Preface

Stochastic orders have been a topic of study in many areas of probability and statistics. Using properties associated with distribution functions (dfs), density functions, hazard rates and whatnot, many useful stochastic orders have been defined and these give insights into the relationships among classes of random variables (rvs) with specified properties. These facilitate comparison of rvs, their probability distribution / density functions and other functions associated with them like, for example, hazard rates, cumulative hazard functions, etcetera. The concept of stochastic ordering extends the idea of comparing two numbers / functions to rvs and functions of rvs. Though many of these comparisons are mathematical artefacts, the urge to study these comparisons have led to many useful results and some of these have been used to propose statistical tests of hypotheses. Comparing means of two dfs may not be very informative quite often even when the means exist. Comparing moments of order greater than 1 whenever these exist may also not yield desired results in certain cases. It is possible that one has more information about two populations than those about the two means or moments. Several stochastic orders of rvs and associated dfs and their functionals have been proposed and studied in the statistical literature. Stochastic orders facilitate comparison of two or more rvs. They are also useful in devising statistical tests of hypotheses.
After a brief introduction on some reliability concepts and stochastic orders, the first chapter following this Preface consists of definitions of some stochastic orders, a few reliability concepts for general lifetime rvs, some definitions of reliability concepts for discrete lifetime rvs with some examples and a few relationships between these orderings and reliability concepts.

Several authors have defined reliability concepts for discrete lifetime rvs. With a view to compare several definitions of the same reliability concept for discrete lifetimes, the second chapter looks at various definitions of reliability concepts given in Xie (2002), Cyril and Olivier (2003) and Kemp (2004). Discrete lifetimes following discrete uniform, shifted geometric and a few other discrete distributions are considered and the lifetimes and associated ageing properties are classified according to the definitions given in these articles. Some new results are also proved comparing stochastic order relations between a discrete lifetime rv and its associated residual life at random time.

This chapter forms the basis for the article Ravi and Prathibha (2012b), which has been accepted for publication in myScience, the science journal of the University of Mysore.

Suppose that $X$ and $Y$ denote two independent lifetime rvs with respective dfs $F$ and $G$, survival functions (sfs) $\bar{F}$ and $\bar{G}$. For $t \geq 0$, let $X_t$ denote the excess or residual or remaining life rv at time $t$ with df $F_t(x) = P(X_t \leq x) = 0, x \leq 0$, and

$$F_t(x) = \frac{F(t + x) - F(t)}{F(t)} = 1 - \frac{\bar{F}(t + x)}{\bar{F}(t)}, x > 0, t > 0;$$

and sf $\bar{F}_t(x) = 1, x \leq 0$; and $\bar{F}_t(x) = \frac{\bar{F}(t + x)}{\bar{F}(t)}, x > 0, t > 0$. Some relationships between stochastic orderings and residual life at fixed time and at random time are explored.
in the third chapter. A few results on the associated equilibrium distributions and ageing classes are also established. More specifically, if the rv \( X \) has finite mean \( \mu_F \), and \( X_1 \) denotes the equilibrium rv with df \( F_1(x) = \frac{1}{\mu_F} \int_0^x F(t)dt, x \geq 0 \) and \( = 0 \) otherwise, then apart from other results, it is shown that if \( F \) is NBUE (New Better than Used in Expectation), then (i) \( X_1 \leq_c X \), (ii) \( X_1 \leq_{utt} X \), (iii) \( X_1 \leq_L X \) where the inequalities denote stochastic orders.

This chapter forms the basis for the article Ravi and Prathibha (2012a), which has been accepted for publication after due refereeing in the e-journal Probstat Forum.

The fourth chapter looks at stochastic orderings between several rvs derived using the concepts of equilibrium rvs and residual life at random time. Given two life time rvs \( X \) and \( Y \), the probability that the residual lifetime \( X_t \) is greater than the residual lifetime \( Y_t \) is obtained in Zardash and Asadi (2010). This chapter also generalizes this result to residual lifetimes at random times and finds the the probability that at random time \( Z \), the residual lifetime \( X_Z \) is greater than the residual lifetime \( Y_Z \). In the last section of this chapter, we prove converses of a couple of stochastic comparison properties proved in Yue and Cao (2000).

The fifth chapter looks at a couple of properties of discrete NBUL lifetimes, closure of the NBUL class under the formation of parallel systems and closure of NBUL class under a shock model leading to a replacement policy comparison. The results fill some gaps in the literature and appear to be new.

Several attempts have been made to define reliability concepts in the bivariate case. In the sixth chapter, we look at classifying bivariate exponential distributions
according to available definitions. We also look at some properties of bivariate lifetimes analogous to those proved in earlier chapters. The results are not exhaustive and there is room for continuing this study more comprehensively.

Klefsjö (1983) defined the \( \mathcal{L} \) class of ageing distributions in reliability as

\[
\mathcal{L} = \left\{ F : F \text{ is a df } \frac{F(0-)}{1+s\mu_F} \leq 1; \; s > 0, \right\}
\]

where \( \mu_F \) is the mean of \( F \). Bhattacharjee and Basu (2002) defined the subclass \( \mathcal{L}_D \) of class \( \mathcal{L} \) of dfs in\( \mathcal{L} \) wherein coefficient of variation equal to 1 characterizes the exponential distribution as \( \mathcal{L}_D = \{ F \in \mathcal{L} : F_1 \in \mathcal{L} \} \), where \( F_1 \) is the equilibrium df derived from \( F \). In the seventh and last chapter, we look at generalizing these classes. A couple of interesting properties of \( \mathcal{L} \) and \( \mathcal{L}_D \) are noted by defining the new classes of ageing distributions.

**After the Bibliography, a copy each of the articles Ravi and Prathibha (2012a) and Ravi and Prathibha (2012b) are enclosed.**

Notations and abbreviations have been listed after this introduction. Notations are also explained in the text, when they appear for the first time, with some possible exceptions. The end of the proof is indicated by \( \square \). All rvs considered in the thesis are lifetime rvs by which we mean that they are non-negative valued rvs.