Abstract

There is a great deal of interest in Haar wavelet analysis from the point of view of applicability of Haar wavelets for solving differential, integral and integro-differential equations. The thesis develops new schemes to solve differential, integral and integro-differential equations with powerful Haar wavelet characteristics and with combination of existing numerical techniques such as Collocation method, Quasilinearization process, Runge-Kutta method and Gauss elimination method. For nonlinear problems, schemes are developed based on quasilinearization process and Runge-Kutta method, using a Haar wavelet function as described in chapter 1 to 6 and chapter 7 respectively. In wavelet analysis, the approximation of the solution of the differential and integral equations by Haar wavelets has an error. In order to minimize this error we choose collocation method at the collocation points with Haar wavelets, where the approximation is exact. On the other hand in Galerkin's method, the error is orthogonal to each wavelet selected (Politis, 1996) and connection coefficients are required to find derivatives and integrals, this process is complicated. There is no need of connection coefficients in our proposed Haar wavelet scheme with quasilinearization process. From the presented analysis some observations are made regarding the Haar wavelet schemes.

The greatness of the scheme lies in the fact that it has too applications in many fields to analyzing various phenomena. The work presented in this thesis dealt with the applications of Haar wavelet methods to the problems that arise in the mathematical modelling of many physical, chemical and biological phenomena. The problems are considered in Chapters 2 to 8 related to many fields such as Oscillation, Astrophysics, Viscous fluid, Generalized Long Wave equation, Benjamin-Bona-Mahony-Burgers equation, Coupled Burgers’ equation and Volterra population growth model which represents a group of physical and biological species.

The presented work in this thesis is organized into following nine chapters, subsequently where agenda for each of the chapter (with facts and details) is briefly illustrated.
In Chapter 1, we provided a brief introduction to the Haar wavelets, applications and different wavelet methods for solving differential, integral and integro-differential equations. The chapter deals with the previous studies reported in literature on the Haar wavelet properties by changing the approaches for different type of phenomena from the wavelet analysis and numerical analysis in differential and integral equations.

Chapter 2 is to present the Haar wavelet based solutions of linear and nonlinear boundary value problems by Haar collocation method and utilizing quasilinearization process. Haar wavelet based operational matrix makes the algorithm and procedure quite simple in solving differential equations. The algorithm has been developed to facilitate easy calculation of operational matrices. Quasilinearization process with composition of Haar wavelet method has been proposed and Gauss elimination method is also used to solve system of linear equations which gives sufficient accuracy without iterations. More accurate solutions are obtained by wavelet decomposition in the form of a multi-resolution analysis of the function which represents solution of boundary value problems.

In addition to this, the test problems of this chapter demonstrates that the Haar wavelet method coupled with quasilinearization approach can successfully compete with analytic one. The main benefits of the Haar approach is simplicity (as a small number of grid points according to the resolution guarantees the necessary accuracy without iterations and refined towards higher accuracy by increasing the level of the Haar wavelets) and universality (as almost the same approach is applicable for a wide class of higher order nonlinear differential equations). Moreover, computationally proposed method is easily applicable than wavelet based Galerkin procedure.


Chapter 3 describes a numerical method using uniform Haar wavelet approximation and quasilinearization process for solving some nonlinear oscillator equations. Some considered
nonlinear oscillator equations such as Duffing, Van der Pol and Duffing-van der Pol with different parameters. In our method, when we increase number of points in terms of $2^j$ then coefficient matrix becomes ill conditioned and direct solutions can be found only on odd points. To overcome the nonlinearities, quasilinearization is used which converts nonlinear differential equations into set of linear algebraic equations. To the best of our knowledge, the quasilinearization process has not been used for above nonlinear oscillators with Haar wavelets. The advantage of method is that one does not have to apply iterative procedure for achieving sufficient accuracy even in those problems which involve abrupt changing functions. The results of the comparison with various methods indicate that the proposed method is feasible and convergent. A part of this chapter is accepted for publication in *Applied Mathematical Modelling* (Elsevier), 2014.

**Chapter 4** represents a technique to investigate the solutions of generalized nonlinear singular Lane-Emden equations of first and second kinds by using Haar wavelet quasilinearization approach. The Lane-Emden equation is widely studied and is treated as a challenging equation in the theory of stellar structure for the gravitational potential of a self gravitating, spherically symmetric polytropic fluid which models the thermal behavior of a spherical cloud of gas acting under the mutual attraction of its molecules and subject to the classical laws of thermodynamics. The proposed method is based on the quasilinearization process and replacement of an unknown function through a truncated series of Haar wavelet series of the function. The method is shown to be very reliable and easy to capture the solutions of generalized nonlinear singular Lane-Emden equations. The applicability of the method is shown by numerical tests on various cases of the generalized Lane-Emden equation and solutions are also reported in the neighborhood of a singular point. A part of this Chapter has been published in *Computer Physics and Communications* (Elsevier) 184(2013), 2169-2177.

**Chapter 5** describes how the proposed Haar wavelet quasilinearization method is used to solve the well known Blasius equation. The method is based on the uniform Haar wavelet operational matrix defined over the interval $[0,1]$. In this method, we have proposed the transformation for converting the problem on a fixed computational domain. The Blasius
equation arises in the various boundary layer problems of hydrodynamics and in fluid mechanics of laminar viscous flows. We have solved Blasius equation for $1 \leq \alpha \leq 2$ and the numerical results are compared with the available results in literature. Finally, we conclude that proposed method is a promising tool for solving the well known nonlinear Blasius equation. A part of this chapter has been published in *Proceedings of Conference World Academy of Sciences, Engineering and Technology* 79(2013), 1397-1402.

In **Chapter 6**, a numerical scheme is developed to find the numerical solutions of general nonlinear partial differential equations. In order to test the efficiency of the scheme well known nonlinear generalized regularized long wave equation, Fitzhugh-Nagumo equation and Benjamin-Bona-Mahony-Burgers equation are solved for different values of parameter. The scheme is based on time discretization of uniform Haar wavelet series and collocation method. The scheme generates system of linear algebraic equations which is solved by MATLAB. The figures show beautiful concentration profiles of $u$ and $v$ at different time level. Some numerical experiments have been conducted in order to illustrating merits of proposed methods for different parameters. Results are compared with available results by finding $L_2$ and $L_\infty$ errors. It is shown that the proposed method is working well and produces the satisfactory results. A part of this chapter is under review in *Communications in Partial Differential Equations* (Taylor and Francis).

In **Chapter 7**, the numerical solutions of the nonlinear coupled Burgers’ equation with appropriate initial and boundary conditions in one space dimension are considered. The coupled Burgers’ equation is a mathematical model to describe various kinds of phenomena such as turbulence and viscous fluid. A numerical method is proposed based on semi discretization along the space direction for solving coupled viscous Burgers’ equation. The semi discretization scheme forms a system of nonlinear ordinary differential equations which is solved by fourth order Runge-Kutta method. Numerical experiments have been conducted in five examples to illustrate the merits of proposed method. The Relative errors $L_2$ and $L_\infty$ are computed of obtained numerical results. The stability of method is discussed. The method is found to be quite satisfactory and suggested that it can used to solve higher
order nonlinear coupled partial differential equations. A part of this chapter is under revision in *International Journal of Computer Mathematics* (Taylor and Francis).

In **Chapter 8**, we have solved the integral equation and nonlinear Volterra’s population model for population growth of a species in a closed system. The model is characterized by the nonlinear Volterra integro-differential equation on finite interval where the integral term represents the effect of toxin. The method used an efficient operational matrix based on Haar wavelets and has been established based on collocation approach. A part of this chapter has been published in *Mathematical Sciences International Research Journal* 2(2013), 677-681.

**Chapter 9** concludes the findings of the present research work and gives a discussion of their implications in the context of effects of method in differential, integral and integro-differential equations. Based on the present study, conclusions are drawn and future research work in this direction is suggested.

During the study nearly 160 reputed research publications, books, thesis and notes have consulted and cited in the references. For numerical computation, C++ and MATLAB programs have developed.