# Table of Contents

Abstract iii

Acknowledgments vi

Table of Contents vii

1 Introduction 1
   1.1 Introduction ................................................. 1
   1.2 Ramanujan’s theta functions and some preliminary results .... 6
   1.3 Work carried out in this thesis ............................. 9

2 Some Results on 3-Cores 18
   2.1 Introduction ................................................. 18
   2.2 Main theorems ................................................ 19

3 Infinite Families of Arithmetic Identities for 4-Cores 27
   3.1 Introduction and preliminary results ........................ 27
   3.2 Identities connecting $u(n)$, $v(n)$, $r_3(n)$ and $a_4(n)$ ... 28
   3.3 Infinite families of arithmetic properties of $a_4(n)$ ........ 34
   3.4 Infinite families of results involving $r_3(n)$ and $t_3(n)$ .. 39
   3.5 Some other new arithmetic identities for $a_4(n)$ ............ 43

4 Two Infinite Families of Arithmetic Identities for 5-Cores 45
   4.1 Introduction .................................................. 45
   4.2 Preliminary results .......................................... 46
4.3 Main results on $a_5(n)$ .................................................. 48

5 Infinite Families of Arithmetic Identities for Self-Conjugate 5-Cores and 7-Cores ................................................. 58

5.1 Introduction ................................................................. 58
5.2 Relations between $\text{asc}_5(n)$ and $r_2(n)$ ......................... 59
5.3 Relations between $\text{asc}_7(n)$ and $r_3(n)$ ......................... 60
5.4 Infinite families of arithmetic properties of $\text{asc}_5(n)$ ............ 61
5.5 Infinite families of arithmetic properties of $\text{asc}_7(n)$ ............ 66

6 Infinite Families of Arithmetic Identities for Doubled Distinct $t$-Cores for $t = 3, 4, \cdots, 10$ ........................................... 74

6.1 Introduction ................................................................. 74
6.2 Relations between $\text{add}_5(n)$ and $t_2(n)$, and between $\text{add}_5(n)$ and $\text{asc}_5(n)$ ............................................ 77
6.3 Relations among $\text{add}_6(n)$, $\text{asc}_4(n)$ and $r_2(n)$ .................. 78
6.4 Relations between $\text{add}_7(n)$ and $t_3(n)$, and between $\text{add}_7(n)$ and $\text{asc}_7(n)$ ............................................ 80
6.5 Relations between $\text{add}_8(n)$ and $r_3(n)$ ......................... 82
6.6 Relations between $\text{asc}_9(n)$ and $\text{add}_9(n)$ ....................... 83
6.7 Infinite families of results on $\text{add}_5(n)$ and $\text{add}_4(n)$ .......... 84
6.8 Infinite families of arithmetic identities for $\text{add}_5(n)$ ................ 86
6.9 Infinite families of results on $\text{asc}_4(n)$ and $\text{add}_6(n)$ .......... 88
6.10 Infinite families of results on $\text{add}_7(n)$ ............................ 89
6.11 Infinite families of arithmetic identities of $\text{add}_8(n)$ ............. 94
6.12 An arithmetic identity for $\text{add}_{10}(n)$ ............................ 96

Bibliography ................................................................. 98