Effect of Ion and Electron Beam & General Distribution Function on Electrostatic Ion Cyclotron Instability

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4.1 Introduction

The effect of the general distribution function along with the transverse inhomogeneous electric field on the EICI has been discussed in the previous chapters. Another important factor to be considered in the auroral acceleration region is the ion and electron beam. Hence, in this chapter the study of EICI in the presence of the general distribution function and the ion and the electron beam is being reported.

There has been much discussion concerning the possible free energy sources needed to drive the electrostatic ion cyclotron waves. Kintner et al (1979) considered the possibility that either an electron drift or the upstreaming ions (ion beams) were the source of free energy for the EIC waves observed from S3-3 satellite data. In a statistical study of S3-3 data, Cattell (1981) concluded that an ambiguous identification of the free energy source for the waves observed by Kintner et al (1979) was not possible and the waves could be driven by a combination of ion beams and electron drifts. In the subsequent work, Kaufmann and Kintner (1982) concluded that many features of the S3-3 could be understood if the observed EIC waves were being driven by ion beams rather by cold drifting electrons. This view is supported by theoretical and numerical analysis of Okuda and Nishikawa (1984). However, Bergmann's (1984) result indicate that the relative importance of the two processes is sensitive to the temperature regime and the current driven ion
cyclotron instability modified by the presence of an ion beam appears to be most likely mechanism for explaining the S3-3 wave data.

Ion and electron beams have been observed over a wide region associated with the whole auroral oval. Over the last decade it has been established that auroral luminosity is due to the impact of an accelerated electron beam coming towards the ionosphere and at the same event an ion beam moving towards the magnetotail (Tiwari and Rostoker, 1984; Mauk and Zenetti, 1987). Laboratory experiments on EIC turbulences showed that ion heating results in a low-density warm core surrounded by a denser hot ion cloud (Bohmer and Fornaea, 1979). Therefore, high-level ion cyclotron turbulence can be sustained in a core of ion gyro-radius scale. Thus, it is possible for an unstable field-aligned current to produce fine structures in an unstable field-aligned region and lead to the formation of auroral arcs embedded in the inverted-V precipitation region.

The parallel potential drop along the auroral field lines may lead to the down coming electrons and upflowing ion beams (Tiwari and Rostoker, 1984; Mauk and Zenetti, 1987). Plasma wave measurements from the various satellites like Hawkeye-1, Imp-6, DE-1, DE-2, Viking etc show that a broad region of intense plasma wave turbulence occurs in the region of the field aligned current on high latitude auroral field lines at altitudes above few thousand kilometers during the periods of substorm activity (Lundin et al, 1994). Observations have shown the presence of ion and electron beams in the plasma sheet boundary layer (Singh and Lakhina, 2001). Recent observations
by the Polar satellite also indicate upgoing ion beams and downcoming accelerated electrons in the low altitude boundary and high altitude of auroral acceleration region (McFadden et al, 1999; Mozer and Hull, 2001).

Recently, Mozer and Hull (2001) have studied upgoing ion beams and downcoming electron beams in the low altitude boundary and high altitude of auroral acceleration region using the data obtained from a Polar satellite. In the present work we have utilized their data to study the interaction of ion and electron beams with EIC waves and hence the auroral acceleration phenomena.

Electrostatic ion cyclotron waves are a common feature of the auroral plasma in the region above field-aligned potential drops. The EIC wave requires a smaller value of magnetic field-aligned current to be unstable (Kindel and Kennel, 1971), can act as an ion heating source (Ganguli, 1997) and provide anomalous resistivity (Dum and Dupree, 1970). Plasma wave measurements from S3-3 satellite, polar satellite and recently by FAST satellite have observed EIC waves in the lower and topside ionosphere, auroral zone and magnetosphere (Mozer et al, 1997; Gavrishchaka et al, 1999).

Ideal Maxwellian or bi-Maxwellian velocity distribution functions have been assumed in most of the theoretical work reported so far (Ganguli, 1997; Gavrishchaka et al, 1997; Bharuthram et al, 1998; Zakhrov and Meister, 1999; Hamrin et al, 2001), ignoring the steep loss-cone feature. In the present work, for the first time, the general distribution function is used to study the EIC waves in the presence of ion and electron beam. The present
analysis is based on Dawson's theory (Dawson, 1961) of Landau damping, which has been further extended by Terashima (1967), Misra and Tiwari (1979), Varma and Tiwari (1992) and Dwivedi et al (2002).

In the present work the particle aspect analysis (Tiwari et al, 1985) of the EIC wave has been studied by incorporating the details of particle trajectories in the presence of upgoing ion beams and downcoming electron beams and the general distribution function. The advantage of this approach is its suitability for dealing with auroral electrodynamics involving the current system, particle acceleration and energy exchange by wave-particle resonant interaction. The method is more accurate than the magnetohydrodynamic approach in dealing with finite gyro-radius effects and temperature anisotropies.

4.2 Basic Assumptions

The basic assumptions are the same as in earlier work by Terashima (1967) and others (Varma and Tiwari, 1992; Dwivedi et al, 2002). The plasma is considered to be homogeneous and collisionless consisting of resonant and non-resonant particles. The ions are supposed to have unit charge. The wave is considered to be propagating obliquely to the static uniform magnetic field \( B_0 \) that is along the z-direction. The non-resonant particles support the oscillatory motion of the EIC wave while the resonant particles participate in energy exchange with the wave. An EIC wave is assumed to start at \( t = 0 \) when the resonant particles are not disturbed. The trajectories of
particles are then evaluated within the framework of linear theory. Using the particle trajectory in the presence of EIC wave, the dispersion relation and the growth rate is derived for different distribution indices.

The wave is assumed to have the form

\[ \mathbf{k} \parallel \mathbf{E}, \quad \mathbf{k} = (k_{\parallel}, 0, k_{\perp}), \quad \mathbf{E} = (E_x, 0, E_z) \]

with \( E_x(r,t) = E_1 \cos(k_{\parallel} x + k_{\perp} z - \omega t) \), \( E_z(r,t) = \kappa E_1 \cos(k_{\parallel} x + k_{\perp} z - \omega t) \) \hspace{1cm} (4.1)

\[ \kappa = \left( \frac{k_{\parallel}}{k_{\perp}} \right) < 1 \]

The amplitude \( E_1 \) is a slowly varying function of \( t \) i.e. \( \frac{1}{E_1} \left( \frac{dE_1}{dt} \right) \ll \omega \).

In the present analysis the EIC instability in the system of hot electrons and hot ions is considered under the condition (Terashima, 1967)

\[ \omega \sim \ell \Omega_i \]

\[ V_{T\parallel i} < \left| \frac{\omega - \ell \Omega_i}{k_{\parallel}} \right| < V_{T\parallel e} \]

\[ 1 > \kappa^2 \equiv \frac{k_{\parallel}^2}{k_{\perp}^2} > \left( \frac{\omega - \ell \Omega_i}{\Omega_i} \right)^2 \]

and \( k_{\perp}^2 \rho_e^2 << k_{\parallel}^2 \rho_i^2 \sim 1 \) \hspace{1cm} (4.2)

where \( V_{T\parallel i,e} \) is the thermal velocity of the ions and electrons respectively along the magnetic field, \( \Omega_i \) is the gyro-frequency of the ion. \( \ell = 1, 2, \ldots \) represents the harmonics of the wave, \( \rho_{i,e} \) is the mean gyro-radii of the ions and electrons
respectively, \( \omega \) represents the wave frequency, \( k_\| \) and \( k_\perp \) are the components of the wave vector along and across the magnetic field respectively.

### 4.3 Particle Trajectories and Velocities

In the present mathematical analysis the procedure adopted by Terashima (1967) and Dwivedi et al (2002) is followed. The equation of motion of a particle is given by

\[
m \left( \frac{dv}{dt} \right) = q \left[ E + \left( \frac{1}{c} \right) v \times B_0 \right]
\]

where symbols have their usual meaning.

If \( E \) is considered to be a small perturbation, velocity \( v \) can be expressed in terms of unperturbed velocity \( V \) and perturbed velocity \( u \). The perturbed velocity \( u \) is determined by

\[
\frac{du_\|}{dt} = \frac{q \kappa E_1}{m} \sum_{n=0}^{\infty} J_n(\mu) \cos(\Lambda_n t + \Psi_n^0)
\]

\[
\frac{du_\perp}{dt} + i \Omega u_\perp = \frac{qE_1}{m} \sum_{n=0}^{\infty} J_n(\mu) \cos(\Lambda_n t + \Psi_n^0)
\]  \hspace{1cm} (4.4)

where \( u_\perp = u_x + iu_y \) represents the perturbed velocity in transverse direction and \( u_\| \) represents the perturbed velocity in parallel direction. The basic trajectories are the same as derived by Terashima (1967). The resonance criteria is given by
\[ \Lambda_n(V_{\parallel}=V_r) = k_1 V_r \frac{\omega - \ell \Omega_i}{k_1} = 0; \quad \ell = \pm 1, \pm 2, \ldots \quad (4.5) \]

where \( V_r \) is the resonance velocity of the particles and the particles with parallel unperturbed velocity \( 'V_{\parallel}' \) near to \( V_r = \frac{\omega - \ell \Omega_i}{k_1} \) are the resonant particles which, in this case, are the ions. This resonance condition means that for ions the wave appear to be independent of ‘t’ in the particles frame. \( J_n(\mu) \) and \( J_\ell(\mu) \) are Bessel’s function which arise from the different periodical variation of charged particles trajectories and \( \mu = \frac{k_1 V_{\perp}}{\Omega_i} \). The term represented by the Bessel’s function indicates the reduction in the field intensity due to finite gyro-radius effect. The oscillatory solution of \( u(t) \) is given by

\[
u_x(r,t) = \frac{qE_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \times \left[ \frac{\Lambda_n}{\Lambda_n^2 - \Omega_i^2} \sin \chi_{n\ell} - \frac{\delta}{2\Lambda_{n+1}} \sin(\chi_{n\ell} - \Lambda_{n+1}t) - \frac{\delta}{2\Lambda_{n-1}} \sin(\chi_{n\ell} - \Lambda_{n-1}t) \right] \]

\[
u_y(r,t) = \frac{qE_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \times \left[ \frac{\Omega_i}{\Lambda_n^2 - \Omega_i^2} \cos \chi_{n\ell} + \frac{\delta}{2\Lambda_{n+1}} \cos(\chi_{n\ell} - \Lambda_{n+1}t) - \frac{\delta}{2\Lambda_{n-1}} \cos(\chi_{n\ell} - \Lambda_{n-1}t) \right] \]

\[
u_z(r,t) = \frac{qkE_1}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \frac{1}{\Lambda_n} \left[ \sin \chi_{n\ell} - \delta \sin(\chi_{n\ell} - \Lambda_n t) \right] \quad (4.6) \]

where \( \chi_{n\ell} = k.r - \omega t + (n-\ell)(\Omega_i t - \theta) \quad (4.7) \)

\( \delta = 0 \) for non-resonant particles and \( \delta = 1 \) for resonant particles.
4.4 Density Perturbation

To find out density perturbation associated with the velocity perturbation \( u(r,t) \) we consider the equation (Terashima, 1967)

\[
\frac{dn_1}{dt} = -N(V) (\nabla \cdot u)
\] (4.8)

Expressing the right hand side of equation (4.8) as the function of \( t \) and the initial parameters and integrating we get \( n_1(r,t) \) the perturbed density, for the non-resonant and resonant particle as

\[
n_1(r,t) = -\frac{qE_1N}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \left[ \frac{k_\perp}{\Lambda_n^2 - \Omega_i^2} + \frac{\kappa^2 k_\perp}{\Lambda_n^2} \right] \sin \chi_{n\ell}
\] (4.9)

\[
n_1(r,t) = -\frac{qE_1N\kappa^2 k_\perp}{m} \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_\ell(\mu) \frac{1}{\Lambda_n^2}
\]

\[\times \left[ \sin \chi_{n\ell} - \sin(\chi_{n\ell} - \Lambda_n t) - \Lambda_n t \cos(\chi_{n\ell} - \Lambda_n t) \right]
\] (4.10)

provided that \( \omega \sim \ell \Omega_i \)

and \( \kappa^2 = \left( \frac{k_\parallel}{k_\perp} \right)^2 \left| \frac{\Lambda_n^2}{\Lambda_n^2 - \Omega_i^2} \right| \approx \frac{\Lambda_n^2}{\Omega_i^2} \) (4.11)

4.5 General Distribution Function

To calculate the dispersion relation and growth rate the general loss-cone distribution function of the following form (Dory et al, 1965; Wong et al, 1985; Varma and Tiwari, 1992; Gaelzer et al, 1997) is used
\[ N(y, V) = N_0 \left\{ 1 - \varepsilon \left( y + \frac{V_x}{\Omega_i} \right) \right\} f_\perp(V_\perp) f_\parallel(V_\parallel) \]  
\hfill (4.12)

where \( f_\perp(V_\perp) = \frac{V_\perp^{2J}}{\pi V_{T_\perp}^{2(J+1)I}} \exp \left\{ - \frac{V_\perp^2}{V_{T_\perp}^2} \right\} \)

and \( f_\parallel(V_\parallel) \) is defined by the drifting Maxwellian

\[ f_\parallel(V_\parallel) = \frac{1}{\sqrt{\pi V_{T_\parallel}}} \exp \left\{ - \frac{(V_\parallel - V_{Dj})^2}{V_{T_\parallel}^2} \right\} \]  
\hfill (4.13)

where \( N_0 \) is the background plasma density, \( \varepsilon \) is a small parameter of the order of inverse of 'density gradient scale length' and is zero for homogeneous plasma under consideration, \( J=0,1,2,... \) is the distribution index, also known as the steepness of the loss-cone. For \( J=0 \) this distribution represents a bi-Maxwellian distribution and for \( J=\infty \) this reduces to Dirac-Delta function. \( V_{T_\parallel}^2 = 2T_\parallel / m \) and \( V_{T_\perp}^2 = (J+1)^{-1}(2T_\perp / m) \) are the squares of parallel and transverse thermal velocities with respect to the external magnetic field. Index 'J' characterizes the width of the loss-cone. Moreover, \( f_\perp(V_\perp) \) is peaked about \( J^{1/2}V_{T_\perp} \) and has a half width of \( \Delta V_\perp \sim J^{1/2}V_{T_\perp} \). \( V_{Dj} \) defines the beam velocity of the particles and subscript \( j \) stands either for electrons or ions.

### 4.6 Dispersion Relation

Applying the charge neutrality condition, \( \tilde{n}_i \approx \tilde{n}_e \), where \( \tilde{n}_{i,e} \) are the integrated perturbed densities for the non-resonant particles, and using equations (4.9) and (4.12) we obtain
\[\tilde{n}_e \equiv \left( \frac{1}{k_\perp d_{ie}^2} \right) \frac{E_1}{4\pi e} \sin(kr - \omega t)\]  
(4.14)

\[
\tilde{n}_i \equiv -\frac{k_\perp \kappa^2 \omega_{pi}^2}{\left[ \omega - k_{||} V_{Di} - \ell \Omega \right]^2} \left( J_\ell^2 \right) \frac{E_1}{4\pi e} \sin(kr - \omega t) 
(4.15)
\]

where \( \omega_{pi,e}^2 = \frac{4\pi n_0 e^2}{m_{i,e}} \)

and \( <J_\ell^2> = \int_0^\infty 2\pi V_\perp dV_\perp J_\ell^2(\mu) f_\perp(V_\perp) = \exp\left(-\frac{k_\perp^2 \rho_i^2}{2}\right) I_\ell\left(\frac{k_\perp^2 \rho_i^2}{2}\right) \)  
(4.16)

where \( I_\ell\left(\frac{k_\perp^2 \rho_i^2}{2}\right) \) is modified Bessel function.

The Debye length 'd_{ie}' corresponding to mean parallel energy is given by

\[
d_{ie}^2 = \frac{T_{||e}}{m_e \omega_{pe}^2} \]  
(4.17)

Using the Poisson’s equation

\[\nabla \cdot E = -k_\perp (1 + \kappa^2) E_1 \sin(kr - \omega t) = 4\pi e (\tilde{n}_i - \tilde{n}_e)\]  
(4.18)

and perturbed ion and electron density \( \tilde{n}_i \) and \( \tilde{n}_e \) the dispersion relation is obtained as

\[
1 + \left( \frac{1}{1 + \kappa^2}\right) \left( \frac{1}{k_\perp^2 d_{ie}^2} \right) - \left( \frac{\kappa^2}{1 + \kappa^2} \right) \left[ \frac{\omega_{pi}^2}{\left[ \omega - k_{||} V_{Di} - \ell \Omega \right]^2} \right] \left( J_{\ell-1}^2 + J_{\ell+1}^2 \right) \equiv 0 \] 
(4.19)

For \( \ell = 1 \), \( <(J_0^2 + J_2^2)> = 1 - (J+1)b_i \); \( <(J_0 + J_2)^2> = 1 - \frac{1}{2}(J+1)b_i \); \( b_i = \frac{k_\perp \rho_i^2}{2} \)  
(4.20)
For $J=0$ and $V_{Di}=0$ this dispersion relation reduces to that given by Terashima (1967).

### 4.7 Energy Balance and Growth Rate

The wave energy density $W_w$ per unit wavelength is the sum of pure field energy and the changes in the energy of the non-resonant particles i.e. $W_w = \frac{\lambda E_i^2}{8\pi} + W_e + W_i$, which comes out to be

$$W_w = \frac{\lambda E_i^2}{8\pi} + \frac{\lambda E_i^2}{16\pi} \frac{\omega_{pi}^2}{\left(\left(\omega - k_{||} V_{Di}\right) - \ell \Omega_i\right)^2} \kappa^2 \frac{1}{2} \left(J_{\ell-1}^2 + J_{\ell+1}^2\right) + \frac{\lambda E_i^2}{16\pi} \left(\frac{1}{k_{||}^2 d_{||e}^2}\right)$$

(4.21)

here, the ions contribution is dominant unless $k_{||}^2 d_{||e}^2 < 1$.

The transverse energy and parallel energy of the resonant ions are calculated to be

$$W_{r\perp} = \left(\frac{\lambda E_i^2}{8}\right) \left(\frac{\omega_{pi}^2}{\Omega_i^2}\right) \frac{\omega_i}{k_{||} V_{thi}} \sqrt{2\pi} \exp \left(-\frac{1}{2} \left(\frac{\omega_i}{k_{||} V_{thi}}\right)^2 \left(1 - \frac{\ell \Omega_i}{\omega_i}\right)^2\right)$$

$$\times \left(\frac{1}{2} \left(J_{\ell-1}^2 + J_{\ell+1}^2\right)\right) \left[1 - \left(R \left(\frac{\ell \Omega_i}{\omega} - 1\right)\right) \frac{T_{\perp i}}{T_{\parallel i}}\right]$$

(4.22)
\[ W_{r\parallel} = \left( \frac{\lambda E_i^2}{8} \right) \left( \frac{\omega_{pi}^2}{\Omega_i^2} \right) \left( \frac{\omega_i}{k_{||} V_{thi}} \right) \frac{\Omega_i t}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{\omega_i}{k_{||} V_{thi}} \right)^2 \left( 1 - \frac{\ell \Omega_i}{\omega_i} \right)^2 \right\} \left[ \frac{T_{\perp i}}{T_{|| i}} \right] \]

\[ \times \frac{1}{2} \left( J_{\ell-1} + J_{\ell+1} \right)^2 \left[ \frac{(1 - \frac{\ell \Omega_i}{\omega_i})^2}{\frac{\ell \Omega_i}{\omega_i}} \right] \] (4.23)

where \( \omega_i = \omega - k_{||} V_{Di} \) and \( R = \frac{\left\langle (J_{\ell-1} + J_{\ell+1})^2 \right\rangle}{\left\langle J_{\ell-1}^2 + J_{\ell+1}^2 \right\rangle} \) (4.24)

Using the law of conservation of energy

\[ \frac{d}{dt} (W_w + W_r) = 0 \] (4.25)

The growth rate is derived (Terashima, 1967) as

\[ \gamma = \frac{1}{E_i} \left( \frac{dE_i}{dt} \right) = -\frac{dW_r}{dt} / 2W_w \] (4.26)

Also \( \left| \frac{dW_{r\parallel}^i}{dt} \right| \gg \left| \frac{dW_{r\perp}^i}{dt} \right| \) (4.27)

Hence the growth rate defined in equation (4.26) is given by

\[ \frac{\gamma}{\omega} = \sqrt{\frac{\pi}{2}} \left( \frac{\omega_i}{k_{||} V_{thi}} \right) \left( 1 - \frac{\ell \Omega_i}{\omega_i} \right)^2 \exp \left\{ -\frac{1}{2} \left( \frac{\omega_i}{k_{||} V_{thi}} \right)^2 \left( 1 - \frac{\ell \Omega_i}{\omega_i} \right)^2 \right\} \]

\[ \times \left[ \frac{R \left( \frac{\ell \Omega_i}{\omega_i} - 1 \right)}{T_{\perp i}} \frac{T_{\perp i}}{T_{|| i} - 1} \right] \] (4.28)
where $\ell=1,2,3\ldots$ is to be substituted and $R$ is given by equation (4.24). Here it is noted that the distribution index $J$ and the ion beam affects the growth rate. For $J=0$ and $V_{Di}=0$ the result is the same as derived by Terashima (1967).

### 4.8 Marginal Instability

For the marginal unstable condition $\gamma=0$, we then arrive at the result

$$V_{Di} = \frac{e\Omega_i}{k_\parallel} \left[ 1 - \left( \frac{T_{\parallel}}{T_\perp R} \right) \right] - \frac{\omega}{k_\parallel}$$

(4.29)

which shows that ion beam may be a source of EIC wave generation besides the temperature anisotropy and the steep loss-cone. When both $T_{\perp}/T_{\parallel}$ and $R$ are greater than unity and $\omega<\Omega_i$ wave generation by ion beam is possible. For the plasma parameters mentioned in the result and discussion and $\omega=309\,\text{s}^{-1}$ the estimated value of $V_{Di}$ for the wave generation in the auroral acceleration region is of the order of $10^6 \,\text{m/s}$ which is in accordance with the observations on the auroral field lines (Mozer and Hull, 2001).

### 4.9 Results and Discussion

In the present analysis the expressions for the dispersion relation, resonant energies and growth rate are derived in the presence of an upgoing ion beam, downcoming electron beam and the steepness of the loss-cone index. The following parameters relevant to the auroral acceleration region (Mozer et
al, 1997; Mozer and Hull, 2001; Dwivedi et al, 2002) are used to evaluate the
dispersion relation, resonant energies and growth rate:

$B_0=4300\text{nT at } 1.4R_E, \Omega_i=412\text{s}^{-1}, \ell=1, \lambda=300\text{m}, E_1=50\text{mV/m}, \omega_{pi}^2/\Omega_i^2=2,$
\[b_i=0.1, k_{||}=0.00002\text{m}^{-1}, T_{1i}/T_{||i}=50 \text{ and } \Omega_i t=10.\]

The effect of ion beam velocity ($V_{Di}$) and distribution index ($J$) on the growth rate and the resonant energies transferred is depicted in Figures-4.1, 4.2 and 4.3. Figure-4.1 depicts the variation of growth rate ($\gamma/\omega$) with the wave frequency ($\omega/\Omega_i$) for different values of ion beam velocity ($V_{Di}$) and distribution index ‘$J$’ at constant electron beam velocity for the first harmonic of the ion cyclotron wave. It is assumed that the ion beam is directed from the ionosphere towards the magnetotail and therefore, the ion beam velocity is negative. It is observed that the effect of increasing ion beam velocity is to reduce the growth rate that may be due to the shifting of resonance condition. The effect of higher distribution index is to enhance the growth rate. Thus, the mirror like structure of the magnetosphere with a steep distribution index may be unstable for the EIC wave emission. It is also observed that the growth rate decreases with the increasing values of $\omega/\Omega_i$ which may be due to the shifting of the resonance condition. Hence the wave energy is being transferred to the particles.

Figure-4.2 shows the variation of transverse resonant energy ($W_{r1}$), in joules, with the wave frequency ($\omega/\Omega_i$) of the wave for different values of $V_{Di}$ and ‘$J$’ at constant $V_{De}$ for the first harmonic of the ion cyclotron wave. It is observed that the effect of increasing $V_{Di}$ is to increase the
transverse resonant energy. Thus the perpendicular acceleration of charged particles is possible through the ion cyclotron wave at the cost of ion beam energy. The effect of increasing distribution index is to decrease the transverse resonant energy. Thus, the steep loss-cone distribution of the magnetosphere stabilizes the transverse resonant energy. It is also observed that \( W_{r\perp} \) increases with the increasing values of \( \omega/\Omega_i \). The increase in heating of the particles by the ion beam is supported by the decrease in growth rate, as the wave energy is being transferred to the particles by the resonance interaction processes.

Figure-4.3 depicts the variation of parallel resonant energy \( (W_{r\parallel}) \), in joules, with wave frequency \( (\omega/\Omega_i) \) for different values of \( V_{Di} \) and for the increasing values of the distribution index at constant \( V_{De} \) for the first harmonic of the ion cyclotron wave. Here it is observed that \( W_{r\parallel} \) decreases when \( \omega<\Omega_i \), becomes minimum when \( \Omega_i=\omega \), and for \( \omega>\Omega_i \), \( W_{r\parallel} \) increases i.e. for \( \omega<\Omega_i \) the parallel energy is being transferred for perpendicular energisation and the wave energy is being transferred to the parallel resonating particles only for \( \omega>\Omega_i \). For \( \omega>\Omega_i \) effect of \( V_{Di} \) is to increase the parallel resonant energy.

Thus the heating of resonant ions parallel to magnetic field may be enhanced by the ion beam. The effect of increasing values of the distribution index is to decrease the parallel resonant energy. Thus, the steep loss-cone distribution of the magnetosphere stabilizes the parallel resonant energy of the EIC wave.
Figure 4.1

Variation of $\gamma/\omega$ with $\omega/\Omega_i$ for different values of $V_{Di}$ and $J$.
Figure 4.2

Variation of $W_{r\perp}$ with $\omega/\Omega_i$ for different values of $V_{Di}$ and $J$
Figure 4.3

Variation of $W_{r_{||}}$ with $\omega/\Omega_i$ for different values of $V_{Di}$ and $J$
The effect of electron beam velocity \( V_{De} \) and the distribution index at constant \( V_{Di} \) on the growth rate and the energy transferred for the first harmonic of the wave is depicted in Figures-4.4, 4.5 and 4.6.

Figure-4.4 shows the variation of growth rate with wave frequency for different values of \( V_{De} \) and \( J \). It is observed that the growth rate increases with the electron beam velocity that may be due to the shifting of resonance condition. Since the EIC waves have a finite wave number component along the magnetic field the electron beam streaming along the magnetic field may destabilize them. However, very high electron beam velocity leads to saturation of instability and the growth rate is slightly affected by the increase of the beam velocity. This may be due to the fact that the beam velocity slightly above the phase velocity of the wave is most effective in its interaction with the wave in the resonance condition. The effect of \( J \) is also to increase the growth rate as is discussed earlier.

Figure-4.5 shows the variation of \( W_{r\perp} \) (in joules) with wave frequency for different electron beam velocity \( (V_{De}) \) and \( J \) at constant \( V_{Di} \). The effect of \( V_{De} \) is to reduce \( W_{r\perp} \) that may be due to the ion cyclotron interaction. \( W_{r\perp} \) decreases with increasing \( J \) as discussed earlier.

From Figure-4.6 it is observed that \( V_{De} \) shows reducing effect on \( W_{r\parallel} \) (in joules) which decreases with \( J \) also as discussed earlier.
Figure 4.4

Variation of $\gamma/\omega$ with $\omega/\Omega_i$ for different values of $V_{De}$ and $J$
Figure-4.5

Variation of $W_{r\perp}$ with $\omega/\Omega_i$ for different values of $V_{De}$ and $J$
Figure 4.6

Variation of $W_{r||}$ with $\omega/\Omega_i$ for different values of $V_{De}$ and $J$
The reduction in transverse and parallel energy by the electron beam velocity \(V_{De}\) is supported by the increase in the growth rate, as the particle energy is being transferred to the wave by the resonance interaction processes. Thus, the wave may be generated by extracting energy from the resonant particles in the presence of the electron beam.

Thus, the electron beam acts as a source of free energy for the EIC waves. It modifies the wave-particle resonance condition and leads to weakening of Landau damping effects and enhances growth rate. The wave extracts the electron beam energy through its electric field directed parallel to the magnetic field. However, the effect of the ion beam is to reduce the growth rate of the wave but to increase the transverse acceleration of the ions. The results are consistent with the findings of Singh et al (1985), Sugawa and Utsunomiya (1989) and Hwang and Okuda (1989) and recent rocket and satellite observations (Mozer and Hull, 2001).

The effect of distribution index is also to increase the growth rate of the wave but to decrease the transverse acceleration of the ions. The destabilizing effects due to the steep loss-cone on different instabilities have also been reported by various workers (Varma and Tiwari, 1992; Gaelzer et al, 1997; Baronia and Tiwari, 2000; Dwivedi et al, 2001a). The steep loss-cone structures are analogous to mirror like devices with higher mirror ratio that may accelerate the charged particles moving perpendicular to the magnetic field (Gaelzer et al, 1997). Thus more energetic particles may be available to provide energy to the wave by wave-particle interaction.
Recently a FAST satellite (Carlson et al, 1998) has observed intense EIC wave turbulences up to 1000 Hz in association with ion and electron beam in the upward current auroral region (Gavrishchaka et al, 2000). Coherent ion cyclotron waves with amplitude up to 50 mV/m in association with upgoing ~1 keV ion beam and downgoing ~ 800 eV electron beam have also been observed recently by a Polar satellite (Acuna et al, 1995) in the auroral zone (Mozer et al, 1997). Local ion cyclotron waves with frequency ~ 100 Hz in association with downgoing electrons (≤100 eV) and upflowing ion beams have also been observed by the same satellite in an upward field-aligned current region. The same Polar satellite has also observed local hydrogen ion cyclotron wave of amplitude 500 mV/m in association with upward 1 keV accelerated ion beam and downward 5 keV accelerated electron beam at lower altitudes. EIC waves of frequency ~ 105 Hz in association with upgoing ion beam of 2 keV energy and downcoming electron beam of 1 keV energy have also been observed in the magnetosphere by the satellite.

The results obtained may be useful to study the electrodynamics of the auroral acceleration region. The EIC turbulence has been considered as a possible source of anomalous resistivity. When the instability occurs, the field energy grows exponentially therefore, the loss of kinetic energy of electrons is also exponential and the current carried by these electrons is suddenly disrupted (Palmadesso et al, 1974). Such instabilities can produce anomalous resistivity. Recently transversely accelerated ions and their association with the ion cyclotron waves at altitudes up to a few thousand kilometers has been
reported by the analysis of the Freja satellite data (Wahlund et al, 1998). The anomalous resistivity produced by the EIC wave leads to an anomalous version of the Joule heating effect in the topside ionosphere or lower magnetosphere and transfer of energy occurs from the EIC wave to ion thermal motion. Owing to this heating in this cyclotron motion the total energy of the ions rapidly increases and this becomes subject to the gradient \( B \) or mirror force by means of which they are ejected for the perpendicular energisation. The process of perpendicular ion energisation gives rise to ion distribution, which are known as transverse acceleration of ions (TAI). The ion beam enhances the heating rate of TAI whereas the electron beam and the steep loss cone controls the heating rate of TAI through the EICI in the presence of ion and electron beams in the auroral acceleration region and transfers this energy to the wave via inverse Landau damping.

Recently, a Polar satellite has observed EIC waves in the auroral acceleration region containing ion conics (Mozer et al, 1997). EIC wave heating is one of the major candidates for ion conic formation. The ion conic distributions have been interpreted as resulting from perpendicular heating of ions at low altitudes followed by parallel upward adiabatic motion due to the magnetic mirror force (Satyanarayan et al, 1989). The turbulent resistivity produced by EIC instability allows parallel electric field to develop along the parallel field lines. This parallel electric field may lead to upflowing ion beam and downcoming electron beam which may generate the EIC wave.
Recent observations by the Viking (Lundin et al, 1994) and the Freja satellite (Wahlund et al, 1998) indicate that EIC waves and the ion and electron beams in the plasma sheet boundary layer give rise to broad band extremely low frequency (BB-ELF) instabilities (Singh and Lakhina, 2001). At least at altitudes from 1000 km up to several 1000 km, most of the ion energisation is associated with BB-ELF waves.

The EIC turbulence plays an important role in the loss-cone current-potential relationship. It leads to spatial variations in the double layer potential and thereby produces thin auroral arcs embedded in inverted-V precipitation (Mozer et al, 1977; Mozer et al, 1980). It has also been suggested that the loss-cone effect can enhance the anomalous resistivity for a given turbulence level. Since the steep loss-cone distribution in the presence of EIC wave and the electron beam enhances the growth rate, the anomalous resistivity and transport resulting from this instability is likely to play crucial role in the auroral acceleration region. The equilibrium dipolar magnetic field of the earth is curved in the meridional plane and introduces loss-cone effects in the particle distribution function (Gaelzer et al, 1997). Thus the behavior studied for the EIC wave may be of importance in the electrostatic emission in the auroral acceleration region.

This theory may be useful to study the electrodynamics of auroral ionospheric region. The presence of field-aligned currents in the auroral ionosphere can permit the EIC waves to grow at lower altitudes. The converging magnetic field lines in the higher latitude ionosphere may be
considered suitable for the use of generalized distribution function. Upflowing ion beam and downcoming electron beam may excite EIC waves. This study may be useful for the experimental devices with current carrying plasma.

4.10 Summary

The effect of ion and electron beam and general distribution function on electrostatic ion cyclotron instability is studied. The expressions for the dispersion relation, resonant energy transferred, growth rate and marginal instability criteria for the electrostatic ion cyclotron wave with general loss-cone distribution function are derived in the presence of upgoing ion beam and downcoming electron beam in the auroral acceleration region. It is found that the effect of upgoing ion beam is to stabilize the wave and enhance the transverse acceleration of ions whereas the downcoming electron beam acts as a source of free energy for the electrostatic ion-cyclotron wave and enhances the growth rate. Effect of steepness of loss-cone is also to enhance the growth rate and decrease the transverse acceleration of ions. The applicability of the findings is discussed for the auroral acceleration region.