CHAPTER 7
MULTI-ITEM INVENTORY MODEL WITH DEMAND DEPENDENT UNIT COST, STORAGE SPACE AND LEAD TIME CONSTRAINTS

7.1 INTRODUCTION

In most of the existing literature, inventory related costs are assumed to be deterministic and are represented as real numbers. But, in real situation, the inventory costs are usually imprecise in nature due to the influence of various uncontrollable factors. For example, costs may depend on some foreign monetary unit. In such a case, due to exchange rates, the costs are often not known precisely. Inventory carrying cost may also depend on some parameters like interest rate and stock keeping unit’s market price, which are not precise. Therefore, the cost parameters are described as ‘approximately equal to some certain amount’ and so it is more reasonable to characterize these parameters as fuzzy.

Since the development of EOQ model by Harris (1915), lot of research works have been carried out in inventory control system. In the existing literature, inventory models are generally developed under the assumption of constant or stochastic lead time. A number of research papers have already been published in this direction. Kalpakam & Sapan (1995) studied a perishable inventory model with stochastic lead time. But in real life situations, the lead time is normally vague and imprecise i.e., uncertain in
non-stochastic sense. It will be more realistic to consider the lead time as fuzzy in nature.

Roy & Maiti (1998) developed a multi objective inventory model for deteriorating items with stock dependent demand under two restrictions in fuzzy environment. They solved the problem with infinite time horizon not considering shortages.

In this chapter a multi-item inventory model with limited storage space, finite investment and limited orders is considered with demand, number of orders and leading time as decision variables. Here the unit price is dependent on demand and leading time crashing cost is dependent on lead time. The model is solved using Karush Kuhn-Tucker conditions approach to find the annual total cost.

7.2 ASSUMPTIONS OF THE MODEL

The mathematical model in this chapter is developed on the basis of the following assumptions

(1) Time horizon is finite

(2) No shortages are allowed

(3) Unit production cost is inversely related to the demand rate.

\[ p_i = A_i D_i^{-\beta} \quad i = 1,2,3,\ldots\,, A > 0 \,, \beta \geq 1 \]

Where \( A_i \), \( \beta \) are real constants, selected to provide the best fit of the estimated cost function.

(4) Lead time crashing cost is related to the lead time by a function of the form \( R(l_i) = \alpha L_i^{-b} \), \( i = 1,2,\ldots\,, \alpha > 0 \,, \quad 0 < b \leq 0.5 \)
7.3 OBJECTIVE FUNCTION OF THE INVENTORY MODEL

The annual relevant total cost (sum of production, order, inventory carrying and lead time crashing costs) which according to the basic assumptions of the EOQ model is formulated as

\[ TC(D_i, Q_i, L_i) = \sum_{j=1}^{n} \left( p_i D_i + \frac{S_i D_i}{Q_i} + \left[ \frac{Q_i}{2} + K \sigma \sqrt{L_i} \right] h_i + \frac{D_i}{Q_i} r(L_i) \right) \]  \hspace{1cm} (7.1) 

Substituting \( p_i \) and \( R(L_i) \) in (7.1) gives

\[ TC(D_i, Q_i, L_i) = \sum_{j=1}^{n} \left[ A D_i^{\gamma \beta} + \frac{S_i D_i}{Q_i} + \left[ \frac{Q_i}{2} + K \sigma \sqrt{L_i} \right] h_i + \frac{D_i}{Q_i} \sigma L_i^{\gamma \beta} \right] \]

7.4 CONSTRAINTS OF THE MODEL

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

(i) There is a limitation on the available warehouse floor space where the items are to be stored

\[ \text{i.e} \quad \sum_{i=1}^{n} w_i Q_i \leq W \]

(ii) Investment amount on total production cost cannot be infinite, it may have an upper limit on the maximum investment

\[ \text{i.e} \quad \sum_{i=1}^{n} p_i Q_i \leq B \]

(or) \[ \sum_{i=1}^{n} A D_i^{\gamma \beta} Q_i \leq B \]
(iii) An upper limit on the number of orders that can be made in a
time cycle on the system

\[ \text{i.e. } \sum_{j=1}^{n} \frac{D_j}{Q_j} \leq t \]

Thus the above mentioned model can be stated as

\[
\text{Min} T(C(D_i, Q_i, L_i)) = \sum_{j=1}^{n} \left[ AD_j^{1-\beta} + \frac{S_j D_j}{Q_j} + \left( \frac{Q_j}{2} + K \sigma \sqrt{L_j} \right) H_j + \frac{D_j}{Q_j} \alpha L_j^{\gamma_i} \right]
\]  \hspace{1cm} (7.2)

subject to the inequality constraints

\[ \sum_{i=1}^{n} w_i Q_i \leq W \]  \hspace{1cm} (7.3)

\[ \sum_{j=1}^{n} AD_j^{-\beta} Q_j \leq B \]  \hspace{1cm} (7.4)

\[ \sum_{j=1}^{n} \frac{D_j}{Q_j} \leq t \]  \hspace{1cm} (7.5)

7.5 FUZZIFICATION OF COST PARAMETER

In the crisp environment, all parameters in the total inventory cost
such as holding cost, ordering cost, setup cost, purchasing cost, deterioration
rate, demand rate and production rate etc., are known and have definite value
without ambiguity. Some of the business situations fit such conditions, but in
most of the situations and in the day-by-day changing market scenario the
parameters and variables are highly uncertain or imprecise.
In this chapter, the unit production cost $p_i$ has been defined under fuzzy environment. The membership function for the fuzzy variable $p_i$ is defined as follows

$$
\mu_{\tilde{p}_i}(X) = \begin{cases}
1, & p_i \leq U_{i_l} \\
\frac{U_{i_u} - p_i}{U_{i_u} - L_{i_u}}, & L_{i_l} \leq p_i \leq U_{i_i} \\
0, & p_i \geq U_{i_u}
\end{cases}
$$

Here $U_{i_u}$ and $L_{i_u}$ are upper limit and lower limit of $p_i$ respectively.

### 7.6 Karush Kuhn-Tucker Conditions Approach for the Model

For only one item the objective function reduces to the following form

$$
TC(D,Q,L) = AD^\beta + \frac{SD}{Q} + \left[ \frac{Q}{2} + K\sigma \sqrt{L} \right] H + \frac{D}{Q} \alpha L^b
$$

subject to the constraints

$$
wQ \leq W
$$

$$
AD^\beta Q \leq B \quad \text{and} \\
\frac{D}{Q} \leq I
$$

By introducing the slack variables $s_1$, $s_2$ and $s_3$ the objective function and the constraints of the Lagrangean function can be written as

$$
G = AD^\beta + \frac{SD}{Q} + \left[ \frac{Q}{2} + K\sigma \sqrt{L} \right] H + \frac{D}{Q} \alpha L^b - \lambda_1 \left(W - wQ - s_1^2\right) - \lambda_2 \left(B - AD^\beta Q - s_2^2\right) - \lambda_3 \left(I - DQ^\gamma - s_3^2\right)
$$

(7.6)
Differentiating (7.6) partially with respect to the decision variables D, Q and L gives

\[
\frac{\partial G}{\partial D} = (1 - \beta) AD^{-\beta} + SQ^{-1} + \alpha L^\beta Q^{-1} - \lambda_2 \beta AD^{-\beta} \beta Q + \lambda_5 Q^{-1}
\]

\[
\frac{\partial G}{\partial Q} = -SDQ^{-2} + 0.5H - D\alpha L^\beta Q^{-2} + \lambda_4 \alpha w + \lambda_5 AD^{-\beta} - \lambda_7 DQ^{-2}
\]

\[
\frac{\partial G}{\partial L} = 0.5K\sigma L^{-0.5} - b\alpha L^{\beta - 1} DQ^{-1}
\]

By applying the conditions of KKT

\[
\frac{\partial G}{\partial D} = 0 \Rightarrow (1 - \beta) AD^{-\beta} + SQ^{-1} + \alpha L^\beta Q^{-1} - \lambda_2 \beta AD^{-\beta} \beta Q + \lambda_5 Q^{-1} = 0 \quad (7.7)
\]

\[
\frac{\partial G}{\partial Q} = 0 \Rightarrow -SDQ^{-2} + 0.5H - D\alpha L^\beta Q^{-2} + \lambda_4 \alpha w + \lambda_5 AD^{-\beta} - \lambda_7 DQ^{-2} = 0 \quad (7.8)
\]

\[
\frac{\partial G}{\partial L} = 0 \Rightarrow 0.5K\sigma L^{-0.5} - b\alpha L^{\beta - 1} DQ^{-1} = 0 \quad (7.9)
\]

Solving these Equations (7.7), (7.8) and (7.9) gives the optimal solutions for the decision variables D, Q and L.

### 7.7 NUMERICAL EXAMPLE

The decision variables namely the optimal order quantity \( Q_1 \), optimal demand rate \( D_1 \) and optimal lead time \( L_1 \) whose values determine the minimum annual relevant total cost are computed for different values of \( \beta \).
In order to illustrate the above solution procedure, an inventory system with the following data is considered.

The parameters of the model are shown in Table 7.1.

**Table 7.1 Input data for different values of parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of items</td>
<td>n</td>
<td>1</td>
</tr>
<tr>
<td>Constant</td>
<td>A₁</td>
<td>15</td>
</tr>
<tr>
<td>Setup cost of the item 1</td>
<td>S₁</td>
<td>$200</td>
</tr>
<tr>
<td>Holding cost of the item 1</td>
<td>H₁</td>
<td>$0.80</td>
</tr>
<tr>
<td>Total investment cost</td>
<td>B</td>
<td>$300</td>
</tr>
<tr>
<td>Storage space of the item 1</td>
<td>w₁</td>
<td>2</td>
</tr>
<tr>
<td>Lower limit of the unit cost of the item 1</td>
<td>l₁</td>
<td>$3</td>
</tr>
<tr>
<td>Upper limit of the unit cost of the item 1</td>
<td>u₁</td>
<td>$8</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>σ</td>
<td>6 unit/year</td>
</tr>
<tr>
<td>Constant</td>
<td>k</td>
<td>2</td>
</tr>
<tr>
<td>Real constant</td>
<td>α</td>
<td>1</td>
</tr>
<tr>
<td>Real constant</td>
<td>b</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The following table shows the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

The optimal values of the production batch $Q_t$, demand rate $D_t$, lead time $L_t$ and minimum total cost are given in Table 7.2.
Table 7.2 Optimal values for the proposed model without shortages when 
\(\alpha = 1 \& b = 0.1\)

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(D_1)</th>
<th>(Q_1)</th>
<th>(L_1)</th>
<th>(p_1)</th>
<th>(\mu_p)</th>
<th>Min TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.399</td>
<td>26.47</td>
<td>1.2 (\times 10^5)</td>
<td>7.664</td>
<td>0.0672</td>
<td>32.078</td>
</tr>
<tr>
<td>2.2</td>
<td>1.503</td>
<td>27.63</td>
<td>1.25 (\times 10^5)</td>
<td>6.12</td>
<td>0.376</td>
<td>31.333</td>
</tr>
<tr>
<td>2.4</td>
<td>1.562</td>
<td>28.12</td>
<td>1.25 (\times 10^5)</td>
<td>5.143</td>
<td>0.5714</td>
<td>30.597</td>
</tr>
<tr>
<td>2.6</td>
<td>1.594</td>
<td>28.46</td>
<td>1.29 (\times 10^5)</td>
<td>4.463</td>
<td>0.7074</td>
<td>29.907</td>
</tr>
<tr>
<td>2.8</td>
<td>1.612</td>
<td>28.58</td>
<td>1.31 (\times 10^5)</td>
<td>3.939</td>
<td>0.8122</td>
<td>29.272</td>
</tr>
</tbody>
</table>

From the above table it is observed that the optimal solution is 
\(D_1 = 1.612\), \(Q_1 = 28.58\), \(L_1 = 1.31 \times 10^5\) and Min TC = $29.272, which 
corresponds to the maximum membership function 0.8122. It has been seen 
that as \(\beta\) value increases, the lot size \(Q_1\), the demand \(D_1\), lead time \(L_1\) increases 
whereas the unit price \(p_1\) and the minimum total cost decreases.

7.8 SENSITIVITY ANALYSIS

The change in the values of parameter may happen due to 
uncertainties in any decision-making situation. In order to examine the 
implications of these changes, the sensitivity analysis will be of great help in 
decision-making. Using the numerical examples given in the preceding 
section, the sensitivity analysis of various values of parameter \(\beta\) has been 
done.

In the real life and global market situations some parameters like 
ordering cost, holding costs and deteriorating cost fluctuate with their actual 
values. So the parameters are not assumed to be constant.

The exact solutions are plotted in Figure 7.1, 7.2, 7.3 and 7.4.
**Figure 7.1** Graphical representation of demand with respect to β

The above Figure 7.1 shows that if the value of β increases from 2 to 2.8 then the demand of the total expenditure also increases from 1.399 to 1.612.

**Figure 7.2** Graphical representation of lot size with respect to β

The above Figure 7.2 shows that if the value of β increases from 2 to 2.8 then the lot size also increases from 26.47 to 28.58.
Figure 7.3 Graphical representation of total cost with respect to $\beta$

The above Figure 7.3 shows that if the value of $\beta$ increases from 2 to 2.8 then the total expenditure decreases from $32,078 to $29,272.

Figure 7.4 Graphical representation of unit price with respect to $\beta$

The above Figure 7.4 shows that if the value of $\beta$ increases from 2 to 2.8 then the unit price decreases from $7,664 to $3,939.
7.9 **SUMMARY**

In this chapter a concept of the optimal solution of the inventory problem with fuzzy cost price per unit item was proposed. An inventory model with demand dependent unit cost and lead time dependent on leading time crashing cost with limited lot size, warehouse and investment is solved using Karush Kuhn-Tucker Conditions. Here the optimal solution is calculated with fuzzy unit price per item. The result reveals the minimum expected annual total cost of the inventory model. The model can be extended for more than one item. Also it can be solved for various constraints like limited budgetary, setup cost, etc.

This method is quite general and can be extended to other similar inventory models including with shortages and deteriorating items. A few additional aspects that is intended to take into account in the near future are imposing promotion and pricing through a new optimization model.

Many researchers solved fuzzy multi-item multi objective inventory especially using Geometric programming method. Here we solved the model using Karush Kuhn-Tucker conditions with unit production cost under fuzzy environment.

This chapter focuses only on inventory models with demand dependent unit cost in which unit cost is a fuzzy parameter. But some parameters of inventory models may have variations in other characteristics, such as random variables or random fuzzy variables. Therefore, in future study, models should be developed to support other parameters.