Appendix A
Overview of Radon Transform

This appendix discusses the Radon Transform, which is used for solving the problem of reconstruction irrespective of the solution technique.

When a CT scan is made, the X-rays are absorbed by the tissues and bones in the body. The absorption is described by a linear attenuation function, which at a fixed point has the value of the linear attenuation coefficient of the tissue. The linear attenuation function is approximately proportional to the density of the body, so it is this function which is to be displayed form the data obtained. Radon transform plays very vital role to display the images.

A.1 Mathematical Framework

When an X-ray beam is sent through a body, it is assumed that it follows a straight line, $L$ from a source $S$ to a detector $R$. The linear attenuation function is denoted by $f$, and is a function of a position $x = (x_1, x_2)$ in a Cartesian coordinate system. The intensity of beam at a point $x$ is expressed by $I(x)$, the attenuation of the intensity within a distance $dl$ along the line $L$, is written as:

$$dl(x) = -f(x) I(x) dl \ (A.1)$$

$$\frac{dl(x)}{I(x)} = -f(x) dl \ (A.2)$$

The solution of the above equation is given by Beer’s Law as shown below:

$$\ln(I(x)) = \int_{L} -f(x) dl \ (A.3)$$

The intensity of the beam denoted as $I_s$, at the source, is known as difference in intensity from where the beam has emerged to a point on the line $L_s$ is expressed as:
\[
\ln(I_s) - \ln(I(x)) = -\int_L -f(x) \, dl \quad (A.4)
\]
\[
\int_L -f(x) \, dl = \ln \left( \frac{I_s}{I} \right) \quad (A.5)
\]

This described process is repeated on parallel lines. During the number of lines, corresponding to the number of source and detectors will be finite. Hence, the inverse problem is to recover \( f(x) \) from these equations, which are popularly known as reconstruction problem. Obtaining the integrals means that a slice of the object is being investigated in a certain direction. The integrals represent the projection of \( f(x) \). The next step is to define the new direction and through that the other projections are obtained. Graphically it shown in the figure A-1:

\[\text{Figure A-1: Example of Projections}\]

When a CT scan is performed, and the sources and receivers move as described, there will be a point of rotation in the middle of gantry. The origin 0 is selected in \( x - \text{plane} \), considering a single slice. The Cartesian axes are chosen such that \( x_1 - \text{axis} \) is parallel to the initial direction of the source detector pair movement, as shown in the figure A-2. Figure illustrates new coordinate system which appears when the source detectors are moved at an angle \( \emptyset \in [0,2\pi] \) from the \( x_1 - \text{axis} \).
The direction is defined by the vector $\theta$ as:

$$\theta = \{\cos \phi, \sin \phi\} \quad (A.6)$$

$$\theta^\perp = \{-\sin \phi, \cos \phi\} \quad (A.7)$$

The $s - axis$ will be parallel to the line where the sources are placed and $t$ is the line parallel to the lines of the rays. The dotted line $x$ represents one of these lines. From equation A.6 and A.7, it can be inferred that for a given line $s$, the Cartesian coordinated system is written as:

$$x = s\theta + t\theta^\perp \; ; t \in R \quad (A.8)$$

Hence, the projection of $f$ in the direction $\theta$ is given by:

$$\left( R_{\phi}f \right)(s) = \int_{-\infty}^{\infty} f(s\theta + t\theta^\perp) dt \quad (A.9)$$

where $R_{\phi}$ denotes the Radon transform for the specific direction of $\theta$. Considering all the possible directions of $\theta$, the function is defined by two variables as:

$$\left( Rf \right)(s, \phi) = \left( R_{\phi}f \right)(s) \quad (A.10)$$
Hence, all inverse problems are consisting of recovering function of two variables from its line integrals. Expressing the equation A.10 in terms of \( s \) and \( \phi \) is given by:

\[
(Rf)(s,\phi) = \int_{-\infty}^{\infty} f(s \cos \phi - t \sin \phi, s \sin \phi + t \cos \phi) \, dt (A.11)
\]

Plotting the value of \( R_f \) as the function of \( s \) and \( \phi \) is known as Sinogram, when the values of \( f \) are represented as grey scale level. Practically, during the scan, the values of Radon transform are discrete and noisy.

It is assumed that density function \( f \) is assumed to have the support within a disc of radius \( a \). Thus, the sinogram will have support of a rectangle \(-a \leq s \leq a\) and \(-\pi < \phi < \pi\). For every point in the sinogram, there is corresponding straight line in the \( x-y \) plane.

For all the lines passing through a fixed point \( x_0 = (x_1^0, x_2^0) \), the corresponding points in the sinogram are described as sinusoidal curve in the \((s, \phi)\) plane. This curve is described by:

\[
s = x \cdot \theta = x_1^0 \cos \phi + x_2^0 \sin \phi \tag{A.12}
\]

**A.2 Implementation**

The realization of this problem was done on the MATLAB® platform for two images:

a) Image with only one non zero pixel

b) Standard Sheep Logan Phantom

The MATLAB® Code developed for:

a) **Image with only one non zero pixel is given below:**

```matlab
% Making a 256x256 matrix, where only one pixel is non-zero:
X = zeros(256);
X(100,100) = 255;
figure(1)
```
subplot(2,2,1)
imagesc(X), colormap(gray), title('Single Object'), axis square
xlabel('x_1'), ylabel('x_2')
theta = 0:359;
Y = radon(X,theta);
subplot(2,2,2)
imagesc(Y), colormap(gray), xlabel('phi'), ylabel('s'), axis square
title('Sinogram of the single object');

b) Sinogram for Sheep Logan Phantom

% Using the Phantom-image to see more sinograms in one image
X = phantom(256);
subplot(2,2,3)
imagesc(X), colormap(gray), title('Phantom'), axis square
xlabel('x_1'), ylabel('x_2')
Y = radon(X,theta);
subplot(2,2,4)
imagesc(Y), colormap(gray), xlabel('phi','LineWidth',5), ylabel('s'), title('Sinogram of Phantom');

The results obtained are presented in four figures as under:
Figure A-3: Image with single pixel

Figure A-4: Sinogram of the Image with single pixel

Figure A-5: Image of Sheep Logan Phantom.

Figure A-6: Sinogram of Sheep Logan Phantom.
Appendix B
List of Publications and Presentations

This appendix provides the details pertaining to the publications, presentations and awards received based on this research work.

Publication Based on Research Work for Ph.D.


3. N. D. Shah, S. K. Shah “Radon Transform Based Improved Algorithm For Computed Tomography To Reconstruct Volume From Real 2D Data Gathered Over Arbitrary Path using MATLAB® With Graphical User Interface”, a paper submitted and is accepted for publication Journal of Institution of Engineers, India.


**International Presentation**


**National Presentation**


**Technical Workshop**


**National Conferences & Technical Events**


Awards Received

1. Awarded Second Prize for Paper in Research Category for,” A Novel Approach for Computed Tomography Reconstruction Algorithm Using Unsupervised Neural Network” at Target Technologies in Computing, Automation and Communication. TTCAC_2014, Organised by Institute of Electronics and Telecommunication Engineers and Institute of Engineering