Preface

Statistical theory of reliability deals with the study of life times of manufactured products and services helpful in formulating warranties, maintenance policies and whatnot while survival analysis deals with the study of analogous aspects for human subjects. Though many concepts are common to both these disciplines, the two have their own unique problems which have to be discussed separately. This thesis contains a small number of original results which are a minuscule contribution to both these disciplines. The results in the thesis throw up a number of interesting questions which need to be explored further. The notion of ageing plays a prominent role in the theory of reliability and maintenance and survival analysis. While ‘no ageing’ means that ageing has no effect on the residual life of a component or system, ‘positive ageing’ refers to the adverse effects ageing has on the residual life. The dual concept of ‘negative ageing’ refers to the favourable effects on residual life. In the statistical theory of reliability, several ageing classes of distributions have been defined and their properties studied. These classes give insights into modelling reliability of a component or a system and the models help in the estimation of reliability. Typically, the ageing classes are defined in terms of the failure or hazard rate (FR or HR), conditional survival (reliability) function (sf), the mean residual lifetime (MRL) and a few other transformations of the unknown life time random variable (rv) $X$ and its distribution function (df) $F$. We assume throughout that the life time rv $X$ is non-negative valued. The notion of FR is crucial in reliability analysis, survival analysis and many other areas of applied probability. FR, also termed as HR, is the reciprocal of Mills’ ratio in the case of normal distribution. Barlow and Proschan (1996), is called the force of mortality or the mortality
rate by actuaries and demographers Finkelstein Maxim (2008) and is also called as intensity
function Gumbel, E. J. (1958). The FR is also used by social, medical and the biological
scientists dealing with lifetime data, see Lawless J.F. (2003) for examples of such lifetime
data.

Since the MRL function is intimately related to the FR, various investigations have been
done on the monotonicity of the MRL function in relation to the monotonicity of the FR.
It is well known that in the case of increasing failure rate (IFR) (or the decreasing failure
rate, DFR), the MRL function is decreasing (increasing). A fascinating aspect about MRL
function is its tremendous range of applications: Bjerkedal (1960) studies the life length
of guinea pigs injected with different amounts of tubercle bacilli. Guinea pigs are known
to have a high susceptibility to human tuberculosis which is one reason for choosing this
presents further references on the MRL function and burn-in as well as a brief history on
research in burn-in. Actuaries apply the MRL function to setting rates and benefits for life
insurance. In biomedical setting, researchers analyse survivorship studies using the MRL
function as in Ellant-Johnson and Johnson (1980), and Gross and Clark (1975). Social
scientists use the MRL function for studies on job mobility, length of wars, duration of
strikes, etc. Morrison (1978) mentions that the increasing MR or IMR distributions have
been found useful as models in the social sciences for the life lengths of war and strikes.
Bhattacharjee and Krishnaji (1981) present applications of the MRL function in investigat-
ing landholding. Bhattacharjee (1982) observes that the MRL functions occur naturally
in areas such as optimal disposal of an asset, renewal theory, dynamic programming and
branching processes. Bhattacharjee (1984) uses the new better than used in expectation
(NBUE) distributions for developing optimal inventory policies for perishable items with
random shelf life and variable supply.

After a brief introduction on some reliability concepts and ageing, as a first chapter, several
concepts used in reliability and survival analysis are gathered in this chapter. The concepts
discussed are associated with positive and negative ageing classes and their importance in
reliability. Definitions and properties of ageing classes are discussed, and in particular, the
following chain of implications is looked into.

\[
\text{IFR} \Rightarrow \text{IFRA} \Rightarrow \text{NBU} \Rightarrow \text{NBUFRA} \Rightarrow \text{NBUFRA} \\
\downarrow \quad \downarrow \\
\text{IFR2} \quad \text{NBUC} \\
\downarrow \quad \downarrow \\
\text{DMRL} \Rightarrow \text{NBUE} \Rightarrow \text{HNBUE} \Rightarrow \mathcal{L}.
\]

We also discuss reliability properties of equilibrium distributions generated by lifetime rvs. Several authors have discussed reliability concepts which are not covered in the first chapter, and some of these are Barlow et al. (1963), Bracquemond et al. (2001), Roy et al. (1992), Shaked et al. (1995), Xie (2002). Though not a complete survey, the first chapter helps in identifying new problems in the area of the thesis.

The second chapter looks at some reliability properties of extreme value, generalized Pareto and transformed distributions. System lifetimes of series and parallel systems are, respectively, the minimum and maximum of independent component lifetimes. Barlow and Proschan (1975) discuss the limiting distributions of such system lifetimes assuming that the number of components become large. The limiting distributions turn out to be the well known extreme value distributions. Generalized Pareto distributions are limit laws of linearly normalized conditional excesses over a high threshold as the threshold tends to infinity. These have been used to model exceedances over high thresholds and were first introduced and studied by Balkema and de Haan (1974). It is well known that the extreme value laws are related to the generalized Pareto laws. In this chapter, we discuss some reliability properties associated with these dfs and some transformed ones. Among other things, the behaviour of the hazard rates and reverse hazard rates of the dfs are discussed. This chapter forms the basis of the article Ravi and Mirza A. A. (2012a): A note on some reliability properties of extreme value, generalized Pareto and transformed distributions, which has been accepted for publication in the science journal of
The third chapter looks at FR and reversed hazard rate (RHR) of lifetime random variable $Y_N$ for series and parallel systems when the system size is random having distribution shifted geometric or shifted Poisson or shifted negative binomial. Many authors have worked on the stochastic comparisons of random minima and maxima which happen to be system lifetimes of series and parallel systems when the system size is random. For example, Shaked and Wong (1997) obtain some stochastic comparison results, Bartoszewicz (2001) gives some relations between the random extremes and classes of life distributions with monotone FR, Li and Zuo (2004) have shown that the right spread order and the increasing convex order are both preserved under random maxima, and that the total time on test transform order and the increasing concave order are preserved under random minima, Li and Yam (2005) prove preservation properties of some negative ageing concepts under a similar set-up, Ahmad and Kayid (2007) obtain preservation properties of the right spread order, total time on test order and increasing convex (concave) order under random minima and maxima. In this chapter, we discuss the behaviour of FR (HR) and RHR of series and parallel systems when the system size is random having distribution shifted geometric or shifted Poisson or shifted negative binomial. Observing that the ageing classes IFR and DFR are not closed under the formation of coherent systems generally, we show in this chapter that for systems with random number of components, some of the closure properties are satisfied given that the components satisfy appropriate ageing properties. Some illustrative examples are also given.

This chapter forms the basis of the article Ravi and Mirza A. A. (2012b): On reliability properties of series and parallel systems, which has been accepted for publication in the journal Open Journal of Statistics (OJS), after due refereeing.

The fourth chapter looks at an inequality in applied probability and reliability. In this chapter, under some conditions, we derive inequalities for the pdf of a non-negative valued rv in terms of its df. These inequalities can also be interpreted as inequalities for the FR
(HR) and RHR of the rv in terms of its sf and df respectively. Analogous inequalities in the bivariate case are given along with some illustrative graphs. The inequalities derived are applicable in several areas of applied probability like, for example, in queueing theory, and whatnot.

The fifth chapter looks at coefficient of variation in the univariate and the multivariate case. Some properties related to compound distributions of a class of ageing distributions defined using coefficient of variation are derived and a possibly new definition of bivariate coefficient of variation is given. Some consequences of the new definition are studied and coefficient of variation using the new definition is computed for a few bivariate distributions. The definition is extended to multivariate random vectors and the coefficient of variation of multinomial random vector is computed. It is proposed to look into the new definition in greater detail in future.

Notations and abbreviations have been listed after this introduction. Notations are also explained in the text, when they appear for the first time, with some possible exceptions. The end of the proof is indicated by □. All rvs considered in the thesis are lifetime rvs by which we mean that they are non-negative valued rvs.