CHAPTER - 6

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The Deep Inelastic Scattering (DIS) of leptons on nucleons indicates that the nucleon consists of three valence quarks, sea quarks and gluons, confined within a small volume. Based on this observation, several phenomenological models [1-3] have been proposed to obtain the parton distribution functions in the valence quark region but they fail miserably in the small \( x \) (sea and gluon) region, to describe which the sea quarks and gluons have to be put in by hand. This artificial input is avoided in this statistical based model for the nucleon, developed by Ganesamurthy et al [4-5]. It is a modified form of MIT bag consisting of quark-gluon gas.

The theoretical concept of a Fermi-gas may be applied for systems of weakly interacting fermions, i.e. particles obeying Fermi-Dirac statistics leading to the Pauli Exclusion Principle. The free electron gas is an example of solid state physics, where electrons move quasi-freely in a background of positively charged ions. In nuclear physics, protons and neutrons are considered as moving quasi-freely within the nuclear volume. The binding potential is generated by all nucleons. In a simple
picture of the nucleus, protons and neutrons are distinguishable fermions and are therefore situated in two separate potential wells. Hence the Fermi distribution function is used for describing the quarks and the Bose distribution function for describing the gluons. The number density defines the number per unit volume.

6.1: Number density of valence quarks

Treating the quarks as particles of zero rest mass, the number density of u-quarks with momentum lying between \( p \) and \( p+dp \) at temperature \( T \) is given by the Fermi distribution function.

\[
n_u(p) = \frac{g}{(2\pi)^3} \frac{1}{e^{(\epsilon - \mu_u)/T} + 1}
\]  

(6.1)

where \( \epsilon \) is the energy and \( \mu_u \) the chemical potential of the u-quark. The degeneracy factor \( g \) is 6 which is the number of degrees of freedom (3 due to color and 2 due to spin) available for each flavor of quarks. Similar equations can be written for the d-quarks and the antiquarks. The chemical potential for the d-quark \( \mu_d \) is different from that u-quark. The chemical potential for the antiquark is of opposite sign to the chemical potential of the quark.

The chemical potential is a fundamental quantity in statistical mechanics which characterizes the many-particle systems in thermal
equilibrium [6]. In the second law of thermodynamics, the change in total energy \(dU\) of the system exchanging the number of particles \(dN\) with the reservoir at a temperature \((T)\) and under pressure \((P)\) is given by 
\[dU = T\, dS - P\, dV + \mu\, dN,\]
where \(dS\) is the change in entropy by \(T\), \(dV\) is the change in volume by \(P\) and \(\mu\) is the chemical potential. When \(S\) and \(V\) of the system are fixed, then \(\mu\) is defined as 
\[
\mu = \left. \frac{dU}{dN} \right|_{S,V}.
\]
In the ground state, \(S\) vanishes and \(\mu\) is obtained under the condition that the number of particles in volume \(V\) does not depend on temperature [7]. The chemical potential is the energy necessary to add one particle to the system without changing both the entropy and volume. The quantum theory in solids and low-dimensional systems is essentially the physics deduced from the ideal Fermi gas model where the physical properties at a finite temperature are expressed in terms of the Fermi Dirac distribution function characterized by \(\mu\). At absolute zero, the chemical potential equals the Fermi energy. Various techniques are described in the literature [8] for evaluating \(\mu\) of the non-interacting Fermi gas at low-temperature limit.

With this observation, one can write down the distributions functions for d-quarks and also for antiquarks of u and d [9] as,
\[ n_d(p) = \frac{g}{(2\pi)^3} \frac{1}{e^{(e-\mu_d)/T} + 1} \] (6.2)

\[ n_\pi(p) = \frac{g}{(2\pi)^3} \frac{1}{e^{(e+\mu_\pi)/T} + 1} \] (6.3)

\[ n_\bar{\pi}(p) = \frac{g}{(2\pi)^3} \frac{1}{e^{(e+\mu_\pi)/T} + 1} \] (6.4)

From the distribution functions, the number density of each flavor of quarks is obtained by integration over the momentum.

\[ n_u = \int n_u(p) \, d^3p, \quad n_d = \int n_d(p) \, d^3p, \quad n_\pi = \int n_\pi(p) \, d^3p, \quad n_\bar{\pi} = \int n_\bar{\pi}(p) \, d^3p \]

For the proton, the number of u-valence quarks is 2 and the number of d-valence quarks is 1. If \( V \) is the volume of proton, then

\[ (n_u - n_\pi)V = 2; \quad (n_d - n_\bar{\pi})V = 1. \]

In Thermodynamical Bag Model, the above valence quark content can be evaluated as follows:

\[ n_u = \frac{6}{(2\pi)^3} \int \frac{1}{e^{\beta(e-\mu_u)/T} + 1} \, d^3p \] (6.5)

\[ n_\pi = \frac{6}{(2\pi)^3} \int \frac{1}{e^{\beta(e+\mu_\pi)/T} + 1} \, d^3p \] (6.6)

where \( \beta = 1/T \) and the volume integral \( d^3p = 4\pi \, p^2 \, dp \). Their difference can be found out as follows:

\[ n_u - n_\pi = \frac{3}{\pi^2} \left[ \int_0^\infty \frac{p^2 \, dp}{e^{\beta(p-\mu_u)} + 1} - \int_0^\infty \frac{p^2 \, dp}{e^{\beta(p+\mu_\pi)} + 1} \right] \] (6.7)
Substituting \( x = \beta(p - \mu_u) \), one can find \( p = \frac{1}{\beta}(x + \beta\mu_u) \).

Similarly, substituting \( y = \beta(p + \mu_u) \), we get \( p = \frac{1}{\beta}(y - \beta\mu_u) \). With these substitutions in (6.7) one gets,

\[
\begin{align*}
\frac{n_u - n_{\pi}}{\pi^2} &= \frac{3}{\beta^3} \left[ \int_{-\beta\mu_u}^{\infty} \frac{(x + \beta\mu_u)^2}{e^x + 1} - \int_{0}^{\infty} \frac{(y - \beta\mu_u)^2}{e^y + 1} \right] \\
\frac{n_u - n_{\pi}}{\pi^2} &= \frac{3}{\beta^5} \left[ \int_{-\beta\mu_u}^{0} \frac{(x + \beta\mu_u)^2}{e^x + 1} + \int_{0}^{\infty} \frac{(x + \beta\mu_u)^2}{e^x + 1} \\
&\quad \quad \quad + \int_{0}^{\beta\mu_u} \frac{(y - \beta\mu_u)^2}{e^y + 1} - \int_{0}^{\infty} \frac{(y - \beta\mu_u)^2}{e^y + 1} \right].
\end{align*}
\]

The integrals can be evaluated pair wise, taking second and fourth together and then first and third together. Therefore,

\[
\begin{align*}
\int_{0}^{\infty} \frac{(x + \beta\mu_u)^2}{e^x + 1} - \int_{0}^{\infty} \frac{(y - \beta\mu_u)^2}{e^y + 1} &= \\
\int_{0}^{\infty} \frac{(x^2 + \beta^2\mu_u^2 + 2x\beta\mu_u)}{e^x + 1} - \int_{0}^{\infty} \frac{(y^2 + \beta^2\mu_u^2 - 2y\beta\mu_u)}{e^y + 1} &= 4\gamma \beta \mu_u \int_{0}^{\infty} \frac{x}{e^x + 1}.
\end{align*}
\]

Putting \( x = y \), since they are dummy variables,

\[
\frac{\pi^2}{3} \beta \mu_u, \text{ since } \int_{0}^{\infty} \frac{x}{e^x + 1} = \frac{\pi^2}{12}.
\]
\[ \int_{-\beta u}^{0} \frac{(x + \beta \mu_u)^2}{e^x + 1} \, dx + \frac{\beta \mu_u}{e^y + 1} \int_{0}^{\beta \mu_u} (y - \beta \mu_u)^2 \, dy \]

\[
= \int_{-\beta u}^{0} \frac{(x + \beta \mu_u)^2}{e^x + 1} \, dx - \int_{0}^{\beta \mu_u} \frac{(x + \beta \mu_u)^2}{e^{-x} + 1} \, dx, \quad \text{putting } y = -x,
\]

\[
= \int_{-\beta u}^{0} \frac{(x + \beta \mu_u)^2}{e^x + 1} \, dx + \int_{-\beta u}^{0} \frac{(x + \beta \mu_u)^2}{e^{-x} + 1} \, dx,
\]

\[
= \int_{-\beta u}^{0} \frac{(x + \beta \mu_u)^2}{e^x + 1} \left( \frac{1}{e^x + 1} + \frac{e^x}{e^x + 1} \right) dx
\]

since \( \frac{1}{e^{-x} + 1} = \frac{e^x}{e^x + 1} \).

\[
= \int_{-\beta u}^{0} (x + \beta \mu_u)^2 \, dx
\]

\[
= \frac{1}{3} (\beta \mu_u)^3
\]

(6.11)

Substituting the results (6.11) and (6.10) in (6.9), one immediately gets

\[
\mu_u - n_\pi = \frac{3}{\pi^2} \beta^2 \left[ \frac{\pi^2}{3} \beta \mu_u + \frac{1}{3} (\beta \mu_u)^3 \right]
\]

\[
= \frac{\mu_u}{\beta^2} + \frac{\mu_u^3}{\pi^2}
\]

Substituting \( \beta = \frac{1}{T} \)

\[
n_u - n_\pi = \mu_u T^2 + \frac{\mu_u^3}{\pi^2}
\]

(6.12)
By the same procedure one can obtain for d-quark,

\[ n_d - n_{\bar{d}} = \mu_q T^2 + \frac{\mu_d^3}{\pi^2} \]  \hspace{1cm} (6.13)

The degeneracy factor for the gluons is 16, of which 8 is due to color degree of freedom and 2 due to the transverse components of spin.

**6.2: Energy density of quark flavors**

In a similar way, one can find the energy density \( \varepsilon_q \) of each flavor of quarks and antiquarks \((u,d,\bar{u} \text{ and } \bar{d})\) and each of their contributions.

Since the quarks are said to have zero rest mass, the energy of the quark is numerically equal to its momentum in the natural units \((\hbar = c = 1) \varepsilon = p\),

\[ \varepsilon_q = \int \frac{6}{(2\pi)^3} \frac{p}{e^{(p-\mu_q)/T} + 1} d^3 p \]  \hspace{1cm} (6.14)

\[ \varepsilon_q = \int \frac{6}{(2\pi)^3} \frac{p}{e^{(p+\mu_q)/T} + 1} d^3 p \quad q = u, d \]  \hspace{1cm} (6.15)

The sum of energy densities of \(u\) and \(\bar{u}\), treated as quarks of zero rest mass is

\[ \varepsilon_u + \varepsilon_{\bar{u}} = \int_0^\infty \frac{6}{(2\pi)^3} \frac{p}{e^{\beta(p-\mu_u)} + 1} d^3 p + \int_0^\infty \frac{6}{(2\pi)^3} \frac{p}{e^{\beta(p+\mu_u)} + 1} d^3 p \]

\[ = \frac{3}{\pi^2} \left[ \int_0^\infty \frac{p^3}{e^\beta(p-\mu_u) + 1} dp + \int_0^\infty \frac{p^3}{e^\beta(p+\mu_u) + 1} dp \right] \]  \hspace{1cm} (6.16)
Substituting \( x = \beta(p - \mu_u) \), one can find easily that \( p = \frac{1}{\beta}(x + \beta\mu_u) \).

Similarly, substituting \( y = \beta(p + \mu_u) \), one gets \( p = \frac{1}{\beta}(y - \beta\mu_u) \). With these substitutions in (6.16), it becomes

\[
\varepsilon_u + \varepsilon_{\pi} = \frac{3}{\pi^2} \left[ \frac{1}{\beta^4} \int_{-\beta\mu_u}^{\infty} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx + \frac{1}{\beta^4} \int_{-\beta\mu_u}^{\infty} \frac{(y - \beta\mu_u)^3}{e^y + 1} dy \right] 
\]

(6.17)

\[
\varepsilon_u + \varepsilon_{\pi} = \frac{3}{\pi^2} \beta^4 \left[ \int_{0}^{\infty} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx + \int_{0}^{\infty} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx 
+ \int_{0}^{\infty} \frac{(y - \beta\mu_u)^3}{e^y + 1} dy - \int_{0}^{\infty} \frac{\beta\mu_u}{e^y + 1} dy \right] 
\]

(6.18)

The integrals can be evaluated pair wise, taking first and third together and then second and fourth together. Therefore,

\[
\int_{0}^{\infty} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx + \int_{0}^{\infty} \frac{(y - \beta\mu_u)^3}{e^y + 1} dy = \int_{0}^{\infty} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx + \int_{0}^{\infty} \frac{(x - \beta\mu_u)^3}{e^x + 1} dx 
\]

by putting \( y = x \).

\[
= 2 \int_{0}^{\infty} \frac{x^3}{e^x + 1} dx + 6 \beta^2 \mu_u \int_{0}^{\infty} \frac{x}{e^x + 1} dx 
\]

\[
= \frac{7}{60} \pi^4 + \pi^2 \beta^2 \mu_u^2 
\]

(6.19)

after using the values of standard integrals.
Similarly the other pair of terms can be evaluated.

\[
\int_{-\beta\mu_u}^{0} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx - \int_{0}^{\beta\mu_u} \frac{(y - \beta\mu_u)^3}{e^y + 1} dy
\]

\[
= \int_{-\beta\mu_u}^{0} \frac{(x + \beta\mu_u)^3}{e^x + 1} dx + \int_{0}^{-\beta\mu_u} \frac{(x + \beta\mu_u)^3}{e^{-x} + 1} dx \quad \text{after substituting } y = -x.
\]

\[
= \int_{-\beta\mu_u}^{0} (x + \beta\mu_u)^3 \left( \frac{1}{e^x + 1} + \frac{1}{e^{-x} + 1} \right) dx
\]

\[
= \int_{-\beta\mu_u}^{0} (x + \beta\mu_u)^3 dx
\]

\[
= \frac{\beta^4 \mu_u^4}{4}, \quad \text{since } \frac{1}{e^{-x} + 1} = \frac{1}{e^x + 1}.
\]

Substituting the results of (6.19) and (6.20) in (6.18) one obtains

\[
\varepsilon_u + \varepsilon_{\bar{u}} = \frac{3}{\pi^2 \beta^4} \left[ \frac{7}{60} \pi^4 + \frac{\pi^2}{2} \beta^2 \mu_u^2 + \frac{\beta^4 \mu_u^4}{4} \right].
\]

Substituting \( \beta = \frac{1}{T} \)

\[
\varepsilon_u + \varepsilon_{\bar{u}} = \frac{7}{20} \pi^2 T^4 + \frac{3}{2} \mu_u^2 T^2 + \frac{3 \mu_u^4}{4 \pi^2}
\]

6.3: Energy density of gluon

Gluons are elementary particles which act as the exchange particles (or gauge bosons) for the color force between quarks, analogous to the exchange of photons in the electromagnetic force between two
charged particles. Figure 6.1 shows its representation by Feynman’s diagram. The gluon is a vector boson; like the photon, it has a spin of 1. While massive spin-1 particles have three polarization states, massless gauge bosons like the gluon have only two polarization states because gauge invariance requires the polarization to be transverse. In quantum field theory, unbroken gauge invariance requires that gauge bosons have zero mass. The gluon has negative intrinsic parity.

B.E statistics was introduced for photons by Bose and generalized to atoms by Einstein. The expected number of particles in an energy state \( i \) for B.E. statistics is

\[
 n_i = \frac{g_i}{e^{(\epsilon_i - \mu)/kT} - 1}
\]

with \( \epsilon_i > \mu \) and where \( n_i \) is the number of particles in state \( i \), \( g_i \) is the degeneracy of state \( i \), \( \epsilon_i \) is the energy of the \( i \)-th state, \( \mu \) is the chemical potential, \( k \) is the Boltzmann constant, and \( T \) is absolute temperature. Using the Bose-Einstein distribution for the gluons, we can write down its energy density as

\[
\epsilon_g = \int \frac{16}{(2\pi)^3} \frac{p}{e^{p/\mu} - 1} d^3 p,
\]

(6.21)

Since \( d^3 p = 4\pi p^2 dp \),
\[ \epsilon_g = \frac{8}{\pi^2} \int_0^\infty \frac{p^3 \, dp}{e^{\beta p} - 1} \]

Using the standard integral \( \int_0^\infty \frac{x^3 \, dx}{e^x - 1} = \frac{\pi^4}{15} \) the gluon energy density becomes, Substituting \( \beta = \frac{1}{T} \)

\[ \epsilon_g = \frac{8}{15} \pi^2 T^4. \]

The energy density due to all the quarks and gluons is the sum,

\[ \epsilon = \epsilon_u + \epsilon_{\bar{u}} + \epsilon_d + \epsilon_{\bar{d}} + \epsilon_g. \]

\[ \epsilon(T) = \frac{3}{4\pi^2} (\mu_u^4 + \mu_{\bar{u}}^4) + \frac{3}{2} T^2 (\mu_u^2 + \mu_{\bar{d}}^2) + \frac{37}{30} \pi^2 T^4 \]

(6.22)

This is a function of temperature.

### 6.4: Equations of state for thermodynamical bag

In physics and thermodynamics, an equation of state is a relation between state variables. More specifically, an equation of state is a thermodynamic equation describing the state of matter under a given set of physical conditions. It is a constitutive equation which provides a mathematical relationship between two or more state functions associated
with the matter, such as its temperature, pressure, volume, or internal energy. Equations of state are useful in describing the properties of fluids, mixtures of fluids, solids, and even the interior of stars.

In TBM equations of state for the proton can be written as follows:

$$\varepsilon(T)V + BV = W$$  \hspace{1cm} (6.23)

$$n_u - n_{\bar{u}} = \frac{2}{V} = \mu_u T^2 + \frac{\mu_u^3}{\pi^2}$$  \hspace{1cm} (6.24)

$$n_d - n_{\bar{d}} = \frac{1}{V} = \mu_d T^2 + \frac{\mu_d^3}{\pi^2}$$  \hspace{1cm} (6.25)

$$P = \frac{1}{3} \varepsilon(T) - B = 0.$$  \hspace{1cm} (6.26)

So, this bag model describes the nucleon not only in the ground state \(T=0\) but also in the excited states at higher temperature. Hence it can be truly described as the *Thermodynamical Bag Model* of the nucleon. The bag constant is denoted by \(B\) and the volume of the bag by \(V\). The mass of the nucleon \(M\) in this model corresponds to \(T=0\) and \(W\) denotes the mass of the excited nucleon at some finite temperature \(T\). The equation \(P = \frac{1}{3} \varepsilon(T) - B = 0\) arises from the pressure balance condition or the energy minimization condition with respect to the bag volume.

Equations (6.24) and (6.25) specify that the number of u-valence quarks is 2 and the number of d-valence quark 1 for a proton. The four
equations (6.23) – (6.26) determine uniquely the four quantities, given the mass of the nucleon \( M = 938.4\text{MeV} \) and the temperature of the system \( T = 0 \).

The temperature dependence of bag constant is obtained by using minimization condition for energy with respect to the bag radius. It is known that energy of a heated excited spherical baryon with radius \( R \) is given by [11]

\[
E = \frac{C}{R} + BV + DT^4V
\]

where \( C = \sum_i \omega_i n_i \) with \( n_i \) being the occupation number of the quark in state \( \omega_i \).

\[
\left. \frac{\partial E}{\partial R} \right|_{R_0} = 0 = -\sum_i \frac{\omega_i n_i}{R_0^2} + 4\pi R_0^2 B + 4\pi R_0^2 DT^4
\]

\[
B = \frac{1}{4\pi R_0^2} \left[ \sum_i \frac{\omega_i n_i}{R_0^2} - 4\pi R_0^2 DT^4 \right]
\]

\[
B = B_0(T) \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]
\]

where \( B_0(T) = \sum_i \frac{\omega_i n_i}{4\pi R_0^4} \) and \( T_C = \left( \frac{B_0(T)}{D} \right)^4 \).

Therefore, the bag constant is known to decrease as the temperature increases as given by the relation,
Here $T_C$ refers to the critical temperature determined from the pressure balance equation (6.26) by imposing the condition that the chemical potential vanishes at the critical temperature

$$T_C = \left( \frac{90B}{37 \pi^2} \right)^{1/4} \approx 102.6 \text{ MeV}. \quad (6.28)$$

The invariant mass $W$ of the final hadronic state depends only on $x$ and $Q^2$ that characterize the DIS. In this model, assuming the value of the bag constant, the parameters $T, \mu_u, \mu_d, V$ are determined uniquely, by solving the four equations of state (6.23) – (6.26) self-consistently. Figure 6.2 shows the temperature dependence of Bag constant and figure 6.3 shows the temperature dependence of the chemical potential. The thermodynamical bag is intuitively appealing, since it suggests an intimate relation between $x$ and $T$. The smaller the value of $x$, the greater is the temperature, thereby explaining the copious production of sea quarks and gluons at smaller values of $x$. The theory obtained above is free from any arbitrary parameter and all the dynamical variables of the bag are obtained by solving the bag equations self-consistently.

For example, with $M = 938.4$ MeV at $T = 0$, the four quantities determined self-consistently from the four equations of state are:
\[ \mu_u = 335.9 \text{ MeV}, \mu_d = 266.6 \text{ MeV}, B^{\frac{1}{4}} = 145.68 \text{ MeV}, R = 0.985 \text{ fm}. \]

In particle physics, helicity \( h \) is the projection of the spin \( \vec{s} \) on the direction of momentum \( \vec{p} \).

\[ h = \vec{s} \cdot \vec{p} \quad \text{and} \quad \vec{p} = \frac{\vec{p}}{|\vec{p}|}. \]

If a particle’s spin vector points in the same direction as the momentum vector, the helicity is positive. If they point in opposite directions, the helicity is negative. The helicity of a particle is a Lorentz invariant. Because the eigenvalues of spin with respect to an axis have discrete values, the eigenvalues of helicity are also discrete. For a particle of spin \( S \), the eigenvalues of helicity are \( S, S-1, \ldots, -S \). The measured helicity of a spin \( S \) particle will range from \(-S\) to \(+S\). For massless spin-1/2 particles, helicity is equivalent to the chirality operator multiplied by \( \frac{\hbar}{2} \). Devanathan and McCarthy [5] extended the thermodynamical bag model to include the helicity states of the quarks. Then there are six equations of state and the number of variables to be determined also increased by two.

\[ \varepsilon(T)V + BV = W \]

\[ 3V \left[ n^\uparrow_{u} + n^\downarrow_{u} \right] - \left[ n^\uparrow_{d} + n^\downarrow_{d} \right] = 2 \quad (6.29) \]
\[
3V \left[ \left( n_\uparrow^d + n_\downarrow^d \right) - \left( n_\uparrow^u + n_\downarrow^u \right) \right] = 1
\] (6.30)

\[
3V \left[ \left( n_\uparrow^u + n_\uparrow^\pi \right) - \left( n_\uparrow^\pi + n_\uparrow^\pi \right) \right] = a
\] (6.31)

\[
3V \left[ \left( n_\uparrow^d + n_\uparrow^\pi \right) - \left( n_\uparrow^\pi + n_\uparrow^\pi \right) \right] = b
\] (6.32)

\[
P = \frac{1}{3} \varepsilon(T) - B = 0.
\]

The multiplicative factor 3 denotes the color degeneracy. The energy density of the quark-gluon gas can be written explicitly in terms of the energy densities of the constituents.

\[
\varepsilon(T) = 3 \left( \varepsilon_\uparrow^u + \varepsilon_\downarrow^u + \varepsilon_\uparrow^\pi + \varepsilon_\downarrow^\pi \right) + 3 \left( \varepsilon_\uparrow^d + \varepsilon_\downarrow^d + \varepsilon_\uparrow^\pi + \varepsilon_\downarrow^\pi \right) + 16 \varepsilon_g.
\]

Equations (6.31) and (6.32) have a deep physical meaning. They denote the separate spin contributions from u and d quarks to the spin of the proton. If the entire spin of the proton arises from the spin of the constituent quarks, then \( a + b = 1 \). Choosing \( a = 1.1 \) and \( b = -0.7 \) such that \( a + b = 0.4 \), the model explains completely the spin structure of both the proton and the neutron and yields the Bjorken sum rule accurately.

Using the above equations of state for the bag, the parton distribution functions are obtained.

This model was used by Ganesamurthy [13] to study the nucleon-nucleon potential in terms of quarks and the static properties of hadrons. This model offers a clear insight into the transition of state properties of
the nucleon into its dynamical properties [14-15], as observed in DIS of leptons.

The doubts that may be raised against TBM and the clarifications are as follows:

1. It may be argued that it is unreasonable to identify the final state of the hadronic system with an excited bag since a conventional parton picture suggests a different configuration such as string like one for the final hadronic system. Parton picture being consistent with the factorization theorem of perturbative QCD, one parton (quark or antiquark) in the target hadron (or nucleus) is scattered off with large $Q^2$. Since the scattered parton carries a color while $Q^2$ is large, it is most probable that a string like color dipole configuration may be formed as an intermediate state of the final hadronic system.

   But the excited bag of TBM will have more or less spherical shape and will have no appreciable color dipole. Also, the formation of excited bag by TBM implies that thermal equilibrium is realized in the DIS process. It may be asked that whether this is consistent with the fact that the relevant QCD coupling constant is small at large $Q^2$. Even if it were possible, the equilibrium process would have taken a time which is much longer than the characteristic time of the initial hard process. Then the
distributions of the final hadronic system described by TBM will have no
direct relevance to the structure functions of the initial hadron.

The persuasive arguments are as follows: The parton distribution
function is a function of $x$ and $Q^2$. Once the parton distribution is
obtained the nucleon structure functions and the cross sections are
calculated using the model. All the consequences of the parton picture -
one parton in the target medium being scattered off with large $Q^2$ and the
string like configuration of the final hadronic system will follow. Some
empirical formulae are available for the parton distribution functions in
the literature. The TBM is only an attempt to deduce the parton
distribution functions by phenomenological model. Once the parton
distribution functions are obtained, all the consequences of the parton
picture will follow without any change. However it is to be admitted that
the TBM used to obtain the parton distribution function cannot be carried
over to explain the mechanism of DIS for which the impulse
approximation coupled with the parton model are most suited.

2. One may raise a question regarding the convolution procedure
adopted in TBM and the physical significance of the parameter $\eta(Q^2)$.

The explanation is as follows: The parton distribution is shown to
depend on $x$ and $Q^2$ of the probe. The invariant mass of the final hadronic
state $W$ is also a function of $x$ and $Q^2$. So in some way the parton distribution should depend on the invariant mass of the final hadronic state. The TBM envisages such a connection. Here we make the important postulate that the temperature $T$ can be determined by identifying the invariant mass $W$ of the final hadronic state with the mass of the excited nucleon. Since the invariant mass $W$ depends upon $x$, there is an intimate relation between the Bjorken variable $x$ and the temperature $T$ of the thermodynamical bag. The smaller the value of $x$, the greater is the temperature, thereby explaining the copious production of sea quarks and gluons in the region of small $x$. Since the quarks are treated as Fermi gas, it is appropriate to choose the Fermi distribution to describe the distribution of quarks inside the nucleon. Usually, the Fermi distribution function is given for fixed $T$, but here, each point of the distribution function corresponds to different values of $T$. This disturbs the normalization and the renormalization is done by introducing the single parameter $\eta(Q^2)$. The resultant distribution function is obtained by recognizing the dual role of $x$. ($x$ as the inelasticity parameter in the laboratory frame and $x$ as the fraction of the nucleon momentum carried by the quark in the infinite momentum frame) and it is the realistic distribution function corresponding to the ground state of the nucleon. It
exhibits the correct asymptotic behavior. It vanished as $x \to 1$ and shows Regge behavior as $x \to 0$. A single parameter $\eta(Q^2)$ is used in the renormalization procedure where $\eta(Q^2) = \eta(Q_0^2) \left[ 1 - a \ln \left( \frac{Q^2}{Q_0^2} \right) \right]$ with $\eta(Q_0^2 = 4 GeV^2) = 0.625$ and $a = 0.09$. The variation of $\eta$ with $Q^2$ is similar to the QCD evolution equation (e.g. Altarelli-parisi equation). So there is no need to use the QCD evolution here.

3. The authors such as Clemens and Thews, mac and Ugaz assume that a nucleon is a bag that contains quark-gluon gas at a finite effective temperature and calculate the distributions of the constituents in the infinite momentum frame to obtain the structure functions of the nucleon. The result is in reasonable agreement with the experimental data only in the large $x$ region. On the other hand there is big deficit of the sea quarks in the small $x$ region. The wrong $x$ behavior may be rectified by identifying the distributions with those given at some small $Q^2 = Q_0^2$, the initial value of the QCD evolution equation in $Q^2$. These initial distributions are then sued to calculate the distributions at large $Q^2$ by using the QCD evolution equations to get the correct $x$-dependence with sufficient amount of sea quarks.
The comments on such a viewpoint are as follows: Cleymans & Thews, Mac and Ugaz chose the Fermi distribution function at some arbitrary temperature $T$, which yields to some extent the shape of the quark distribution function but not the absolute values. Large QCD corrections are invoked to reduce the calculated values to almost half their values to agree with the expected distribution function. Also their asymptotic behaviors as $x \to 1$ and $x \to 0$ are wrong. To overcome these defects, TBM is proposed only to obtain the quark distribution function.

In summary, the naive use of the Fermi distribution function for quark distribution is found to be inadequate. Instead the Fermi distribution should be modulated in accordance with the structure of the hadronic current-current interaction $W_{\mu\nu} \sim \sum_{h} \langle p | J^\dagger_{\mu} | h \rangle \langle h | J_{\nu} | p \rangle$, where the summation is overall the possible final hadronic states of the invariant mass $W$. Since the hadronic tensor is expressed in terms of the nucleon structure functions, it is very suggestive that the quark distribution functions which describe the DIS should have the above inbuilt structure.

To conclude, it is to be reiterated that the TBM is used only to obtain the quark distribution functions which include the effect of quark interactions. The thermal equilibrium envisaged in the TBM is only for the purpose of deducing the quark distribution functions and nor for
explaining the mechanism of lepton interaction with the nucleon. For the later, we retain the original parton model and the DIS of lepton on nucleon is treated as the inherent sum of elastic scattering of lepton on quarks. In an inelastic event, this allows the energy transfer to a single quark which eventually hadronized to form a jet or more than one jet if it emits gluons. Thus the phenomenological model known as the thermodynamical bag model is described to explain the features observed in the deep inelastic scattering of leptons on nucleons. This model is highly successful in explaining the wealth of experimental data of both unpolarized and polarized nucleon structure functions.
REFERENCES


**Figure 6.1:** Feynman diagram representing gluon by spiral.

**Figure 6.2:** Bag constant as a function of Temperature.
Figure 6.3: Chemical potential as a function of Temperature.