CHAPTER - 7

UNPOLARIZED EMC EFFECT
The distribution of quarks in a nucleus differs significantly from the distribution of quarks in the nucleon. This became clear when nearly three decades ago the EMC measured the ratio of structure functions of Iron to Deuterium in the Deep Inelastic Scattering (DIS) of muons [1]. This discovery gave way to a variety of theoretical models on both the nucleon and nuclear nucleon structure functions. Initially, in general the nuclear dependence is accounted in one of the following ways: conventional nuclear physics employing nucleon’s degrees of freedom, quark clusters [2] and rescaling model [3]. Detailed reviews of data and models of the EMC effect are also available [4]. While many of the models have had some success, they typically reproduce only part of the observed enhancement or suppression, explain a limited range or differ from other measurements. In the present work, the quarks momentum distributions are evaluated in nuclear medium ranging from light nuclei to heavy nuclei as a function of Bjorken variable ‘x’ at fixed value of $Q^2 = 5\text{GeV}^2$ using phenomenological model of statistical distribution known as the Thermodynamical Bag Model (TBM) [5-7]. Based on the
evaluation, the unpolarized EMC effect is studied in nuclei such as $^9$Be, $^{27}$Al, $^{40}$Ca, $^{63}$Cu, $^{108}$Ag and $^{197}$Au and results are compared with experimental values obtained in deep inelastic scattering by SLAC-E139 and NMC. The quenching effect observed in momentum distribution of nuclear matter proves significance of present study.

### 7.1: Statistical distribution functions in rest frame

The rest frame of compound objects is taken to be the frame of reference in which the average momentum of the particles which make up the substance is zero (the particles may individually have momentum, but collectively have no net momentum). The rest frame of a container of gas, for example, would be the rest frame of the container itself, in which the gas molecules are not at rest, but are no more likely to be traveling in one direction as another. Since the quarks are fermions, it is appropriate to use the Fermi-Dirac statistical distribution function to describe their motion in the rest frame of the nucleon. For the gluons we use Bose-Einstein statistics. Treating the quarks as particles of zero rest mass, the number of u-quarks with momentum lying between $p_r$ and $p_r + dp_r$ is

$$n_u(p_r) = \frac{6V_r}{(2\pi)^3} \frac{1}{e^{(E_r - \mu_u)/T} + 1},$$

(7.1)
where $V_r$ denotes the nucleon volume, $T$ the temperature, $\epsilon_r$ the energy, and $\mu_u$ the chemical potential for $u$-quark – all measured in the rest frame of the nucleon. The degeneracy factor 6 is the number of degrees of freedom (3 due to color and 2 due to spin) available to each flavor of quarks. Similar equations can be written for the $d$-quarks and antiquarks. The chemical potential for the $d$-quark is in general different from that of $u$-quark and it is distinguished by suffix indicating the flavor. The chemical potential for the antiquark is of opposite sign to the chemical potential for the quark. Hence

\begin{align}
 n_d(p_r) &= \frac{6V_r}{(2\pi)^3} \frac{1}{e^{(\epsilon_r-\mu_u)/T} + 1} \\
 n_\bar{u}(p_r) &= \frac{6V_r}{(2\pi)^3} \frac{1}{e^{(\epsilon_r-\mu_d)/T} + 1} \\
 n_\bar{d}(p_r) &= \frac{6V_r}{(2\pi)^3} \frac{1}{e^{(\epsilon_r+\mu_u)/T} + 1}
\end{align}

The temperature $T$ and the chemical potentials $\mu_u, \mu_d$ can be considered as the Lagragian multipliers which are determined by invoking the energy conservation and the number conservation of the valence quarks for the composite nucleon system. The approach of constructing the Lagrangians and setting its gradient to zero is known as the method of Lagrange multipliers. In mathematical optimization [8,9], the method of Lagrange
multipliers (named after Joseph Louis Lagrange) provides a strategy for finding the maxima and minima of a function subject to constraints. For the gluons, there is no number conservation and hence the chemical potential for the gluon is zero. The distribution function for the gluons is

\[
n_g(p_r) = \frac{16V_r}{(2\pi)^{3}} \frac{1}{e^{E_r/T} - 1}
\]

The degeneracy factor for the gluons is 16, of which 8 is due to the color degree of freedom and 2 due to spin.

7.2: Transformation to Infinite Momentum Frame (IMF)

A frame of reference [10] is a system of measuring axes used by an observer to measure the surrounding space by providing coordinates. A moving frame is then a frame of reference which moves with the observer along a trajectory. The method of the moving frame, in this simple example, seeks to produce a "preferred" moving frame out of kinematic properties of the observer.

The DIS of leptons on nucleons is characterized by two kinematical variables \( x \) and \( Q^2 \), of which the Bjorken variable \( x \) acquires a nice physical interpretation in the parton model of the nucleon in the IMF. In IMF, \( x \) denotes the fraction of the momentum carried by the parton. So it becomes necessary to investigate how the statistical distribution functions
transform when one goes from the rest frame to the IMF. For this, the
method of Mac and Ugaz [11] is followed and the parton distribution
functions in IMF are obtained after integrating over transverse
components of the momentum. Let the nucleon be moving along the
z-axis with very high momentum. In the moving frame, the Fermi
distribution function of the u-quark can be written as

\[ n_u(p) = \frac{6V_m}{(2\pi)^3} \frac{1}{e^{(e-\mu_{u,m})/T_m} + 1} \]  

(7.6)

where \( p \) and \( \epsilon \) denote momentum and energy of quark, \( \mu_{u,m} \) the
chemical potential of u-quark, \( V_m \) volume of the nucleon and \( T_m \) the
temperature – all measured in moving frame. The exponential factor is a
dimensionless quantity and is invariant under Lorentz transformation.
Therefore,

\[ \frac{\epsilon_r - \mu_u}{T} = \frac{\epsilon - \mu_{u,m}}{T_m} \]  

(7.7)

Since the direction of motion of the nucleon is the z-axis and quarks are
treated as particles with zero rest mass, the energy of the quark in the
moving frame can be expressed in terms of its longitudinal and transverse
momenta \( p_z \) and \( p_t \).

\[ \epsilon = p = \left( p_z^2 + p_t^2 \right)^{1/2} \approx p_z \left[ 1 + \frac{p_t^2}{2p_z^2} \right]. \]  

(7.8)
Taking the scalar product of the four-vector of the nucleon with that of quark,

\[ M \epsilon_r = E \epsilon - P p_z, \quad (7.9) \]

where \( E \) denotes the energy, \( M \) the mass and \( P \) the momentum of nucleon in IMF. From (7.9), it follows that

\[ \frac{\epsilon_r}{T} = \frac{E \epsilon - P p_z}{MT}. \quad (7.10) \]

Now some simple algebraic manipulations can be done using (7.8). The longitudinal momentum \( p_z \) of quark is fraction \( x \) of nucleon momentum \( P \). Making the substitutions,

\[ p_z = xP \quad (7.11) \]

\[ y = \frac{E p_t^2}{2MTxP}, \quad (7.12) \]

We obtain

\[ \frac{\epsilon E}{MT} = \frac{p_z E}{MT} \left(1 + \frac{p_t^2}{2p_z^2}\right) = \frac{x EP}{MT} + y \quad (7.13) \]

Equation (7.10) now becomes

\[ \frac{\epsilon_r}{T} = \frac{x EP}{MT} + y - \frac{xP^2}{2T} + y. \quad (7.14) \]

Further the volumes transform under Lorentz transformation. Mac and Ugaz obtained the formula for the transformation of volume \( V \) in rest frame to volume \( V_m \) in moving frame.
Substituting the relations (7.14) and (7.15) in equation (7.6), the Fermi distribution function in IMF becomes,

\[ n_u(p) = \frac{6VM}{(2\pi)^3 P} \frac{1}{e^{(c+y)} + 1}, \]  

where \( \varepsilon = \frac{xM}{2T} - \frac{\mu_u}{T} \)  

An integration is to be performed over transverse momentum in order to obtain distribution function in terms of longitudinal momentum.

\[ n_u(p_z) = \int n_u(p) d^2 p_t. \]  

From (7.12) it follows that

\[ d^2 p_t = 2\pi p_t dp_t = 2\pi \pi dy, \]  

with

\[ a = \frac{MTxP}{E}. \]

Evaluating the integral

\[ \int \frac{dy}{e^{(c+y)} + 1} = \ln \left( 1 + e^{-c} \right), \]

and invoking that in IMF, and normalizing \( P = 1 \) we get the quark momentum distribution function as

\[ u(x) = \frac{6M^2 VxT}{4\pi^2} \ln \left[ 1 + \exp \left( \frac{1}{T} \left( \mu_u - \frac{xM}{2} \right) \right) \right]. \]
Hereafter, the quark distribution functions will be denoted by their flavor for simplicity. Similar expressions for the distribution functions for the $d, \overline{u}$ and $\overline{d}$ quarks are obtained. Thus the quark distribution functions or in general term the Parton Distribution Functions (PDFs) are given by

\[
\begin{align*}
\pi_u(x) &= \frac{6M^2V_xT}{4\pi^2} \ln \left[ 1 + \exp \left\{ \frac{1}{T} \left( \mu_u - \frac{xM}{2} \right) \right\} \right] \\
\pi_d(x) &= \frac{6M^2V_xT}{4\pi^2} \ln \left[ 1 + \exp \left\{ \frac{1}{T} \left( \mu_d - \frac{xM}{2} \right) \right\} \right] \\
\pi_{\overline{u}}(x) &= \frac{6M^2V_xT}{4\pi^2} \ln \left[ 1 + \exp \left\{ \frac{1}{T} \left( - \mu_u - \frac{xM}{2} \right) \right\} \right] \\
\pi_{\overline{d}}(x) &= \frac{6M^2V_xT}{4\pi^2} \ln \left[ 1 + \exp \left\{ \frac{1}{T} \left( - \mu_d - \frac{xM}{2} \right) \right\} \right]
\end{align*}
\]

The distribution functions for the antiquarks are obtained by simply reversing the sign of the chemical potential ($\mu \rightarrow -\mu$) in the corresponding quark distribution functions.

Following a similar procedure, the gluon distribution function can be obtained by transforming the Bose distribution function to the IMF and accordingly it becomes,

\[
g(x) = \frac{-16M^2V_xT}{4\pi^2} \ln \left[ 1 - \exp \left( - \frac{xM}{2T} \right) \right].
\]

The PDFs given by (7.23) involve the parameters $V, T, \mu_u$ and $\mu_d$. These are not free parameters and they have to be determined by solving
the equations of state for the nucleon imposing the constraints on energy and the number of valence u- and d-quarks. The distribution functions for the u- and d-quarks are depicted in figure 7.1 for the proton for two different sets of parameters, so obtained. For the protons, it is seen that u-quarks dominate over the d-quarks whereas for the antiquarks, the opposite feature is observed. The dominance of the $\bar{u}$-quarks over the $\bar{d}$-quarks, a feature so essential for the explanation of the Gotfried Sum Rule (GSR)-comes out naturally in this formalism. This feature is observed because the chemical potential $\mu_u$ is greater than $\mu_d$ as a consequence of 2 valence u-quarks and 1-valence quark for the proton.

Since the PDFs have been obtained by transforming the Fermi distribution functions from the rest frame to the IMF, it is to be expected that their integrals should yield the total number of quarks of a particular flavor. For instance,

$$N_u = \int_0^1 n_u(p_r) d^3 p_r$$

where $N_u$ denotes the total number of u-quarks and $p_r$ denotes momentum in the rest frame. Similar expressions can be written for $N_d, N_{\bar{u}}$ and $N_{\bar{d}}$. These relations have been verified by numerical integration for a given set of parameters $T, V, \mu_u$ and $\mu_d$. Calculations have been performed with the following two sets of parameters:
Set I: $T = 85.9 \text{ MeV}, V = 51.02 \text{ fm}^3, \mu_u = 39.9 \text{ MeV}, \mu_d = 20.3 \text{ MeV}.$

Set II: $T = 84.9 \text{ MeV}, V = 25.69 \text{ fm}^3, \mu_u = 76.7 \text{ MeV}, \mu_d = 40.6 \text{ MeV}.$

It has been observed that the number of quarks of a particular flavor obtained by performing the integration in the rest frame or in the IMF is essentially the same for the proton yielding the value 2 for the $u$-valence quarks $(N_u^v)$ and 1 for the $d$-valence quarks $(N_d^v)$. In IMF, the values obtained are slightly less since the distribution functions extend to a small extent beyond the physical region $x=1$ due to the neglect of second order terms in the transformation of the Fermi statistical distribution function from the rest frame to the IMF. If the upper limit of the integration is increased slightly beyond $x=1$, we obtain once again the correct number of valence quarks. This ensures the correctness of the quark distribution function obtained by transformation of the Fermi statistical distribution function from the rest frame to the IMF.

The PDFs (7.23) in IMF involve the mass $M$ whereas the Fermi statistical distribution functions do not depend explicitly on $M$. The effect of changing $M$ in the quark distribution functions is investigated and presented in figure 7.2 as an example. The different values of mass $M$ chosen are 0.9384 GeV, 1.5 GeV, 2.5 GeV and 3.5 GeV. The other parameters chosen are the same as given in figure 7.1. It is found that as
the value of $M$ is increased, the curve shift towards the lower value of $x$ but the area enclosed by the curve remains a constant and is equal to the number of quarks of that flavor. This is a very important observation and this has suggested the formulation of the thermodynamical bag model to obtain the parton distribution functions of the correct asymptotic behavior.

7.3: Nucleon structure functions

In the parton model, the quark distribution functions serve as the basis for the study of nucleon structure functions. Just as measurements of elastic form factors provided us with information on the size of the proton, measurements on the inelastic structure function at large $Q^2$ reveal the quark structure of the proton. The distribution function of the quark momenta is denoted by $q_i(x)$ and it is the expectation value of the number of quarks of type $i$ in the hadron whose momentum fraction lies within the interval $[x, x + dx]$. The momentum distribution of antiquarks is denoted by $\bar{q}_i(x)$. Instead of parameterized [12-15] structure function $F_2$, the sum of the momentum distributions weighted by $x$ and $z_i^2$ is used [16] and hence

$$F_2(x) = x \sum_i z_i^2 [q_i(x) + \bar{q}_i(x)]$$

(7.24)
The sum in the above relation runs over the charged partons and hence for proton and neutron the structure functions are given by

\[
F_2^p(x) = x \left[ \left( \frac{2}{3} \right)^2 \{ u(x) + \bar{u}(x) \} + \left( \frac{1}{3} \right)^2 \{ d(x) + \bar{d}(x) \} \right] \\
F_2^n(x) = x \left[ \left( \frac{1}{3} \right)^2 \{ u(x) + \bar{u}(x) \} + \left( \frac{2}{3} \right)^2 \{ d(x) + \bar{d}(x) \} \right]
\] (7.25) (7.26)

These equations include the contributions of both valence and sea quarks to the nucleon structure functions. To assess the contributions from valence quarks alone, the following equations are used.

\[
F_2^p(x) \bigg|_{\text{valence}} = x \left[ \left( \frac{4}{9} \right) u_v(x) + \left( \frac{1}{9} \right) d_v(x) \right] \\
F_2^n(x) \bigg|_{\text{valence}} = x \left[ \left( \frac{1}{9} \right) u_v(x) + \left( \frac{4}{9} \right) d_v(x) \right]
\] (7.27) (7.28)

where \( u_v(x) = u(x) - \bar{u}(x) \) and \( d_v(x) = d(x) - \bar{d}(x) \). In TBM, it is verified that at small \( x \), the sea quarks begin to contribute and dominate as \( x \) decreases.

It is rather quite obvious to assume that a deuteron is to a good approximation just a free proton and free neutron. The binding of the two nucleons is so weak that it can be neglected if compared to the binding of the nucleons in heavier nuclei. Thus the ratio of the two structure functions should demonstrate the influence of the nucleon binding on the
momentum distribution of quarks in nucleon. Structure functions taken on heavy targets are always compared to those on a deuterium target in order to minimize the corrections due to the isospin. Hence the structure function of the deuteron $F_2^D$ is assumed as the sum of the proton and neutron structure functions and is given by

$$F_2^D \approx F_2^p + F_2^n. \quad (7.29)$$

7.4: Nuclear structure functions and EMC effect

The nucleus is a bound system of hadrons. The partons are confined within these hadrons. The momentum distribution of partons in a bound state plays prominent role in structure function evaluation of nuclear medium. In terms of single nucleon contribution, nuclear structure function is

$$F_2^A(x) = \int \frac{d^3p}{(2\pi)^3} s(p)F_2^N(x)$$

where $F_2^N(x)$ is single nucleon structure function, $s(p)$ is the spectral function that gives a measure of the four momentum distribution of nucleons in nucleus. $s(p)$ is extracted from quasi-free (ee’p) process and in terms of single particle levels, the function is written as

$$s(p) = \sum_\lambda \left| \phi_\lambda(p) \right|^2 2\pi \delta(x - p^+/m)$$
where \( \phi_j(p) \) is the single particle wave function in momentum space. This spectral function \( s(p) \) is normalized to allow for the number of nucleons for a given nucleus as

\[
\frac{1}{(2\pi)^4} \int s(p) d^4 p = A.
\]

In the convolution model, the calculation of \( F^A_z(x) \) becomes

\[
F^A_z(x) = \int dz f^A(z) F^N_z(x/z) \tag{7.30}
\]

where \( f^A(z) \) describes momentum and energy distribution of nucleons that satisfies normalization conditions and is given by

\[
f^A(z) = \int d^4 p s(p) 2\pi \delta(z - p^+ / m).
\]

For description of \( F^N_z(x/z) \) in terms of quark degrees of freedom, the distance scale \( x \) is modified to the rescaling variable \( x/z \), which increases with mass number \( A \). In order to calculate the effect of nuclear binding on the structure function, the momentum spectrum of target nucleons has to be evaluated. In simple Fermi gas model, momentum distribution is constant up to the maximum Fermi momentum \( k_f \) and is zero above \( k_f \). The momentum distribution inside the nucleus within the Fermi momentum can be written as

\[
f^A(z) = (3/4)(M/k_f)^3 \left[ (k_f/M)^2 - (z - \eta)^2 \right], \text{ for } 1 - k_f/M < z < 1 + k_f/M \tag{7.31}
\]
In present analysis, initially u- and d-quarks distributions in nuclear media are evaluated. The momentum distributions are then studied. Based on the convolution model, the theoretical evaluation of quarks distributions in nuclear matter is carried out by using

\[
u^A(x) = \int f^A(z) u(x/z) dz,
\]

\[u^A(x) = \int f^A(z) u(x/z) dz,
\]

and

\[d^A(x) = \int f^A(z) d(x/z) dz.
\]

Finally the structure function ratio of various nuclei to deuterium are obtained for \(Q^2 = 5 GeV^2\) as

\[
\frac{F_2^A}{F_2^D} = \frac{\int f^A(z) F_2^N (x/z) dz}{\int F_2^D (x)}.
\]

7.5: DIS and resonance region

In general in Deep Inelastic Scattering (DIS) region, where the final square of the invariant hadronic mass \(W\) is given by \(W^2 > 3 GeV^2\), the \(Q^2\) dependence of the structure functions is predicted by perturbative QCD. The additional scaling violations, target mass corrections and also higher twist effects are said to occur at lower \(Q^2\) and \(W^2\) values. Hence data initially in the resonance region would not innocently be expected to have the same EMC effect as that of the DIS region. As the resonance involves instant excited states, the effect of nuclear medium on these immediate nuclear excitations becomes significant compared to DIS
region. Inside the nuclear environment though the resonance production shows different effects, a deeper connection between DIS and resonance regions cannot be ruled out. But EMC effect still remains a fascinating and mysterious one [17-20]. The success of $\xi$–scaling region opens up an interesting possibility. In the Bjorken limit, the parton model predicts that the structure functions will scale, and that the scaling curves are directly related to the quark distributions. At finite (but large) $\nu$ and $Q^2$, scaling is observed and it is therefore assumed that the structure functions are sensitive to the quark distributions. It is not clear that this assumption must be correct, but the success of scaling is taken as a strong indication that it is true. In nuclei, we see a continuation of the DIS scaling even where the resonance strength is a significant contribution to the structure function. This opens up the possibility of measuring quark distributions in nuclei at lower $Q^2$ or higher $x$. If one requires that measurements be in the deep inelastic regime (typically defined as $W^2 > 4\text{GeV}^2$, where $W^2$ is the invariant mass squared of the final hadron state), data at large values of $x$ can only be taken at extremely high values of $Q^2$. Because the quark distributions become small at large $x$, and the cross section drops rapidly with $Q^2$, it can be very difficult to make these high-$x$ measurements in the DIS region. However, the observation of $\xi$-scaling indicates that one
might be able to use measurements at moderate values of $Q^2$, where the contributions of the resonances are relatively small compared to the DIS contributions and where these contributions have the same behavior (on average) as the DIS. We also concentrate on EMC effect in resonance region to compare with the series of electron scattering experiments in Hall C at J Lab. Nachtmann variable $\xi$ is related to Bjorken variable $x$ by means of the following relation

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2/Q^2}}. \quad (7.35)$$

The quarks momentum distributions are evaluated in nuclear medium such as Iron and Gold as a function of Nachtmann variable ‘$\xi’ in the resonance region $1.2 < W^2 < 3\text{GeV}^2$ and also $Q^2 < 5\text{GeV}^2$ using TBM. The results of Iron and Gold from our measurements are compared with J Lab E89-008 data taken in the above resonance region.

7.6: Analysis of outcomes

Initially momentum distributions of u- and d-quarks in nuclear media as a function of Bjorken variable ‘$x’ are evaluated. In nuclear matter from $^9\text{Be}$ to $^{197}\text{Au}$, in the order of increasing mass number $A$, it is observed that for $x > 0.3$, quenching of momentum distribution occurs. Hence the momentum distribution of quarks in a free nucleon is higher
than that of a bound nucleon in the region of $0.3 < x < 0.7$. Figures 7.3 and 7.4 represent the comparison of momentum distribution of u-quark with that of a bound u-quark, in two nuclear media $^{9}$Be and $^{197}$Au. These two nuclei were studied by SLAC and they also show a wide variation of $A$. Figure 7.5 shows the corresponding effect for the case of d-quark distribution in the case of $^{9}$Be. These three figures clearly indicate the momentum distribution of bound quarks differing from that of a free quark of a nucleon and the quenching of momentum distribution especially in the EMC region gains significance.

The momentum carried by u-quark is higher than that of d-quark in the same bound nucleon. Also we observe that as mass number increases the quenching of momentum distribution increases. This is observed from the figure 7.6.

The ratio of the quark distributions in nuclear medium to the corresponding quark distribution in a free nucleon ($u^A/u$ and $d^A/d$) also show nuclear quenching effect on individual quark flavors. The nature of the curves resembles EMC effect. Figures 7.7 and 7.8 show the ratios of momentum distribution for the cases of nuclear media $^{9}$Be and $^{56}$Fe. As $^{56}$Fe has been taken into account not only by EMC but also SLAC for experimental studies, figure 7.8 deals with $^{56}$Fe. This model approach for EMC effect yields the findings of the observations of Stanford Linear
Accelerator Center Experiment (SLAC E-139) and New Muon Collaboration (NMC).

Hence the theoretical evaluation of structure function ratios using TBM is plotted in the figure along with SLAC-E139 and NMC data. The experimental data of available averaged $Q^2$ values [21-23] are compared with values of TBM at $Q^2 = 5 \text{ GeV}^2$. The figures from 7.9 to 7.15 depict the theoretically evaluated values compared with the experimentally available data of $^9\text{Be}$, $^{27}\text{Al}$, $^{40}\text{Ca}$, $^{63}\text{Cu}$, $^{108}\text{Ag}$ and $^{197}\text{Au}$. From the analysis of our theoretical approach a close agreement with experimental data is observed. The results of TBM for various nuclear media show appreciable comparison with other related findings [24-31]. A slight increase at low value of ‘$x$’ may be attributed to the sea quark and gluon contribution. Unlike other models, TBM has an inbuilt mechanism for sea quarks even at small $x$, especially at $x < 0.1$, and shows the dominance of sea quarks and gluons. At medium range ($x > 0.3$) the EMC effect is observed and at higher values ($x > 0.7$) the steady increase may be attributed to Fermi motion. At medium $x$ ($0.3 < x < 0.7$), the behavior shows a higher fraction of momentum being carried by the valence quarks in the nucleon than in the nuclear medium. Thus the slight increase in the high fractional momentum carried by a free nucleon (as observed from Deuterium) at medium $x$, show the occurrence of EMC
effect. The comparison of EMC effect in different nuclei using TBM with data of SLAC shows that an increase in atomic number has a dip at medium $x$ and also an increase in the copious production of sea quarks and gluons at low $x$.

Table 7.1 gives the ratio of the structure function for iron to deuterium as measured by the BCDMS collaboration [31] using a 200 GeV muon beam. The results are compared to TBM values.

The areas enclosed by the curves of $u(x)$ and $d(x)$ remain a constant and are equal to the number of quarks of that flavor. This observation leads to the formulation of TBM to obtain the quark distribution functions of the correct asymptotic behavior. The value of $\int_{-k_f}^{+k_f} f^A(z)dz = 1$ is also verified and it proves the conservation of momentum. The difference between $x$ and $\xi$ is often ignored in high energy scattering or at low $x$, but cannot be ignored at large $x$ or low $Q^2$ to obtain the conservation of valence quark numbers. The EMC ratio of Iron to Deuterium calculated using Nachtmann variable $\xi$ for $Q^2 = 4\text{GeV}^2$ is compared with the extracted data [26] from Jefferson Lab ($1.2 < W^2 < 3\text{GeV}^2$, $Q^2 \approx 4\text{GeV}^2$) and is given in Table 7.2. Figure 7.16 shows ratio of structure functions of Iron to Deuterium using TBM as a function of Nachtmann variable.
From the observation, the data are in close agreement with previous measurements of the EMC effect. The crossover point, where the structure function ratios become larger than unity, occurs at larger $\xi$ for Gold than Iron. At large $\xi$ the EMC effect is dominated by Fermi motion. This is consistent with J Lab and DIS SLAC data. Slight deviation in Au may be attributed to the deviation from isoscalar configuration. We conclude that the TBM is a very successful model in explaining the quark momentum distributions which is used for extracting the nuclear effects.
REFERENCES


