Chapter 6

Assignment Problem in Fuzzy Membership Functions

Abstract

In this chapter, we analyze the fuzzification of assignment problem in the way of trapezoidal fuzzy membership functions and the optimal solution is justified in terms of same membership functions.

\footnote{1The contents of this chapter form the material of the paper published in “International Journal of Fuzzy Mathematics”, International Institute of Fuzzy Mathematics, Los Angeles, USA, Vol. 19, No. 3, pp. 551 - 560 (2011).}
6.1 Introduction

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in industry and other applications. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem $C_{ij}$ denotes the cost of assigning the jth job to the ith person. We assume that one person can be assigned exactly one job; also each person can do at most one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum.

Nowadays Assignment Problem (AP) is solved depending on the measurement of efficiency criteria of a person related to his/her qualification. So the problem is how to assign the person to the particular job with maximum efficiency for that job. So when the assignment problem is analyzed one has to consider the efficiency parameter in such a way that the estimation may remain subjective. This type of assignment with fuzzy parameters is totally a new concept. Moreover, it includes the problem that some person has no qualification for some jobs. Here the efficiency has been measured in terms of fuzzy costs. We may also consider the restriction on the maximum cost that can be spent for each job.

Moreover, this restriction in terms of fuzzy numbers has not been explored so far. Here a composite judging matrix is constructed taking into consideration both the constraints in order to show the
existence of the solution. The assignment problem (AP) is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate. All the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. However, due to it’s highly degeneracy nature, a specially designed algorithm, widely known as Hungarian method proposed by Kuhn [53], is used for its solution.

However, in real life situations, the parameters of AP are imprecise number instead of fixed real numbers because time/cost for doing a job by a facility (machine/person) might vary due to different reasons. Examples of these types of problems may be the case of assigning men to offices, crews (drivers and conductors) to buses, trucks to delivery routes etc. Over the past 50 years, many variations on the classical AP are proposed e.g. bottleneck assignment problem, generalized assignment problem, quadratic assignment problem etc. Zadeh [90] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems.

The organization of the chapter is detailed below. In section 6.2, we discussed some comparison with the transportation problem in
fuzzified form. In section 6.3, we introduced the assignment problem in terms of fuzzified form. We consider a whole problem in terms trapezoidal fuzzy numbers of assignment problem by suitability of each assignment and each assignment cost. Later we have proposed the method of fuzzy Hungarian to find out the optimal solution for the total fuzzy minimum cost and also this total cost is verified with the aid of trapezoidal fuzzy membership functions which are stated in section 6.4. A relevant numerical example with results and discussion is also provided in this chapter.

6.2 Comparison with Fuzzy Transportation problem

Fuzzy assignment problem may be regarded as a special case of fuzzy transportation problem. Here fuzzy facilities represent the 'fuzzy sources' while the fuzzy jobs represent the 'fuzzy destinations'.

The fuzzy supply available at each fuzzy source is \([-2\delta, 0, 2\delta, 4\delta]\) where \(\delta\) is small + ve number, i.e., \(a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}] = [-2\delta, 0, 2\delta, 4\delta]\) for all \(i\). Similarly, the fuzzy demand at each fuzzy destinations is \([-2\delta, 0, 2\delta, 4\delta]\) i.e., \(b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}] = [-2\delta, 0, 2\delta, 4\delta]\) for all \(j\). The fuzzy cost of transporting (assigning) fuzzy facility \(i\) to fuzzy job \(j\) is \(C_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]\). The resulting fuzzy transportation problem can be represented as in the below table (here \(X_{ij} = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]\)).
Table 6.1: Fuzzy jobs and fuzzy facilities

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2...</th>
<th>n</th>
<th>Fuzzy capacity ($a_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C_{11}$</td>
<td>$C_{12}$...</td>
<td>$C_{1n}$</td>
<td>$a_1$</td>
</tr>
<tr>
<td></td>
<td>$X_{11}$</td>
<td>$X_{12}$...</td>
<td>$X_{1n}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$C_{21}$</td>
<td>$C_{22}$...</td>
<td>$C_{2n}$</td>
<td>$a_2$</td>
</tr>
<tr>
<td></td>
<td>$X_{21}$</td>
<td>$X_{22}$...</td>
<td>$X_{2n}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>$C_{m1}$</td>
<td>$C_{m2}$...</td>
<td>$C_{mn}$</td>
<td>$a_m$</td>
</tr>
<tr>
<td></td>
<td>$X_{m1}$</td>
<td>$X_{m2}$...</td>
<td>$X_{mn}$</td>
<td></td>
</tr>
<tr>
<td>Fuzzy demand ($b_j$)</td>
<td>$b_1$</td>
<td>$b_2$...</td>
<td>$b_n$</td>
<td>$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$</td>
</tr>
</tbody>
</table>
6.3 The Assignment problem in Fuzzified Form

The fuzzy assignment problem can be expressed as follows;

Let

\[
\begin{cases}
[-2\delta, -\delta, \delta, 2\delta] & \text{if the ith fuzzy facility is not assigned to jth fuzzy job} \\
[-2\delta, 0, 2\delta, 4\delta] & \text{if the ith fuzzy facility is assigned to jth fuzzy job}
\end{cases}
\]

Then the problem is given by

Minimize

\[
Z = [z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]
\]

\[
= \sum_{j=1}^{m} \sum_{i=1}^{n} [c_{ij}^{(1)} c_{ij}^{(2)} c_{ij}^{(3)} c_{ij}^{(4)}] [x_{ij}^{(1)} x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}]
\]

Subject to the constraints

\[
\sum_{j=1}^{m} [x_{ij}^{(1)} x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}] = [-2\delta, 0, 2\delta, 4\delta],
\]

for \( i = 1, 2, \ldots, n \)

\[
\sum_{i=1}^{n} [x_{ij}^{(1)} x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}] = [-2\delta, 0, 2\delta, 4\delta],
\]

for \( j = 1, 2, \ldots, m \) and

\[
[x_{ij}^{(1)} x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}] = [-2\delta, 0, 2\delta, 4\delta] \text{ or } [-2\delta, -\delta, \delta, 2\delta]
\]

we see that if the last condition is replaced by \( [x_{ij}^{(1)} x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}] \geq [-2\delta, -\delta, \delta, 2\delta] \), we have fuzzy transportation problem with all fuzzy requirements and available fuzzy resources equal to \([-2\delta, 0, 2\delta, 4\delta]\).
However, fuzzy transportation technique cannot be used to solve this problem because of fuzzy degeneracy. Whenever we make a fuzzy assignment, we automatically satisfy row and column fuzzy requirements simultaneously (rim requirements being equal to $[-2\delta, 0, 2\delta, 4\delta]$), resulting in fuzzy degeneracy. This special structure of fuzzy assignment problem allows a more convenient method of solution.

The technique used for solving fuzzy assignment problem makes use of two theorems.

6.3.1 Theorem I

In an fuzzy assignment problem, if we add or subtract a fuzzy constant to every element of a row (or column) in the cost fuzzy matrix, then an fuzzy assignment which minimizes the total cost on the fuzzy matrix also minimizes the total cost on the other fuzzy matrix.

**Proof:**

If fuzzy constants $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$ are subtracted from ith row and jth column respectively, then the new fuzzy cost elements will become

$$C'_{ij} = [c_{ij}^{(1)'}, c_{ij}^{(2)'}, c_{ij}^{(3)'}, c_{ij}^{(4)'}]$$

$$= [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] - [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] - [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}]$$

and the new objective function will be:

$$Z' = [z^{(1)'}, z^{(2)'}, z^{(3)'}, z^{(4)'}]$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} [c_{ij}^{(1)'}, c_{ij}^{(2)'}, c_{ij}^{(3)'}, c_{ij}^{(4)'}, x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$$

$$Z' = \sum_{j=1}^{m} \sum_{i=1}^{n} \{[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] - [u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}] - [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}] \} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$$
\[ Z' = \{ \sum_{j=1}^{m} \sum_{i=1}^{n} \{ [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] \} - \]
\[ \{ \sum_{i=1}^{n} [u_{i}^{(1)}, u_{i}^{(2)}, u_{i}^{(3)}] \sum_{j=1}^{m} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] \} - \]
\[ \{ \sum_{j=1}^{m} [v_{j}^{(1)}, v_{j}^{(2)}, v_{j}^{(3)}, v_{j}^{(4)}] \sum_{i=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] \} \]

since from the fuzzy constraints of the problem;
\[ \sum_{i=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = \sum_{j=1}^{m} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [-2\delta, 0, 2\delta, 4\delta] \]
\[ Z' = \sum_{j=1}^{m} \sum_{i=1}^{n} \{ [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] - \]
\[ \sum_{i=1}^{n} [u_{i}^{(1)}, u_{i}^{(2)}, u_{i}^{(3)}, u_{i}^{(4)}] - \sum_{j=1}^{m} [v_{j}^{(1)}, v_{j}^{(2)}, v_{j}^{(3)}, v_{j}^{(4)}] \} \]
\[ Z' = z - \text{constant} \]

This shows that the minimization of the new fuzzy objective function \([z^{(1)}', z^{(2)}', z^{(3)}', z^{(4)}']\) yields the same solution as the minimization of original fuzzy objective function \(z\).

### 6.3.2 Theorem II

If all \([c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}] \geq [-2\delta, 0, 2\delta, 4\delta]\) and we can find a set \([X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}, X_{ij}^{(4)}] = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]\) such that \([c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}][x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [-2\delta, -\delta, \delta, 2\delta]\) then the solution is optimal.

The above two theorems indicate that if one can create a new \([c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]\) fuzzy matrix with zero entries, and if these zero elements, or a subset thereof constitute a fuzzy feasible solution is the optimal solution. the actual procedure for solving fuzzy assignment problem will be described by taking a numerical example.
6.4 Formulation and Solution of a Fuzzy Assignment problem

We shall consider a numerical example which will make clear the techniques of formulation and solution of fuzzy assignment problems.

6.4.1 Numerical Example

A machine company decides to make four subassemblies through four contractors in terms of fuzziness. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor is shown in the following table in hundreds of rupees. Assign the different subassemblies to contractors so as to minimize the fuzzy total cost.

<table>
<thead>
<tr>
<th>Table 6.2: Fuzzy contractors (FC) and fuzzy subassemblies (FS)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FS I</strong></td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>[13,14,15,18]</td>
</tr>
<tr>
<td><strong>FS II</strong></td>
</tr>
<tr>
<td><strong>FS III</strong></td>
</tr>
<tr>
<td><strong>FS IV</strong></td>
</tr>
</tbody>
</table>
6.4.2 Formulation of the fuzzy assignment problem

**Step I**: Key fuzzy decision is what to whom, that is which fuzzy subassembly be assigned to which fuzzy contractors or what are the 'n' optimum fuzzy assignments on 1 - 1 basis.

**Step II**: Fuzzy feasible alternatives are n possible arrangements for n x n fuzzy assignment situation.

**Step III**: Fuzzy objective is to minimize the total cost involved.

i.e., minimize $Z = [z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]$.

**Step IV**: Fuzzy constraints

a) fuzzy constraints on subassemblies are

$$[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{11}, x^{(2)}_{11}, x^{(3)}_{11}, x^{(4)}_{11}
\end{array}]
\end{array}] +
[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{12}, x^{(2)}_{12}, x^{(3)}_{12}, x^{(4)}_{12}
\end{array}]
\end{array}] +
[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{13}, x^{(2)}_{13}, x^{(3)}_{13}, x^{(4)}_{13}
\end{array}]
\end{array}] +
[\begin{array}{c}
[\begin{array}{c}
 x^{(1)}_{14}, x^{(2)}_{14}, x^{(3)}_{14}, x^{(4)}_{14}
\end{array}]
\end{array}] = [-2\delta, 0, 2\delta, 4\delta]$$

b) fuzzy constraints on contractors are

$$[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{11}, x^{(2)}_{11}, x^{(3)}_{11}, x^{(4)}_{11}
\end{array}]
\end{array}] +
[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{21}, x^{(2)}_{21}, x^{(3)}_{21}, x^{(4)}_{21}
\end{array}]
\end{array}] +
[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{31}, x^{(2)}_{31}, x^{(3)}_{31}, x^{(4)}_{31}
\end{array}]
\end{array}] +
[egin{array}{c}
[\begin{array}{c}
 x^{(1)}_{41}, x^{(2)}_{41}, x^{(3)}_{41}, x^{(4)}_{41}
\end{array}]
\end{array}] = [-2\delta, 0, 2\delta, 4\delta]$$
Comparing this model to fuzzy transportation method, we find that $a_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}] = [-2\delta, 0, 2\delta, 4\delta]$ and $b_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}] = [-2\delta, 0, 2\delta, 4\delta]$ the fuzzy assignment problem can be represented in the following table. Therefore fuzzy assignment problem is a special case of fuzzy transportation problem in which

(i) all right hand side constants in the fuzzy constraints are unity

$[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}] = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}] = [-2\delta, 0, 2\delta, 4\delta]$

(ii) all coefficients of $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ in the fuzzy constraints are unity.

(iii) $m = n$. 
### Table 6.3: Fuzzy contractors (FC) and fuzzy subassemblies (FS) (continued)

<table>
<thead>
<tr>
<th></th>
<th>FC I</th>
<th>FC II</th>
<th>FC III</th>
<th>FC IV</th>
<th>Fuzzy supply $(a_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS I</td>
<td>[13,14,15,18]</td>
<td>[10,13,14,15]</td>
<td>[11,14,15,16]</td>
<td>[14,15,16,23]</td>
<td>[-2,0,2,4]</td>
</tr>
<tr>
<td>FS II</td>
<td>[9,10,11,14]</td>
<td>[10,11,12,15]</td>
<td>[13,14,15,18]</td>
<td>[10,13,14,15]</td>
<td>[-2,0,2,4]</td>
</tr>
<tr>
<td>FS III</td>
<td>[10,13,14,15]</td>
<td>[10,11,12,15]</td>
<td>[7,10,11,12]</td>
<td>[9,10,11,14]</td>
<td>[-2,0,2,4]</td>
</tr>
<tr>
<td>FS IV</td>
<td>[13,14,15,18]</td>
<td>[14,15,16,23]</td>
<td>[11,14,15,16]</td>
<td>[14,15,16,19]</td>
<td>[-2,0,2,4]</td>
</tr>
<tr>
<td>Fuzzy demand $(b_j)$</td>
<td>[-2,0,2,4]</td>
<td>[-2,0,2,4]</td>
<td>[-2,0,2,4]</td>
<td>[-2,0,2,4]</td>
<td></td>
</tr>
</tbody>
</table>
6.4.3 Solution of the fuzzy assignment problem

Fuzzy Hungarian method

**Step I:** Prepare a fuzzy square matrix - since the situation involves a fuzzy square matrix this step is not necessary.

**Step II:** Reduce the square matrix

**Sub step 1:** Subtract the fuzzy minimum element of each row from all the elements of row. See if there is at least one fuzzy zero in each row and in each column. If it is so, stop here if not, proceed to sub step 2.

**Sub step 2:** Now subtract the fuzzy minimum element of each column from all the elements of column.

In the given situation the fuzzy minimum element in first row is \([10,13,14,15]\), so we subtract \([10,13,14,15]\) from all the elements of first row. Similarly we subtract \([9,10,11,14]\), \([7,10,11,12]\) and \([11,14,15,16]\) from all the elements of row 2, 3 and 4 respectively. This gives at least one zero in each row as shown in table 6.4.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[-2,0,2,8]</td>
<td>[-5, -1,1,5]</td>
<td>[-4,0,2,6]</td>
<td>[-1,1,3,13]</td>
</tr>
<tr>
<td>[-5, -1,1,5]</td>
<td>[-4,0,2,6]</td>
<td>[-1,3,5,9]</td>
<td>[-4,2,4,6]</td>
</tr>
<tr>
<td>[-2,2,4,8]</td>
<td>[-2,0,2,8]</td>
<td>[-5, -1,1,5]</td>
<td>[-3, -1,1,7]</td>
</tr>
<tr>
<td>[-3, -1,1,7]</td>
<td>[-2,0,2,12]</td>
<td>[-4, -1,1,4]</td>
<td>[-2,0,2,8]</td>
</tr>
</tbody>
</table>

Since column 4 contains no zero entry, we go to sub step 2 giving the following fuzzy matrix.
### Table 6.5: Assignment Tableau (Continued)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[-7, -1,3,13]</td>
<td>[-10, -2,2,10]</td>
<td>[-9, -1,3,11]</td>
<td>[-8,0,4,16]</td>
</tr>
<tr>
<td>[-10, -2,2,10]</td>
<td>[-9, -1,3,11]</td>
<td>[-6,2,6,14]</td>
<td>[-11,1,5,9]</td>
</tr>
<tr>
<td>[-7,1,5,13]</td>
<td>[-7, -1,3,13]</td>
<td>[-10, -2,2,10]</td>
<td>[-10, -2,2,10]</td>
</tr>
<tr>
<td>[-8, -2,2,12]</td>
<td>[-7, -1,3,17]</td>
<td>[-9, -2,2,9]</td>
<td>[-9, -1,3,11]</td>
</tr>
</tbody>
</table>

**Step III:** Check if optimal fuzzy assignment can be made in the current solution or not.

Fuzzy basic for making this check is that if fuzzy minimum number of lines crossing all fuzzy zeros is less than \( n \) (in our problem \( n = 4 \)), then an fuzzy optimal assignment cannot be made in the current solution. If it is equal to \( n = 4 \), then fuzzy optimal assignment can be made in the current solution. Approach for finding fuzzy minimum number of lines crossing all zeros consists of the following sub steps.

**Sub step 1:** Examine rows successively until a row with exactly one unmarked fuzzy zero is found. Mark \{[ ]\} this fuzzy zero, indicating that an fuzzy assignment will be made there. Mark \{[*]\} all other fuzzy zeros in the same column showing that they cannot be used for making other fuzzy assignments. Proceed in this manner until all rows have been examined.

In the given situation, row1 has a single unmarked fuzzy zero in column 2. Make an fuzzy assignment as shown. Row 2 has a single unmarked fuzzy zero in column 1, make an fuzzy assignment. Row 4 has a single unmarked fuzzy zero in column 3, make an fuzzy assignment and cross
the second zero in column 3. Now, row 3 has a single unmarked fuzzy zero in column 4. Make an fuzzy assignment here. This is shown in the fuzzy matrix below in the table 6.5.

Sub step 2 : Next examine columns for single unmarked fuzzy zeros, making them \{[ ]\} and also marking \{[*]\} any other fuzzy zeros in their rows.

In case there is no row or column containing single unmarked fuzzy zero (there are more than one unmarked fuzzy zero), mark \{[ ]\} one of the unmarked fuzzy zero arbitrarily and \{[*]\} all other fuzzy zeros in its row and column. Repeat the process till no unmarked zero is left in the fuzzy cost matrix.

Sub step 3 : Repeat sub steps 1 and 2 successively till one of the two things occurs:

a) There may be no row and no column without fuzzy assignment that is, there is one fuzzy assignment in each row and in each column. In such a case the optimal fuzzy assignment can be made in the current solution. i.e., the current fuzzy feasible solution is an fuzzy optimal solution. The fuzzy minimum number of lines crossing all zeros will be equal to ‘n’.

b) There may be some row and/or column without fuzzy assignment. Hence fuzzy optimal assignment cannot be made in the current solution. The fuzzy minimum numbers of lines crossing all fuzzy zeros have to be obtained in this case.

In this problem, sub steps 2 and 3 are not necessary since there is no column left unmarked, since there is one fuzzy assignment in each row
Table 6.6: Assignment Tableau (Continued)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[-7, -1,3,13]</td>
<td>{-10, -2,2,10}</td>
<td>[-9, -1,3,11]</td>
<td>[-8,0,4,16]</td>
</tr>
<tr>
<td>{-10, -2,2,10}</td>
<td>[-9, -1,3,11]</td>
<td>[-6,2,6,14]</td>
<td>[-11,1,5,9]</td>
</tr>
<tr>
<td>[-7,1,5,13]</td>
<td>[-7, -1,3,13]</td>
<td>*{-10, -2,2,10}</td>
<td>{-10, -2,2,10}</td>
</tr>
<tr>
<td>[-8, -2,2,12]</td>
<td>[-7, -1,3,17]</td>
<td>{*-9, -2,2,9}</td>
<td>[-9, -1,3,11]</td>
</tr>
</tbody>
</table>
and in each column, the optimal fuzzy assignment can be made in the current solution.

Therefore, fuzzy minimum total cost is

\[
= Rs \left\{ \left[ c_{12}^{(1)} , c_{12}^{(2)} , c_{12}^{(3)} , c_{12}^{(4)} \right] + \left[ c_{21}^{(1)} , c_{21}^{(2)} , c_{21}^{(3)} , c_{21}^{(4)} \right] + \left[ c_{34}^{(1)} , c_{34}^{(2)} , c_{34}^{(3)} , c_{34}^{(4)} \right] + \\
\left[ c_{43}^{(1)} , c_{43}^{(2)} , c_{43}^{(3)} , c_{43}^{(4)} \right] \right\} \times [25, 75, 125, 175] \\
= Rs \left\{ [10,13,14,15] + [9,10,11,14] + [9,10,11,14][11,14,15,16] \right\} \\
\times [25, 75, 125, 175] \\
= Rs \left\{ [39,47,51,59]*[25,75,125,175] \right\} \\
= Rs [3900,4700,5100,5900].
\]

and the assignment policy is,

fuzzy subassembly I - fuzzy contractor II - [10,13,14,15]
fuzzy subassembly II - fuzzy contractor I - [9,10,11,14]
fuzzy subassembly III - fuzzy contractor IV - [9,10,11,14]
fuzzy subassembly IV - fuzzy contractor III - [11,14,15,16]

Here \( c_{ij} \) are fuzzy cost (matrix) , \( i = j = 1,2,3,4 \).

We have to find fuzzy membership functions (f.m.fs) of \( c_{ij} \) , then the fuzzy assignment cost as follows;

\[
\mu_{c12}(x) (=) \begin{cases} 
\{ \frac{x-10}{3} \} & \text{if } 10 \leq x \leq 13 \\
1 & \text{if } 13 \leq x \leq 14 \\
\{ \frac{x-15}{1} \} & \text{if } 14 \leq x \leq 15 \\
0, \text{ otherwise} 
\end{cases}
\]
To compute the interval of confidence for each level $\alpha$ the trapezoidal shapes will be described by functions of $\alpha$ in the following manner.

Here $\alpha = \frac{(x_1^{(a)} - 10)}{3}$ and $\alpha = \frac{(x_2^{(a)} - 15)}{1}$

Therefore, $c_{12} = [x_1^{(a)}, x_2^{(a)}] = [3\alpha + 10, -\alpha + 15]$  

(6.2)

Exactly in the similar way, we have to write

\[
\mu_{c_{21}}(x) = \begin{cases} 
\{ \frac{x-9}{1} \} & \text{if } 9 \leq x \leq 10 \\
1 & \text{if } 10 \leq x \leq 11 \\
\{ \frac{x-14}{3} \} & \text{if } 11 \leq x \leq 14 \\
0, & \text{otherwise}
\end{cases}
\]

There fore, $c_{21} = [x_1^{(a)}, x_2^{(a)}] = [\alpha + 9, -3\alpha + 14]$  

(6.3)

Here $\mu_{c_{21}}(x) = \mu_{c_{34}}(x)$

which implies that, $c_{21} = c_{34} = [\alpha + 9, -3\alpha + 14]$  

(6.4)
Then

\[
\mu_{c_{43}}(x) = \begin{cases} 
\{\frac{x-11}{3}\} & \text{if } 11 \leq x \leq 14 \\
1 & \text{if } 14 \leq x \leq 15 \\
\{\frac{x-16}{1}\} & \text{if } 15 \leq x \leq 16 \\
0, & \text{otherwise}
\end{cases}
\]

Therefore, \(c_{43} = [x_1^{(\alpha)}, x_2^{(\alpha)}] = [3\alpha + 11, -\alpha + 16]\)

(6.5)

and

\[
\mu_{100}(x) = \begin{cases} 
\{\frac{x-3900}{800}\} & \text{if } 3900 \leq x \leq 4700 \\
1 & \text{if } 4700 \leq x \leq 5100 \\
\{\frac{x-5900}{-800}\} & \text{if } 5100 \leq x \leq 5900 \\
0, & \text{otherwise}
\end{cases}
\]

Therefore, Rs 100 = \([x_1^{(\alpha)}, x_2^{(\alpha)}] = [800\alpha + 3900, -800\alpha + 5900]\)

(6.6)
we can write

fuzzy minimum cost = \[C_{12} + C_{21} + C_{34} + C_{43}\] * 100

(using (6.2),(6.3),(6.4),(6.5) and (6.6))

\[= [8\alpha + 39, -8\alpha + 59] * [800\alpha + 3900, -800\alpha + 5900]\]

\[= [6400\alpha^2 + 62400\alpha + 152100, 6400\alpha^2 - 94400\alpha + 348100]\]

(6.7)

The equations to be solved are

\[6400\alpha^2 + 62400\alpha + 152100 - X_1 = 0\]

(6.8)

\[6400\alpha^2 - 94400\alpha + 348100 - X_2 = 0\]

(6.9)

We are retain only two roots \(\alpha\) in \([0, 1]\)

From (6.8) and (6.9) we get,

\[\alpha = \frac{-62400 + [((62400)^2 - 25600(152100 - X_1)]^{1/2}}{12800}\]

and

\[\alpha = \frac{94400 + [((94400)^2 - 25600(348100 - X_2)]^{1/2}}{12800}\]

Therefore,

\(\mu_{Min.Cost}(x) =\)

\[
\begin{cases}
\frac{-62400 + [(62400)^2 - 25600(152100 - X_1)]^{1/2}}{12800} & \text{if } 3900 \leq x \leq 4700 \\
1 & \text{if } 4700 \leq x \leq 5100 \\
\frac{94400 + [(94400)^2 - 25600(348100 - X_2)]^{1/2}}{12800} & \text{if } 5100 \leq x \leq 5900 \\
0, & \text{otherwise}
\end{cases}
\]
which is the required fuzzy membership functions of fuzzy assignment minimum cost (6.1).

6.5 Results and Discussion

Using the proposed method the total fuzzy assignment minimum cost is $[3900, 4700, 5100, 5900]$, which can be physically interpreted as follows:

1. The least amount of cost is 3900.
2. The most possible amount of cost lies between 4700 and 5100.
3. The greatest amount of cost is 5900.

That is, The optimal fuzzy assignment cost will be always greater than 3900 and less than 5900 and maximum chances are that the cost will be between 4700 and 5100.

The variations in cost with respect to chances are shown in the figure 6.1. Similarly the obtained fuzzy optimal solutions $x_{ij}$ may be physically interpreted.

(i) According to decision maker the total fuzzy assignment minimum cost will be greater than Rs 3900 and less than Rs 5900.

(ii) Decision maker in favour of that the total fuzzy assignment minimum cost will be greater than or equal to Rs 4700 and less than or equal to Rs 5100.

(iii) The percentage of the favourness of the decision maker for the remaining values of total fuzzy assignment minimum cost can be
obtained as follows:

\[ \mu(x) \]

\[ 1 \]

\[ 0 \quad 1000 \quad 2000 \quad 3000 \quad 4000 \quad 5000 \quad 6000 \quad x \]

**Figure: 6.1.** Membership function of fuzzy number representing the total fuzzy assignment minimum cost.

Let \( x \) represent the value of the total fuzzy assignment minimum cost then the percentage of the favourness of the decision maker for \( x = \mu_{\text{min.cost.}}(x) \), Where

\[ \mu_{\text{Min.Cost}}(x) \quad (=) \]

\[
\begin{cases}
\frac{-62400 + [(62400)^2 - 25600(152100 - X)]^{1/2}}{12800} & \text{if } 3900 \leq x \leq 4700 \\
1 & \text{if } 4700 \leq x \leq 5100 \\
\frac{94400 + [(94400)^2 - 25600(348100 - X)]^{1/2}}{12800} & \text{if } 5100 \leq x \leq 5900 \\
0, & \text{otherwise}
\end{cases}
\]