Chapter 5

Effect of Parallel Electric Field & General Distribution Function on Electrostatic Ion Cyclotron Instability

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5.1 Introduction

The effect of the general distribution function, transverse inhomogeneous electric field and ion and electron beam on the EICI has been discussed in chapters 2-4. In this chapter the study of EICI with general distribution function in the presence of the parallel electric field has been reported.

Recently, interest has been devoted to the role of the high-latitude ionosphere in influencing magnetospheric dynamics or morphology. In particular, the problem of high-latitude ionosphere as a source of magnetospheric plasma has come under intense study in the last decade. Several investigators have observed upflowing accelerated ionospheric ions in the auroral zone space plasma (Mozer et al, 1980; Ganguli, 1997). Alfven was first to point out the importance of parallel electric field on auroral field lines.

Electric field plays an important role in the dynamics of plasma in the ionosphere, magnetosphere and the solar wind. In the auroral regions, charged particles are accelerated to very high energies by electric fields parallel to the magnetic field. Measurements of dc and ac electric fields and plasma densities on the S3-3 satellite have shown that the EIC turbulence is associated with significant dc electric fields parallel to auroral field lines, which brings about collisionless wave-particle resonance interactions, resulting in enhanced plasma heating and anomalous transport (Satyanarayan et al, 1985; Ganguli, 1997).
It is well known that upward parallel field accelerates the electrons downwards, in the region of upward field-aligned currents. However, observations also indicate that downward electric fields, supporting potential drops of a few tens to about a few hundred volts, may exist in the region of return currents (Gorney et al, 1985). Gorney et al (1985) suggested that such downward electric fields and the geomagnetic mirror force may keep ions trapped in the region of perpendicular acceleration for a sufficient long time and thus the ions can be energized up to the observed energy levels.

In the auroral zone a quasi-static parallel electric field accelerates ionospheric ions outward, providing a significant source of ions for the magnetosphere. Such an electric field accelerates the ions to, approximately, the same energy irrespective of their mass. However, the corresponding change in velocity is, approximately, inversely proportional to the square root of the ion mass which produces a differential ion flow with the lighter ions having a higher beam speed than the heavier ions. The differential ion flow can lead to an ion-ion streaming instability that can produce strong heating of ions with net energy being transferred from the light ions to the heavy ions (Dusenbery et al, 1988).

A variety of approaches consider the transient phenomena and explain the auroral particle acceleration in terms of EIC waves propagating parallel or obliquely to the ambient magnetic field (Lysak and Dum, 1983; Bajaj and Tiwari, 1992a). These theories mainly describe the field-aligned current and electric field associated with the EIC wave which has originated in
some magnetospheric generator region via impressed perpendicular and time varying electric fields (Tiwari and Rostoker, 1984).

Small-scale intense disturbances of the electric fields are constantly measured by Polar and Freja satellite in the altitude range 900km to $2R_E$ above the auroral ionosphere (Wahlund et al, 1998; Knudson et al, 1998). There is observational evidence for parallel electric fields as large as 100mV/m along auroral field lines (Mozer et al, 1985; Mauk and Zenetti, 1987). Parallel electric fields upto 250 mV/m have been observed recently by a Polar satellite in the low-altitude boundary of the auroral acceleration region whereas higher altitude fields have magnitude of few mV/m (Mozer and Hull, 2001).

Variety of theories have been proposed to explain the development of the parallel electric fields on the auroral field lines such as electrostatic shock model (Mozer et al, 1980), coherent ion cyclotron emissions (Knudson et al, 1998), double layer process (Bostrom et al, 1988), solitary waves (Bostrom et al, 1988; Dovner et al, 1994), anomalous resistivity process (Palmadesso et al, 1974), auroral potential drop and others. The static model considers the dc electric field, which is developed when two different dynamical conditions prevail between the ionosphere and the magnetosphere.

Among the earliest waves found to exist in plasma, the electrostatic ion cyclotron (EIC) waves play an important role in ion acceleration and heating in the earth's magnetosphere (Ganguli, 1997), anomalous transport (Dum and Dupree, 1970), in driving field-aligned currents, in explaining broad band extremely low frequency (BB-ELF)
emissions (Wahlund et al, 1998), ion conic formation (Kintner et al, 1989), in explaining inverted-V structures in magnetosphere-ionosphere coupling (Mozer et al, 1977; Mozer et al, 1980), solar flares and solar winds (Gary et al, 2001). In particular, a series of spacecrafts have directly detected strong EIC wave turbulences associated with particle energisation in the auroral region. Theoretical studies of high latitude transversely accelerated ions and the wave at equatorial to middle latitudes both involve EIC waves. These waves are considered in a number of models as an agent of magnetosphere-ionosphere coupling (Mozer et al, 1977; Mozer et al, 1980; Kintner et al, 1989).

In the present chapter the particle aspect analysis (Tiwari et al, 1985) of the EIC wave has been studied by incorporating the details of particle trajectories in the presence of parallel electric field through the modification of the general distribution function.

In most of the theoretical work reported so far, the velocity distribution functions have been assumed to be either ideal Maxwellian or bi-Maxwellian (Ganguli, 1997; Gavrishchaka et al, 1997; Bharuthram et al, 1998; Zakhrov and Meister, 1999; Hamrin et al, 2001), ignoring the steep loss-cone feature. Plasma in mirror like devices and in the auroral region with curved and converging field lines (Hirose, 1976; Wong et al, 1985) considerably depart from Maxwellian distribution, and have steep loss-cone distribution (Tiwari and Varma, 1993; Gaelzer et al, 1997). In the present work, for the first time, the general distribution function is used to study the EIC waves in the presence of parallel electric field. The present analysis is based on Dawson’s theory
(Dawson, 1961) of Landau damping, which has been further extended by Terashima (1967), Misra and Tiwari (1979), Tiwari and Varma (1993) and Dwivedi et al (2002).

5.2 Basic Assumptions

The basic assumptions are the same as in earlier work by Terashima (1967), Tiwari and Varma (1993), and Dwivedi et al (2002). The plasma is considered to be homogeneous and collisionless consisting of resonant and non-resonant particles. The ions are supposed to have unit charge. The wave is considered to be propagating obliquely to the static uniform magnetic field \( B_0 \) that is along the z-direction. The non-resonant particles support the oscillatory motion of the EIC wave while the resonant particles participate in energy exchange with the wave. An EIC wave is assumed to start at \( t = 0 \) when the resonant particles are not disturbed. The trajectories of particles are then evaluated within the framework of linear theory. Using the particle trajectory in the presence of EIC wave, the dispersion relation and the growth rate is derived for different distribution indices.

The wave is assumed to have the form

\[
k \parallel E, \quad k = (k_{\parallel}, 0, k_{\perp}), \quad E = (E_X, 0, E_Z)
\]

with

\[
E_1(r,t) = E_1\cos(k_{\parallel}x + k_{\perp}z - \omega t), \quad E_2(r,t) = \kappa E_1\cos(k_{\parallel}x + k_{\perp}z - \omega t)
\]

\[
\kappa = \left( \frac{k_{\parallel}}{k_{\perp}} \right) < 1
\]

(5.1)
The amplitude $E_1$ is a slowly varying function of $t$ i.e. 
\[ \frac{1}{E_1} \left( \frac{dE_1}{dt} \right) << \omega. \]

In the present analysis the EIC instability in the system of hot electrons and hot ions is considered under the condition (Terashima, 1967)

$\omega \sim \ell \Omega_i$

$V_{T||i} < \left| \frac{\omega - \ell \Omega_i}{k_\parallel} \right| \ll V_{T||e}$

$1 > \kappa^2 \equiv \frac{k_\parallel^2}{k_\perp^2} > \left( \frac{\omega - \ell \Omega_i}{\Omega_i} \right)^2$

and $k_\perp^2 \rho_{e,2}^2 \ll k_\perp^2 \rho_i^2 \sim 1$ (5.2)

where $V_{T||i,e}$ is the thermal velocity of the ions and electrons respectively along the magnetic field, $\Omega_i$ is the gyro-frequency of the ion. $\ell = 1, 2, \ldots$ represents the harmonics of the wave, $\rho_{i,e}$ is the mean gyro-radii of the ions and electrons respectively, $\omega$ represents the real wave frequency, $k_\parallel$ and $k_\perp$ are the components of the wave vector along and across the magnetic field.

5.3 Particle Trajectories and Velocities

In the present mathematical analysis the procedure adopted by Terashima (1967) and Dwivedi et al (2002) is followed. The equation of motion of a particle is given by
\[
\mathbf{m}\frac{d\mathbf{v}}{dt} = q\left[ \mathbf{E} + \left(\frac{1}{c}\right)\mathbf{v} \times \mathbf{B}_0 \right]
\]

(5.3)

where symbols have their usual meaning.

If \( \mathbf{E} \) is considered to be a small perturbation, velocity \( \mathbf{v} \) can be expressed in terms of unperturbed velocity \( \mathbf{V} \) and perturbed velocity \( \mathbf{u} \). The perturbed velocity \( \mathbf{u} \) is determined by

\[
\frac{du_\parallel}{dt} = \frac{q\kappa E_i}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \cos(\Lambda_n t + \Psi_n^0)
\]

\[
\frac{du_\perp}{dt} + i\Omega u_\perp = \frac{qE_i}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \cos(\Lambda_n t + \Psi_n^0)
\]

(5.4)

where \( u_\parallel = u_x + iu_y \) represents the perturbed velocity in transverse direction and \( u_\parallel \) represents the perturbed velocity in parallel direction. The basic trajectories are the same as derived by Terashima (1967). The resonance criteria is given by

\[
\Lambda_n (V_\parallel = V_r) = k_\parallel V_r - \omega + \ell \Omega_i = 0; \quad \ell = \pm 1, \pm 2, \ldots
\]

(5.5)

where \( V_r \) is the resonance velocity of the particles and the particles with parallel unperturbed velocity \( 'V_\parallel' \) near to \( V_r = \frac{\omega - \ell \Omega_i}{k_\parallel} \) are the resonant particles which, in this case, are the ions. This resonance condition means that for ions the wave appear to be independent of \( 't' \) in the particles frame. \( J_n(\mu) \) and \( J_\ell(\mu) \) are Bessel’s function which arise from the different periodical
variation of charged particles trajectories and \( \mu = \frac{k_{\perp}V_{\perp}}{\Omega_i} \). The term represented by the Bessel's function indicates the reduction in the field intensity due to finite gyro-radius effect. The oscillatory solution of \( u(t) \) is given by

\[
u_x(r, t) = \frac{qE_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_{\ell}(\mu) \times \\
\left[ \frac{\Lambda_n}{\Lambda_n^2 - \Omega_i^2} \sin \chi_{n\ell} - \frac{\delta}{2\Lambda_{n+1}} \sin(\chi_{n\ell} - \Lambda_{n+1}t) - \frac{\delta}{2\Lambda_{n-1}} \sin(\chi_{n\ell} - \Lambda_{n-1}t) \right]
\]

\[
u_y(r, t) = \frac{qE_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_{\ell}(\mu) \times \\
\left[ \frac{\Omega_i}{\Lambda_n^2 - \Omega_i^2} \cos \chi_{n\ell} + \frac{\delta}{2\Lambda_{n+1}} \cos(\chi_{n\ell} - \Lambda_{n+1}t) - \frac{\delta}{2\Lambda_{n-1}} \cos(\chi_{n\ell} - \Lambda_{n-1}t) \right]
\]

\[
u_z(r, t) = \frac{q\kappa E_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_{\ell}(\mu) \frac{1}{\Lambda_n} \left[ \sin \chi_{n\ell} - \delta \sin(\chi_{n\ell} - \Lambda_n t) \right]
\]

\[
(5.6)
\]

where \( \chi_{n\ell} = k \cdot r - \omega t + (n-\ell)(\Omega_i t - \theta) \) \hspace{1cm} \[
(5.7)
\]

\( \delta = 0 \) for non-resonant particles and \( \delta = 1 \) for resonant particles.

### 5.4 Density Perturbation

To find out density perturbation associated with the velocity perturbation \( u(r, t) \) we consider the equation (Terashima, 1967)

\[
\frac{dn_1}{dt} = -N(V \cdot \nabla u)
\]

\[
(5.8)
\]
Expressing the right hand side of equation (5.8) as the function of $t$ and the initial parameters and integrating we get $n_1(r,t)$ the perturbed density, for the non-resonant and resonant particle as

$$n_1(r,t) = -\frac{qE_1N}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \left[ \frac{k_\perp}{\Lambda_n^2 - \Omega_i^2} + \frac{\kappa^2 k_\perp}{\Lambda_n^2} \right] \sin \chi_{n\ell}$$  (5.9)

$$n_1(r,t) = -\frac{qE_1N\kappa^2 k_\perp}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \frac{1}{\Lambda_n^2} \times \left[ \sin \chi_{n\ell} - \sin(\chi_{n\ell} - \Lambda_n t) - \Lambda_n t \cos(\chi_{n\ell} - \Lambda_n t) \right]$$  (5.10)

provided that $\omega \sim \ell \Omega_i$

and $\kappa^2 = \left( \frac{k_\parallel}{k_\perp} \right)^2 \left| \frac{\Lambda_n^2}{\Lambda_n^2 - \Omega_i^2} \right| \approx \frac{\Lambda_n^2}{\Omega_i^2}$  (5.11)

### 5.5 General Distribution Function

To calculate the dispersion relation and growth rate the general loss-cone distribution function of the following form (Dory et al, 1965, Wong et al, 1985; Tiwari and Varma, 1993; Gaelzer et al, 1997) is used

$$N(y, V) = N_0 \left[ 1 - \varepsilon \left( y + \frac{V_x}{\Omega_i} \right) \right] f_\perp(V_\perp) f_\parallel(V_\parallel)$$  (5.12)

where $f_\perp(V_\perp) = \frac{V_\perp^{2J}}{\pi^J V_{T_\perp}^{2(J+1)}} \exp \left\{ - \frac{V_\perp^2}{V_{T_\perp}^2} \right\}$
\[ f_{\parallel}(V_{\parallel}) = \frac{1}{\sqrt{\pi} V_{T\parallel}} \exp \left\{ - \frac{V_{\parallel}^2}{V_{T\parallel}^2} \right\} \]  

(5.13)

with \( V_{T\parallel}^2 = \frac{2T_{\parallel e}}{m} \); \( T_{\parallel e} = T_{\parallel} \left[ 1 + i \frac{eE_{\parallel}k}{k^2 T_{\parallel e}} \right] \); \( k = \sqrt{k^2_{\perp} + k^2_{\parallel}} \)

where \( N_0 \) is the background plasma density, \( \varepsilon \) is a small parameter of the order of inverse of ‘density gradient scale length’ and is zero for the homogeneous plasma under consideration, \( J=0,1,2,..n \) is the distribution index, also known as the steepness of the loss-cone. For \( J=0 \) this distribution represents a bi-Maxwellian distribution and for \( J=\infty \) this reduces to Dirac-Delta function (Tiwari and Varma, 1993). \( V_{T\parallel e}^2 \) and \( V_{T\perp}^2 = (J+1)\frac{1}{2}T_{\parallel e}/m \) are the squares of parallel and transverse thermal velocities with respect to the external magnetic field. Index ‘\( J \)’ characterizes the width of the loss-cone. Moreover, \( f_{\parallel}(V_{\perp}) \) is peaked about \( J^{1/2}V_{T\perp} \) and has a half width of \( \Delta V_{\perp} \sim J^{-1/2}V_{T\perp} \).

The expression for \( T_{\parallel e} \) is originally derived by Pines and Schrieffer (1961) adopting the rigorous treatment of kinetic approach for the collective behavior of solid-state plasma. They arrived at the result that the parallel electric field \( (E_{\parallel}) \) is eliminated by adopting the expression for \( T_{\parallel e} \) as expressed above. The applied electric field parallel to \( B_0 \) modifies the electron thermal velocity in that direction and the temperature \( T_{\parallel} \) in that direction modifies to the complex temperature \( T_{\parallel e} \). Here we followed the technique of Pines and Schrienffer (1961) and Bers and Brueck (1968) where a change in the distribution function is due to the change in the temperature parallel to the
parallel electric field. Misra et al (1979) further considered this method for the investigation of whistler mode instability, Tiwari and Varma (1993) for the Drift waves and Dwivedi et al (2001b) for the investigation of the Alfven waves.

5.6 Dispersion Relation

Applying the charge neutrality condition, $\tilde{n}_i \approx \tilde{n}_e$, where $\tilde{n}_{i,e}$ are the integrated perturbed densities for the non-resonant particles, and using equations (5.9) and (5.12) we obtain

$$\tilde{n}_e \equiv \left( \frac{1}{k_\perp \sigma_{ie}^2} \right) \frac{E_1}{4\pi e} \sin(k_r - \omega t)$$  \hspace{1cm} (5.14)

$$\tilde{n}_i \equiv -\frac{k_\perp \kappa^2 \omega_{pi}^2}{[\omega - \ell \Omega_i]^2} \left( \frac{J^2}{\ell} \right) \frac{E_1}{4\pi e} \sin(k_r - \omega t)$$  \hspace{1cm} (5.15)

where $\omega_{pi,e}^2 = \frac{4\pi n_0 e^2}{m_{i,e}}$, and

$$<J^2> = \int_0^\infty 2\pi V_\perp dV_\perp J^2(\mu)f_{\perp i}(V_\perp) = \exp\left(-\frac{k_\perp^2 \rho_i^2}{2}\right) I_{\ell}\left(\frac{k_\perp^2 \rho_i^2}{2}\right)$$  \hspace{1cm} (5.16)

where $I_{\ell}\left(\frac{k_\perp^2 \rho_i^2}{2}\right)$ is modified Bessel function.

The Debye length 'd_{ie}' corresponding to mean parallel energy is given by
\[
d_{\|e}^2 = \frac{T_{\|e}}{m_e \omega_{pe}^2} \left[ 1 + \frac{e^2 E_{\|}^2}{k^2 \left(K T_{\|e}^2\right)^2} \right] \tag{5.17}
\]

Using the Poisson's equation

\[
\nabla \cdot \mathbf{E} = -k_\perp (1 + \kappa^2) E_1 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) = 4\pi \varepsilon (n_i - n_e) \tag{5.18}
\]

and perturbed ion and electron density \( \bar{n}_i \) and \( \bar{n}_e \) the dispersion relation is obtained as

\[
1 + \left( \frac{1}{1 + \kappa^2} \right) \left( \frac{\kappa^2}{k_{\perp e}^2 d_{\| e}^2} \right) - \left( \frac{\omega_{pi}^2}{\left[ \omega - \ell \Omega_i \right]^2} \right) \left( J_{\ell - 1}^2 + J_{\ell + 1}^2 \right) \equiv 0 \tag{5.19}
\]

For \( \ell = 1 \), \( \langle J_0^2 + J_2^2 \rangle = 1 - (J+1)b_i \); \( \langle J_0 + J_2 \rangle^2 = 1 - \frac{1}{2} (J+1)b_i \); \( b_i = \frac{k_{\perp i}^2 \rho_i^2}{2} \)

\[
\tag{5.20}
\]

For \( J = 0 \) and \( E_{\|} = 0 \) this dispersion relation reduces to that given by Terashima (1967) and Bajaj and Tiwari (1992a).

In this model, the evaluation of dispersion relation and growth rate is based upon the real quantities and the concept of imaginary quantities in the various parameters has not been adopted. Therefore, in density also only real term has been considered else the dispersion relation would have been complex due to that imaginary term which violates the basic principle. It is observed that the essential features of the EIC wave are retained even in this ideal case.
5.7 Energy Balance and Growth Rate

The wave energy density ‘\( W_w \)’ per unit wavelength is the sum of pure field energy and the changes in the energy of the non-resonant particles i.e. \( W_w = \frac{\lambda E_i^2}{8\pi} + W_e + W_i \), which comes out to be

\[
W_w = \frac{\lambda E_i^2}{8\pi} + \frac{\lambda E_i^2}{16\pi} \frac{\omega_{pi}^2}{(\omega - \ell \Omega_i)^2} \kappa^2 \frac{1}{2} \left( J_{\ell-1}^2 + J_{\ell+1}^2 \right) + \frac{\lambda E_i^2}{16\pi} \left( \frac{1}{k^2 d_{||e}^2} \right) \tag{5.21}
\]

where \( d_{||e}^2 \) is given by equation (5.17)

here, the ions contribution is dominant unless \( k^2 d_{||e}^2 < 1 \).

The transverse energy and parallel energy of the resonant ions are calculated to be

\[
W_{\perp} = \left( \frac{\lambda E_i^2}{8} \right) \left( \frac{\omega_{pi}^2}{\Omega_i^2} \right) \frac{\omega}{k_{||} V_{thci}} \frac{\Omega_i t}{\sqrt{2\pi}} \exp \left\{ - \frac{1}{2} \left( \frac{\omega}{k_{||} V_{thci}} \right)^2 \left( \frac{1 - \ell \Omega_i}{\omega} \right)^2 \right\}
\]

\[
\times \frac{1}{2} \left( J_{\ell-1}^2 + J_{\ell+1}^2 \right) \left[ 1 - \left( \frac{R \left( \frac{\ell \Omega_i}{\omega} - 1 \right)}{\ell \Omega_i/\omega} \right) \frac{T_{\perp i}}{T_{\parallel i}} \right] \tag{5.22}
\]
\[ W_{\parallel} = \left( \frac{\lambda E_i^2}{8} \right) \left( \frac{\omega_{pi}^2}{\Omega_i^2} \right) \left( \frac{\omega}{k_{\parallel} V_\text{thci}} \right) \sqrt{2\pi} \exp \left\{ -\frac{1}{2} \left( \frac{\omega}{k_{\parallel} V_\text{thci}} \right)^2 \left( 1 - \frac{\ell \Omega_i}{\omega} \right)^2 \right\} \times \frac{1}{2} \left( \langle J_{\ell-1} + J_{\ell+1} \rangle \right)^2 \left( \frac{\ell \Omega_i}{\omega} \right) \frac{T_{\perp i}}{T_{||ci}} \] (5.23)

where \( R = \frac{\langle J_{\ell-1} + J_{\ell+1} \rangle^2}{\langle J_{\ell-1}^2 + J_{\ell+1}^2 \rangle} \) (5.24)

\[ V_\text{thci} = V_\text{thi} \left[ 1 + \frac{e^2 E_i^2}{k^2 (KT_{||i})^2} \right]^{1/2} \] (5.25)

and \( T_{||ci} = T_{||i} \left[ 1 + \frac{e^2 E_i^2}{k^2 (KT_{||i})^2} \right] \) (5.26)

Using the law of conservation of energy

\[ \frac{d}{dt} (W_w + W_r) = 0 \] (5.27)

The growth rate is derived (Terashima, 1967) as

\[ \gamma = \frac{1}{E_i} \left( \frac{dE_i}{dt} \right) = -\frac{dW_r}{dt} / 2W_w \] (5.28)

Also \( \left| \frac{dW_{r,\perp}}{dt} \right| \gg \left| \frac{dW_{r,\parallel}}{dt} \right| \) (5.29)
Hence, the growth rate defined in equation (5.28) is given by

\[
\frac{\gamma}{\omega} = \sqrt{\frac{\pi}{2}} \left( \frac{\omega}{k_{\parallel} V_{thci}} \right) \left( 1 - \frac{\ell \Omega_i}{\omega} \right)^2 \exp \left\{ - \frac{1}{2} \left( \frac{\omega}{k_{\parallel} V_{thci}} \right)^2 \left( 1 - \frac{\ell \Omega_i}{\omega} \right)^2 \right\} \times \left[ R \left( \frac{\ell \Omega_i}{\omega} - 1 \right) \frac{T_{\perp i}}{T_{\parallel i}} - 1 \right] \tag{5.30}
\]

where \( \ell = 1, 2, 3 \ldots \) is to be substituted, \( R \) is given by equation (5.24), \( V_{thci} \) is given by equation (5.25) and \( T_{\parallel i} \) is given by equation (5.26). Here it is noted that \( J \) and the parallel electric field affects the growth rate. For \( J = 0 \) and \( E_{\parallel} = 0 \) the result is the same as derived by Terashima (1967) and Bajaj and Tiwari (1992a).

### 5.8 Marginal Instability

For the marginal unstable condition \( \gamma = 0 \), we then arrive at the result

\[
E_{\parallel} = \left[ \frac{k^2 (KT_{\parallel i})^2}{e^2} \left\{ \left( \frac{R (\ell \Omega_i - \omega) T_{\perp i}}{\Omega_i} \right) \frac{T_{\perp i}}{T_{\parallel i}} - 1 \right\} \right]^{1/2} \tag{5.31}
\]

which shows that parallel electric field may be a source of EIC wave generation besides the temperature anisotropy and the steep loss-cone. When both \( T_{\perp i}/T_{\parallel i} \) and \( R \) are greater than unity and \( \omega < \Omega_i \) wave generation by parallel electric field is possible. For the plasma parameters mentioned in the result and
discussion and $\omega=309s^{-1}$ the estimated value of $E_||$ for the wave generation in
the auroral acceleration region is of the order of 1mV/m which is in accordance
with the observations on the auroral field lines (Mozer et al, 1997).

5.9 Results and Discussion

In the present analysis the expressions for the dispersion relation,
resonant energies and growth rate are derived in the presence of parallel
electric field and the steepness of the loss-cone index. The following
parameters relevant to the auroral acceleration region (Mozer et al, 1997;
Mozer and Hull, 2001; Dwivedi et al, 2002) are used to evaluate the dispersion
relation, resonant energies and growth rate:

$B_0=4300nT$ at $1.4R_E$, $\Omega_i=412s^{-1}$, $\ell=1$, $\lambda=300m$, $E_i=50mV/m$, $\omega_{pi}^2/\Omega_i^2=2$,

$b_i=0.1$, $k_{||}=0.002m^{-1}$, $k_\perp=0.0025m^{-1}$, $\omega/k_{||} V_{thi}=0.2$, $T_{\perp}/T_{||i}=50$, $K T_{||i}=5eV$

and $\Omega_i t=10$.

The effect of parallel electric field ($E_||$) and distribution index ($J$) on the growth
rate and the resonant energies transferred is depicted in Figures-5.1, 5.2 and
5.3. Figure-5.1 depicts the variation of growth rate ($\gamma/\omega$) with the wave
frequency ($\omega/\Omega_i$) for different values of parallel electric field ($E_||$) and
distribution index ‘$J$’ for the first harmonic of the ion cyclotron wave. It is
observed that the effect of increasing parallel electric field is to reduce the
growth rate that may be due to the shifting of resonance condition. The effect
of higher distribution index is to enhance the growth rate.
Figure 5.1

Variation of $\gamma/\omega$ with $\omega/\Omega_i$ for different values of $E_\parallel$ and $J$.
Figure 5.2

Variation of $W_{r,\perp}$ with $\omega/\Omega_i$ for different values of $E_{r,\parallel}$ and $J$. 

Legend:
- $J=0$, $E_{r,\parallel} = 0$ mV/m
- $J=0$, $E_{r,\parallel} = 10$ mV/m
- $J=0$, $E_{r,\parallel} = 20$ mV/m
- $J=2$, $E_{r,\parallel} = 0$ mV/m
- $J=2$, $E_{r,\parallel} = 10$ mV/m
- $J=2$, $E_{r,\parallel} = 20$ mV/m
- $J=4$, $E_{r,\parallel} = 0$ mV/m
- $J=4$, $E_{r,\parallel} = 10$ mV/m
- $J=4$, $E_{r,\parallel} = 20$ mV/m
Figure-5.3

Variation of $W_{r\parallel}$ with $\omega/\Omega_i$ for different values of $E_\parallel$ and $J$
Thus, the mirror like structure of the magnetosphere with a steep distribution index may be unstable for the EIC wave emission. It is also observed that the growth rate decreases with the increasing values of $\omega/\Omega_i$ which may be due to the shifting of the resonance condition. Hence the wave energy is being transferred to the particles.

Figure-5.2 shows the variation of transverse resonant energy ($W_{r\perp}$), in joules, with the wave frequency ($\omega/\Omega_i$) of the wave for different values of parallel electric field ($E_{\parallel}$) and distribution index ‘$J$’ for the first harmonic of the ion cyclotron wave. It is observed that the effect of increasing ($E_{\parallel}$) is to increase the transverse resonant energy that may be due to the fact that the parallel electric field shifts the resonant criteria due to its effect involved in the modification of resonance condition. The effect of increasing distribution index is to decrease the transverse resonant energy. Thus, the steep loss-cone distribution of the magnetosphere stabilizes the transverse resonant energy. It is also observed that $W_{r\perp}$ increases with the increasing values of $\omega/\Omega_i$. The increase in heating of the particles by the parallel electric field is supported by the decrease in growth rate, as the wave energy is being transferred to the particles by the resonance interaction process.

Figure-5.3 depicts the variation of parallel resonant energy ($W_{r\parallel}$), in joules, with wave frequency ($\omega/\Omega_i$) for different values of parallel electric field ($E_{\parallel}$) and for the increasing values of the distribution index for the first harmonic of the ion cyclotron wave. Here it is observed that $W_{r\parallel}$ decreases when $\omega<\Omega_i$, becomes minimum when $\Omega_i-\omega$, and for $\omega>\Omega_i$, $W_{r\parallel}$ increases i.e.
for $\omega < \Omega_i$ the parallel energy is being transferred for perpendicular energisation and the wave energy is being transferred to the parallel resonating particles for $\omega > \Omega_i$. Effect of $E_\parallel$ is to decrease the parallel resonant energy. The decrease in parallel resonant energy by the parallel electric field ($E_\parallel$) may be accounted for the increase in the perpendicular resonant energy. The effect of increasing values of the distribution index is to decrease the parallel resonant energy. Thus, the steep loss-cone distribution of the magnetosphere stabilizes the parallel resonant energy due to the EIC wave.

Thus, the parallel electric field ($E_\parallel$) acts as a source of free energy for the EIC waves. It modifies the wave-particle resonance condition and leads to Landau damping effects and enhances acceleration of ions transverse to the magnetic field. The wave-particle interaction may cause the perpendicular acceleration and the energy of the parallel potential drop is extracted through the ion cyclotron waves. The results are consistent with the findings of Satyanarayan et al (1985) and other rocket and satellite observations (Mozer et al, 1997; Mozer and Hull, 2001).

The effect of distribution index is to increase the growth rate of the wave but to decrease the transverse acceleration of the ions. The destabilizing effects due to the steep loss-cone on different instabilities have also been reported by various workers (Tiwari and Varma, 1993; Gaelzer et al, 1997; Dwivedi et al, 2002).
EIC waves are often detected in the inverted-V structures of the auroral acceleration region (Mozer et al, 1977, 1980). Recently a FAST satellite (Carlson et al, 1998) has observed intense EIC wave turbulences upto 1000 Hz in association with parallel electric field upto 200 mV/m in the upward current auroral region (Gavrishchaka et al, 2000). Coherent ion cyclotron waves with amplitude upto 50 mV/m in association with parallel electric field ~ 200mV/m have also been observed recently by a Polar satellite (Acuna et al, 1995) in the auroral zone (Mozer et al, 1997). Local ion cyclotron waves with frequency ~ 100 Hz in association with parallel electric field ~ 10 mV/m -20 mV/m have also been observed by the same satellite in an upward field-aligned current region. The same Polar satellite has also observed local hydrogen ion cyclotron wave of amplitude 500 mV/m in association with parallel electric field at lower altitudes. EIC waves of frequency ~ 105 Hz in association with parallel electric field ~ 30 mV/m have also been observed in the magnetosphere by the satellite.

The results obtained here have a direct bearing on the observed space phenomenon. Freja satellite has recently reported the transversely accelerated ions (TAI) and their association with the ion cyclotron waves at altitudes upto few thousand kilometers in the auroral acceleration region (Wahlund et al, 1998). The perpendicular acceleration may be achieved through the resonant wave-particle interactions between the ions and the observed EIC waves and by the parallel resonant energy transferred by the ions in the presence of the parallel electric field The perpendicular acceleration of
ions observed is consistent with the characteristic of auroral ion conics recently observed by a Polar satellite in the auroral acceleration region (Mozer et al 1997).

The parallel acceleration of ions may be accounted for the observed small spatial scale large amplitude electric fields in the shocks and large spatial scale smaller amplitude parallel electric fields present in the regions of the low frequency turbulence, upflowing ions and field aligned currents. Parallel acceleration of ions gives rise to upflowing ion beams that has been observed by Mozer et al (1980) and recently by FAST and Polar satellites (Mozer et al, 1997; Gavrishchaka et al, 2000) in the poleward and equatorward field aligned currents and in upward and downward current regions of auroral acceleration region. The steep loss-cone controls the parallel acceleration of ions as well.

The EIC turbulence has been considered as a possible source of anomalous resistivity. It has been demonstrated that wave-particle interaction and the anomalous transport is necessary to explain a number of phenomenon observed in the ionosphere such as the formation of density cavities and their correlation with plasma waves, large electron temperature etc (Gavrishchaka et al, 1998). The quasistatic parallel electric field produces inverted-V electron distributions and consequently the aurora. The signatures of such a parallel electric field are well known. They include the inverted-V itself, the detailed ion and electron distributions, which includes upflowing ion beams and electrons between the potential, the mirror point and the electrostatic shocks (Mozer et al, 1977, 1980; Hamrin et al, 2001).
The steep loss-cone distribution in the presence of EIC wave enhances the growth rate so the anomalous resistivity and transport resulting from this instability is likely to play crucial role in the auroral acceleration region. The equilibrium dipolar magnetic field of the earth is curved in the meridional plane and may introduce loss-cone effects in the particle distribution function (Gaelzer et al, 1997). Thus the behavior studied for the EIC wave may be of importance in the electrostatic emission in the auroral acceleration region.

In most of the theoretical work the velocity distribution functions have been assumed to be ideal Maxwellian (Ganguli, 1997; Gavrishchaka et al, 1997; Bharuthram et al, 1998), although most turbulent heating experiment have been done in mirror like devices which in general allow non-Maxwellian, particularly loss-cone distribution (Hirose, 1976; Wong et al, 1985). The theory developed in the present work may be applicable to such hot particle mirror experiments. Single particle theory may be able to explain some of the plasma phenomenon that other theories may not.

5.10 Summary

The study of the effect of parallel electric field and general distribution function on electrostatic ion cyclotron instability is presented. The effect of parallel electric field is incorporated in the general distribution function through the modification of the particle thermal velocity parallel to the ambient magnetic field. The dispersion relation, resonant energy
transferred, growth rate and marginal instability criteria of the electrostatic ion cyclotron wave with general loss-cone distribution function in the presence of parallel electric field in the auroral acceleration region are discussed. The parallel electric field modifies the wave-particle resonance condition leading to Landau damping effects and enhanced transverse acceleration of ions whereas the effect of steepness of loss cone is to enhance the growth rate and to decrease the transverse acceleration of ions. The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region.