PREFACE

The thesis entitled “Group theoretic similarity solutions of Diffusion Equations” is an outcome of author’s investigations on theoretical aspects of the different kind of problems of diffusion. It demonstrates the use of Group theoretic similarity solutions of various types of problems associated with diffusion arising in engineering and sciences. It contains five chapters, out of which the first two are devoted to the general introduction of the subject, its Mathematical development and Physico-Mathematical background. The author’s investigations on the diffusion problems are included in subsequent three chapters.

In most of physical phenomena mathematical modeling leads the formation of either ordinary differential equations or partial differential equations. Sometimes leads to a set of simultaneous systems of equations. Ideally one hopes to find the exact solutions of these equations, but it is well known fact that there is hardly a full proof method for the treatment of all such equations. Researchers are using various methods like analytical method, numerical method, etc to find closed form solutions of these equations. The concept of Mathematical similarity analysis has a great role in the basic simplification of the analysis of such methods. We have used the Lie’s theory of infinitesimal group of transformations for solving the problems of diffusion.
Chapter 1 “The Mathematics of Diffusion” represents a sound structure in logical manner to provide wide knowledge of diffusion. Diffusion is useful to engineers, chemists, life scientists and to our daily lives. This chapter deals also with the definition and a brief discussion of fundamental laws introduced by: (1) Fourier, (2) Ohm, (3) Thomas Graham, (4) Adolf Fick and (5) Darcy. In addition, it contains some useful information about the rheological development of these laws. The wonders of diffusion are in general represented by the partial differential equations.

Chapter 2 “Mathematical Background” contains the discussion about the group theoretic similarity techniques in general. Today there are number of similarity techniques found in literature out of which (1) Free Parameter, (2) Separation of Variables, (3) Dimensional Analysis, (4) Group theoretic are frequently used. Among these methods, Group theoretic methods based on finite transformation as well as infinitesimal transformation are more attractive than the other methods, because these methods derived systematically from simple group transformation laws. The various applications of different similarity techniques are given by Hansen and his co-workers. Today, in the modern age, the use of similarity analysis is not restricted to fluid–mechanics or heat transfer, but it is much useful in studying physical, biological, medical and social sciences. In this chapter we discuss the
finite transformations and infinitesimal transformations for the partial
differential equations governing the diffusion. Lie has introduced the
study of continuous group of transformations of ordinary differential
equations based on the infinitesimal properties of the group.

Chapter 3 “Similarity solution of concentration
dependent diffusion equation” deals with the problems of one-
dimensional diffusion equation. The diffusion equation describes the
solute transport due to combined effect of diffusion and convection in a
medium. It is a partial differential equation of parabolic type, derived on
the principle of conservation of mass using Fick’s law. Due to the
growing surface and subsurface hydro environment degradation and the
air pollution, the diffusion equation has drawn significant attention of
hydrologists, civil engineers and mathematicians. Its analytical/numerical
solutions along with an initial condition and boundary conditions help to
understand the contaminant concentration distribution behaviour through
an open medium like air, rivers, lakes and porous medium like aquifer, on
the basis of which remedial processes to reduce the damages may be
enforced. It has wide applications in other disciplines too, like soil
physics, petroleum engineering, chemical engineering and biosciences.
This chapter presents similarity solution of one dimensional concentration
dependent diffusion equation with constant diffusion coefficient. The
solution is obtained by using a technique of infinitesimal transformations
of groups. The solution obtained is physically consistent with results of earlier researchers and which is more classical than other results obtained by the earlier researchers.

**Chapter 4 “The similarity solution of second order nonlinear diffusion equation in an isotropic medium”** discusses about the derivation of one-dimensional partial differential equations, which are associated with diffusion process. It includes the nonlinear diffusion equation. The infinitesimal transformations in searching for the possible groups of transformations to these partial differential equations are developed. The procedure is based on Lie’s theory of infinitesimal continuous group of transformations. This group theory has been applied extensively in recent times by investigators in the field of similarity analysis. By this method, these partial differential equations are reduced to the ordinary differential equations. The obtained ordinary differential equations are solved by the analytical methods.

**Chapter 5 “The classical solution of the phenomenon of longitudinal dispersion in porous media”** concerns with the discussion of miscible displacement. Dispersion is the process by miscible fluids in laminar flow mix in the direction of the flow. The phenomenon is discussed by regarding the cross-sectional flow velocity as time dependent in specific form. The mathematical formulation of the phenomenon using Darcy’s law yields a Burger’s equation. This equation
is transformed into ordinary differential equation using a group of infinitesimal transformations of similarity method. A classical solution of the later is obtained in terms of Hermite functions. The solution obtained is physically consistent with the results of earlier researcher and which is more classical than other results obtained by other researchers.

In conclusion, we have obtained the similarity as well as classical solutions for the equations associated with the diffusion process using infinitesimal group transformations. Thus to sum-up the subject matter of the thesis adds to the existing literature on the investigated topic by making some original contributions giving fresh interpretation to known facts and extending the scope of Mathematical study. Thus, it is my humble attempt to add the valuable contribution of original work in this field.

Lastly, it may be mentioned that the author has been greatly influenced and encouraged by the excellent work of Crank J., Hansen A.G., Blueman G.N., Cole J.D. and Murphy G.N.

Research Scholar

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