An interesting problem in manpower planning is to find the optimal time interval between screening tests for promotions in a compartment manpower system. Recruitment of persons based on their satisfactory performance in screening tests is a common procedure in many organizations. The use of compartment models in manpower planning is also quite common. In this chapter manpower system having three compartments is considered and the transition of persons from one compartment to another is allowed and in between there is a screening test to evaluate the competence of individuals to get into another compartment. For the above model the optimal time interval between successive screening tests is calculated so as to minimize the total cost. The analytical result is numerically illustrated by assuming specific distribution.

12.1 INTRODUCTION

In many organizations recruitment is made based on the satisfactory performance of the person to be recruited in the screening test. There are many mathematical methods to solve optimization problems in manpower planning. Mostly the calculus of maxima and minima is used. In addition to the calculus approach, techniques of operations research such as linear programming, goal programming, dynamic programming and network analysis are also used.
Many authors have used compartment models effectively in manpower planning. An optimum manpower utilization model using mathematical programming has been discussed by Schneider and Kilpatrick (1975) for Health Maintenance Organization. Jaquette and Nelson (1976) have discussed a mathematical model for a military manpower system with a view to determine the optimal steady state, wage rate and force distribution by length of service. The concept of long term horizons involved with manpower decisions and the uncertainty in future manpower requirements have been analysed by Grinold and Marshall (1977) using optimization techniques. An optimal planning of manpower training programmes has been analysed by Goh et al. (1987). The use of dynamic programming to determine the optimal manpower recruitment policies is discussed by Poornachandra Rao (1990). Subramanian (1996) has considered an optimum promotion policy taking into account the cost of promotion of a person belonging to any grade as a function of the number of employees in that grade. Mehlmann (1977a, 1977b, 1979), McClean (1978, 1980, 1991) and Ledermann (1992) have applied the concepts of stochastic process in studying the manpower models.

In this chapter a manpower system having three compartments say $C_1, C_2, C_3$ is considered. The size of $C_1$ is fixed as $n$. Transition of persons from $C_1$ to $C_2$ and from $C_2$ to $C_3$ is allowed and in between there is a screening test to evaluate the competence of the individuals. $C_i$ is the compartment having persons with greater skills than $C_2$ and $C_3$. The objective of the chapter is to find the optimum time for the screening test so that the cost will be minimum. This chapter is organized as follows. In section 12.2, description of the model under consideration is given. The main result on the expected total cost is obtained in section 12.3 and some special cases are also considered in section 12.4 in order to find the optimum time for the screening test. Numerical illustrations are given in section 12.5.
12.2 MODEL DESCRIPTION

Consider the organization having three compartments $C_1, C_2$ and $C_3$. The number of persons in $C_1$ is a fixed positive integer. Persons in $C_1$ are first recruited and kept in the reserve list. Assume that they are given some training to improve their capabilities. Keeping these persons in $C_1$ and training them involves a maintenance cost. The vacancies in $C_3$ are filled from $C_2$ by conducting screening tests. Shortages are permitted in $C_2$ and $C_3$. If $k$ vacancies exist in $C_3$, $r$ out of $k$ are selected each with probability $p_1$ from $C_2$ and remaining $(k-r)$ are selected from outside $C_2$ (direct recruitment) with probability $q_1$ with $p_1 + q_1 = 1$. Let $a$ be the number of vacancies that arises in $C_2$ due to the exit from the system and $r_1 = r + a$. The $r_1$ vacancies that arise in $C_2$ are filled in the following manner: $r_2$ out of $r_1$ are selected from $C_1$ each with probability $p_2$ and the remaining $r_1 - r_2$ are selected from outside $C_1$ (direct recruitment) with probability $q_2$ with $p_2 + q_2 = 1$.

Conducting the test but no person getting entry into $C_3$ involves some cost namely screening test cost. In case no person gets selected and enters into $C_3$, the vacancies in $C_3$ remain unfilled and each such unfilled vacancy gives rise to some shortage cost in terms of loss of productivity. To offset this loss, recruitment of persons from outside is made on an emergency basis.

Now the vacancies in $C_2$ which arise due to transfer of persons to $C_3$ and by any loss is filled from $C_1$ by conducting tests and remaining unfilled vacancies in $C_2$ are filled from outside. The longer the time interval between the screening tests the greater will be the cost of maintenance of persons. Frequent screening tests results in higher test costs. In this chapter the optimal time interval $T$ between successive screening tests is obtained so as to minimize the expected total cost. The result is numerically illustrated.
Notations

\( c_r \) : cost of retention of each person.

\( c_a \) : cost of transition from \( C_i \) to \( C_{i+1} \) per unit time \( i=1,2 \).

\( c_u \) : cost of unfilled vacancy in \( C_i \), \( i=1,2 \) per unit time.

\( T \) : time between two consecutive screening tests.

\( F(.) \) : cumulative distribution function of \( T \).

\( F_k(.) \) : \( k \)-fold convolution of \( F(.) \).

\( f(.) \) : probability density function of \( T \).

\( f_k(.) \) : \( k \)-fold convolution of \( f(.) \).

\( T^* \) : optimum value of \( T \).

12.3 MAIN RESULT

The expected total cost \( E(TC) \) is given as

\[
E(TC) = Tc_{1r} + \sum_{k=1}^{\infty} [F_k(T) - F_{k+1}(T)] \left\{ \sum_{r=0}^{k} C_r p_i^r q_i^{k-r} \right\} (k-r)c_{2r} + [1-F(T)]c_r + \\
Tc_{1s} + \sum_{i=1}^{\infty} [F_i(T) - F_{i+1}(T)] \left\{ \sum_{r_2=0}^{i} C_{r_2} p_2^r q_2^{r_2} \right\} (r_1 - r_2)c_{1s}
\]

\[ \text{(12.3.1)} \]

\[
= T(c_{1r} + c_{2r}) + [1-F(T)]c_r + \sum_{k=1}^{\infty} [F_k(T) - F_{k+1}(T)] \sum_{r=0}^{k} \frac{k(k-1)!}{r!(k-r-1)!} p_i^r q_i^{k-r} c_{2r} + \\
\sum_{i=1}^{\infty} [F_i(T) - F_{i+1}(T)] \sum_{r_2=0}^{i} \frac{r_1(r_1-1)!}{r_2!(r_1-r_2-1)!} p_2^r q_2^{r_2} c_{1s}
\]

i.e.,

\[
E(TC) = T(c_{1r} + c_{2r}) + [1-F(T)]c_r + \sum_{k=1}^{\infty} [F_k(T) - F_{k+1}(T)] kq_i c_{2r} + \\
\sum_{i=1}^{\infty} [F_i(T) - F_{i+1}(T)] r_1 q_2 c_{1s}
\]

\[ \text{(12.3.2)} \]
12.4 SPECIAL CASE

Suppose T follows exponential distribution with parameter $\lambda$. Then the expected total cost becomes

$$E(TC) = T(c_{it} + c_{st}) + e^{-\lambda T} c_r + \lambda T(q_1 c_{2s} + q_2 c_{1s})$$

We now find $T^*$ for which the expected total cost is minimum.

Differentiating (12.4.1) with respect to T and equating it to zero we get

$$c_{it} + c_{st} - \lambda e^{-\lambda T} c_r + \lambda (q_1 c_{2s} + q_2 c_{1s}) = 0$$

Therefore

$$T^* = \frac{1}{\lambda} \log \left[ \frac{c_{it} + c_{st} + \lambda (q_1 C_{2s} + q_2 C_{1s})}{c_{it} + c_{st} + \lambda (q_1 c_{2s} + q_2 c_{1s})} \right]$$

Clearly

$$\left[ \frac{d^2}{dt^2} E(TC) \right]_{T=T^*} = \lambda^2 e^{-\lambda T^*} c_r > 0.$$ 

Therefore $T^*$ given by (12.4.2) is the required optimum value of time between two successive screening tests, minimizing the total cost.

12.5 NUMERICAL ILLUSTRATION

The optimum value of T and the corresponding total cost is calculated by fixing the values for $\lambda$, $q_1$, $q_2$, $C_r$, $C_{1t}$, $C_{2t}$, $C_{1s}$, $C_{2s}$.

For $\lambda=1$, $q_1=0.6$, $q_2=0.4$, $C_r=40000$, $C_{1t}=500$, $C_{2t}=300$, $C_{1s}=1000$, $C_{2s}=1500$

Optimum T is $T^* = 1.259$ years and expected total cost is 8448.55.