his procedure.

In the present chapter, we have developed an inventory model which takes care of shortcomings of the problem and solved it through Dynamic Programming approach to obtain an economically better inventory policy.
5.2 ASSUMPTIONS AND NOTATIONS:

The above problem can be formulated as an inventory model where the production and demand are varying over different periods. We assume that the inventory level is reviewed 'periodically' and the stock is replenished at the beginning of the year and no shortages allowed. Also, delivery lag expressed in terms of fixed number of period is permissible. In order to develop the cost model we shall use the following notations.

Let us define the period by symbol \( i \) where \( i = 1, 2, \ldots, N \). A period might be taken as a quarter of a year if we are studying seasonal fluctuations, or the year itself if we are considering long term productions and demands. Let

\[
\begin{align*}
Z_i &= \text{Total production of pullets in period } i \\
&= \text{(i.e. units of quantity produced or ordered per cycle)}, \\
\xi_i &= \text{amount of demand of pullets in period } i, \\
x_i &= \text{entering inventory (at the beginning of period } i) \\
&= \text{i.e. surplus of pullets at the end of the period } i - 1, \\
h_i &= \text{holding cost per unit of inventory carried forward from period } i \text{ to period } i+1, \text{ This is essentially current maintenance cost over different periods,}
\end{align*}
\]
\[ K_1 = \text{setup cost of the poultry farm,} \]
\[ c_1(z_1) = \text{marginal production cost function given } Z_1 \]
\[ \text{i.e., cost of additional produce.} \]

The function \( c_1(Z_1) \) is the cost which will cause effect only when production cost varies from one period to another or if there are price breaks.

Let
\[ c_1(Z_1) = \delta_1 K_1 + c_1(z_1) \tag{5.2.1} \]

where
\[ \delta_1 = \begin{cases} 0 & , Z_1 = 0 \\ 1 & , Z_1 > 0 \end{cases} \tag{5.2.2} \]

Since no shortages are allowed the objective is to determine the optimal values of \( Z_1 \) which minimize the sum of setup, purchasing and holding costs for all the \( N \) periods.

We are assuming that the holding cost is proportional to the amount of inventory carried forward from period \( i \) to period \( i+1 \), that can be written as follows:

\[ x_{i+1} = x_i + z_i - \bar{c}_i \tag{5.2.3} \]
5.3 FORMULATION OF THE PROBLEM

Here, we are having N-period ('stages') inventory model with discrete units and dynamic deterministic demand. Since, we are having N-stage (discrete) problem, and we have to take decision at each stage, the Dynamic programming approach is considered to be most appropriate procedure for optimization. For the dynamic programming formulation, each period i of production is defined as a 'stage' and the amount of entering inventory \( x_i \) as the 'state' of the system at stage i.

Let \( f_i(x_i) \) be the minimum inventory cost per period \( i, i+1, ..., N \). Then the minimum inventory cost \( f_N(x_N) \) at the final stage can be written as:

\[
f_N(x_N) = \min_{Z_N+x_N = \xi_N} [C_N(x_N)] \tag{5.3.1}
\]

For \( i = 1, 2, ..., N-1 \), we can compute the cost through the following backward recurrence relationship:

\[
f_i(x_i) = \min \left[ C_i(z_{i}) + h_i(x_i+z_i-x_{i+1}) \right.
\]

\[\left. + f_{i+1}(x_{i+1}+z_{i+1}-\xi_{i+1}) \right] \tag{5.3.2}
\]

where

\[
0 \leq x_{i+1} \leq \xi_{i+1} + \cdots + \xi_N \tag{5.3.3}
\]

and

\[
z_i \geq 0 \quad i = 1, 2, ..., N. \tag{5.3.4}
\]
3.4 NUMERICAL ILLUSTRATION AND DISCUSSION:

In order to illustrate the application of the above procedure, we shall consider the following two examples.

Example 1: In the example 1, we compute optimal level of seasonal production in order to meet the average quarterly demands, shown in Table 5.1.

<table>
<thead>
<tr>
<th>Year</th>
<th>January</th>
<th>April</th>
<th>July</th>
<th>October</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>23</td>
<td>27</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>24</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>22</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>23</td>
<td>29</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>28</td>
<td>37</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>35</td>
<td>34</td>
<td>40</td>
<td>33</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
<td>30</td>
<td>39</td>
<td>32</td>
</tr>
</tbody>
</table>

**Table 5.1**: Seasonal Fluctuations in Demand at Bombay (in million)

**Source**: Poultry Industry Year Book and Government of Maharashtra State, Pune (1980).
We take the initial inventory \( IO = 0 \) (zero),
the maximum level of entering inventory \( HI = 26 \),
(i.e. minimum level of demand)
and the maximum production level \( PI = 31 \).
(i.e. maximum level of demand)
Then, for \( n = 4 \) (number of periods) with their corresponding cost components shown in Table 5.2, the optimum inventory and production policy have been obtained.

Table 5.2 : Average Periodical Demands and Costs.

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand ( \xi_i ) (million)</th>
<th>Setup cost ( K_i ) (Rs.('000))</th>
<th>Holding cost ( h_i ) (Rs.('000))</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>27</td>
<td>35</td>
<td>6.42</td>
</tr>
<tr>
<td>April</td>
<td>26</td>
<td>36</td>
<td>5.81</td>
</tr>
<tr>
<td>July</td>
<td>31</td>
<td>39</td>
<td>8.93</td>
</tr>
<tr>
<td>October</td>
<td>28</td>
<td>38</td>
<td>7.22</td>
</tr>
</tbody>
</table>


A computer program has been developed for
finding the minimum cost using equation (5.3.1) and
(5.3.2) subject to the constraints (5.3.3) and (5.3.4).
The optimal cost and corresponding production schedules
are found to be as below:
The minimum total cost(setup, inventory and holding cost) i.e.
\[
\min \sum C_i(Z_i) = \text{Rs. 340('000)}
\]
The optimal amount to produce in different periods are as given below:

<table>
<thead>
<tr>
<th>Period</th>
<th>Amount (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58</td>
</tr>
<tr>
<td>2</td>
<td>2618</td>
</tr>
<tr>
<td>3</td>
<td>9339</td>
</tr>
<tr>
<td>4</td>
<td>10130</td>
</tr>
</tbody>
</table>

It is observed from the above results that there is an increasing trend in the rate of production from period 1 to period 3 but a decrease in the rate of production from period 3 to period 4. This is due to the fact that the demand has fallen down from 31 millions to 28 millions.

A computer check has shown that increase in either maximum of entering inventory level or in maximum production level, enhances the cost and changes the production policy. For instance if we increase the entering inventory to 31, other factors remaining the same, the cost gets enhanced to Rs. 350 (10.00); while changing the periodical production levels to 58, 59, 2618 and 1534 respectively. This kind of inventory policy, no doubt, shows varying seasonal production in relation to demands but, apart from increase in cost, brings down the production level from 2618 millions to 1534 millions, when we move from period 3 to period 4. This policy requires 'panic action' i.e. to destroy almost 40.3% of the pullets at the beginning of the period 4 and this is usually not considered to be desirable policy.
Example 2: In this example we try to apply the same model developed in section 5.3 to find out the inventory (production) level of growing pullets with a minimum cost for varying demands over different years. Data for the numerical computation are shown in Table 5.3.

Table 5.3: Yearly Demand with Setup and Holding Costs.

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand ($q_i$) (million)</th>
<th>Holding cost ($h_i$) (Rs. ('000))</th>
<th>Setup cost ($k_i$) (Rs. ('000))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>60</td>
<td>12.93</td>
<td>35</td>
</tr>
<tr>
<td>1980</td>
<td>66</td>
<td>9.43</td>
<td>39</td>
</tr>
<tr>
<td>1981</td>
<td>72</td>
<td>15.21</td>
<td>43</td>
</tr>
<tr>
<td>1982</td>
<td>79</td>
<td>7.21</td>
<td>46</td>
</tr>
<tr>
<td>1983*</td>
<td>87</td>
<td>8.93</td>
<td>49</td>
</tr>
<tr>
<td>1984*</td>
<td>95</td>
<td>9.40</td>
<td>54</td>
</tr>
</tbody>
</table>

* : Estimated values.

Source: Indian Poultry Industry Year Book 1982 and Government Poultry Farms (on special request).

Keeping demand in view, we take

- the minimum inventory level = 100,
- the maximum production level = 100,
- the initial inventory = 0 (zero),
- number of periods = 6.
With the cost components shown in Table 3.3, equation (5.3.1) to (5.3.4) yields:

The total minimum cost for six periods = Rs. 220 (‘000).

The optimal level of production of pullets in different years are as given below:

<table>
<thead>
<tr>
<th>Period</th>
<th>Production (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20592</td>
</tr>
<tr>
<td>2</td>
<td>32518</td>
</tr>
<tr>
<td>3</td>
<td>15132</td>
</tr>
<tr>
<td>4</td>
<td>29219</td>
</tr>
<tr>
<td>5</td>
<td>07903</td>
</tr>
<tr>
<td>6</td>
<td>60250</td>
</tr>
</tbody>
</table>

From the above example, it is noticed that the optimal amount of production for period one is 20592 units. These units together with the initial inventory will be supplied against the demand of period 1. The units left at the end of the period 1 forms the initial inventory for period 2. This along with the produce of the period 2 i.e. 32518 units will be supplied against the demand of period 2 and so on.

It is further noticed that, for the period 3, we have to produce $X_3 = 15132$ units which is less than the amount produced in period 2, though the demand for period 3 is more than that of the period 2. This can be explained by stating that the number of items left at the end of period 2 is quite high. In fact, this
kind of phenomenon occurs because our objective is to minimize the total cost over entire period. Any attempt to adjust the production as per yearly demand, will increase the total cost as noticed in example 1 and may require panic action.