Preface

The present thesis entitled “Existence and iterative approximation of common solution of generalized equilibrium problems, generalized variational inequality problems and fixed point problems” is an outcome of the studies made by the author at Department of Mathematics, Aligarh Muslim University, Aligarh, India.

It is well known that the theory of variational inequalities and the theory of equilibrium problems provide powerful techniques for studying a large number of problems arising in optimization, transportation, economics equilibrium and other problems of practical interest.

The theory of variational inequalities was initiated independently by Fichera [57] and Stampacchia [155] in the early 1960’s to study the boundary value problems arising in the elasticity and potential theory, respectively. Since then variational inequalities have been extended in various directions using novel and innovative techniques. It is well-known that the variational inequality theory has played a fundamental and important role in the study of a wide range of problems arising in physics, mechanics, elasticity, optimization, control theory, management science, operations research, economics, transportation and other branches of mathematical and engineering sciences. An important generalization of variational inequality is a vector variational inequality introduced by Giannessi [59]. Vector variational inequalities have many applications in transportation and other areas, see for example Giannessi [60].

Equilibrium problems were initially introduced by Zuhovickii, Poljak and Primak [177], Fan [54, 55], perhaps motivated by minimax problems appearing in economic equilib-
rium. A more general result than that in [55] was established by Brézis, Nirenberg and Stampacchia [16]. But, in 1994, the terminology of equilibrium problem was adopted by Blum and Oettli [14]. Since then various generalizations of equilibrium problem considered by Blum and Oettli [14] have been introduced and studied by many authors. It has been shown that the theory of equilibrium problem provides a natural, novel and unified framework for several problems arising in nonlinear analysis, optimization, economics, finance, game theory, physics and engineering. Vector equilibrium problem, an important generalization of equilibrium problem, is introduced and studied independently by Kazmi [82, 84], Konnov [104] and Bianchi et al. [13]. Vector equilibrium problems include vector optimization problems, vector variational inequalities as a special case, and have deep connections with some important areas of nonlinear analysis.

In recent years, much attention has been given for developing efficient and implementable iterative methods including projection method and its variant forms, extragradient method, linear approximation, auxiliary principle method, descent and Newton methods, Helpern iterative method, Mann and Ishikawa iterative methods, viscosity approximation method, hybrid iterative method for the variational inequalities and equilibrium problems.

There is a vast literature is available on iterative methods for studying, separately, variational inequalities, equilibrium problems and fixed point problems for nonlinear mappings, and on iterative methods to approximate common solutions of these problems but to develop and study the iterative methods for approximating common solutions of new generalizations of these problems in the settings of Hilbert spaces and Banach spaces, is still an unexplored field.

The objective of this thesis is to develop and study some iterative methods for approximating common solutions of split generalized vector equilibrium problems, system of unrelated split generalized vector variational inequalities, generalized mixed equilibrium problem, and fixed point problems for a (family of) nonlinear mapping(s) in the setting of Hilbert spaces; and to develop and study some iterative methods for approximating common solutions of generalized vector equilibrium problem, system of unrelated gen-
eralized mixed vector equilibrium problem and fixed point problems for a (family of) nonlinear mapping(s) in the setting of Banach space.

The thesis comprises of six chapters.

In Chapter 1, we review various notations, known definitions and results which are required in carrying out the research work presented in the thesis. Further, we give brief survey of some classes of variational inequalities and equilibrium problems. Furthermore, we give brief survey of some iterative methods for solving fixed point problems, variational inequalities and equilibrium problems.

In Chapter 2, we introduce and study an explicit viscosity Cesàro mean approximation method to approximate a common solution of split generalized vector equilibrium problem and fixed point problems for a finite family of nonexpansive mappings in real Hilbert spaces. We establish a strong convergence theorem for the sequences generated by the proposed iterative scheme. Further, we derive some consequences from the strong convergence theorem. Furthermore, we justify our main result through a numerical example.

In Chapter 3, we consider generalized vector equilibrium problem in real uniformly smooth and uniformly convex Banach space and study some properties of its solution set. We introduce two iterative schemes for finding a common solution of generalized vector equilibrium problem and fixed point problems for relatively nonexpansive mappings. Further, we study strong and weak convergence of the sequences generated by the proposed iterative schemes. We also derive some consequences from our main results.

In Chapter 4, we consider a generalized vector variational inequality problem in Hilbert space and study some properties of its solution set. Further, we develop a hybrid iterative method for approximate a common solution to a system of unrelated split generalized vector variational inequality problems, system of unrelated multi-valued variational inequality problems, and fixed point problems for a family of nonexpensive mappings in real Hilbert spaces. Further, we prove a strong convergence theorem for the sequences generated by the proposed iterative scheme. Furthermore, we derive some
consequences from our main result. We also justify our main result through a numerical example.

In Chapter 5, we introduce and study an explicit hybrid relaxed extragradient iterative method to approximate a common solution to generalized mixed equilibrium problem and fixed point problem for a nonexpansive semigroup in Hilbert space. Further, we prove that the sequence generated by the proposed iterative scheme converges strongly to the common solution to generalized mixed equilibrium problem and fixed point problem for a nonexpansive semigroup. This common solution is the unique solution of a variational inequality problem and is the optimality condition for a minimization problem. Finally, we derive some consequence from our main result.

In Chapter 6, we study some properties of solution set of a generalized mixed vector equilibrium problem in Banach space. Further, we introduce an iterative method based on hybrid method and convex approximation method for finding a common element to the solution set of a system of unrelated generalized mixed vector equilibrium problems and the solution set of fixed point problems for the two families of generalized asymptotically quasi $\phi$-nonexpansive mappings in uniformly convex and strictly convex Banach space. Furthermore, we obtain a strong convergence theorem for the sequences generated by the proposed iterative scheme. We derive some consequences from our main result.

A comprehensive list of references of books, monographs, proceedings and research papers is provided at the end of the thesis.

It has been observed that, in some particular cases our results reduce to some of the existing work available in the literature, which shows that our results and methods are more general than the existing results.

The published/ communicated research papers based on the work of this thesis are as follows:

1. Some iterative schemes for generalized vector equilibrium problems and relatively nonexpasive mappings in Banach spaces, *Mathematical Sciences (Springer)*
(2013), Article 19, 11 pages.


3. Common solutions to some systems of variational inequalities and fixed point problems, Communicated in Fixed Point Theory.

4. Common solutions to some systems of vector equilibrium problems and common fixed point problems in Banach space, Communicated in Afrika Matematika.


Some results of this thesis have been presented in the following National and International conferences:

1. International Conference and Workshop on Mathematical Analysis at University of Putra, Malaysia, May 27-30, 2014.

2. International Conference on Recent Trends in Algebra and Analysis with Applications at Aligarh Muslim University, Aligarh, India, February 12-14, 2014.


