Chapter - V

Simple-4D Hyperchaotic Canonical Van der Pol Duffing Oscillator Using Current Feedback Op-Amp
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SIMPLE-4D HYPERCHAOTIC CANONICAL VAN DER POL DUFFING OSCILLATOR USING CURRENT FEEDBACK OP-AMP

5.1. INTRODUCTION

Of late, application of nonlinear circuits for secure communication purposes has become an active area of theoretical and experimental investigations. In secure communication a signal from a simple chaotic system is used to mask the message to be transmitted. Simple chaotic systems possess only a single positive Lyapunov Exponent (LE). Perez and Cerderia [212] have shown that messages masked by such simple chaotic systems are not always safe. Once intercepted, there is a possibility that the message can be easily extracted using well-known nonlinear signal processing techniques. However, Pecora [213] has suggested that this problem can be overcome by using higher dimensional hyperchaotic systems, which have increased randomness and higher unpredictability. Hyperchaos is defined as the dynamics associated with a chaotic attractor with more than one positive Lyapunov exponents, that is, its dynamics expands not only as a line segment (one-dimensional expansion) but also as a small area (volume) element (two-dimensional expansion). Due to this, hyperchaos improves security and high quality synchronization by giving rise to more complex time signals which are apparently not vulnerable to the unmasking procedures. Therefore, interest in hyperchaos is increasing due to their possible application in improving secure communications [Peng et al. [214]; Lakshmanan and Murali [14]; Tamasevicius et al. [215]; Blakely et al. [216]]. Hyperchaos was first reported by Rossler [61] from the computer simulation experiments of ordinary differential
equations modeling chemical reactions. The experimental realization of this phenomenon from hardware electronic systems was first observed by Matsumoto et al. [62]. This was a fourth-order nonlinear electronic circuit. The above works were followed later by experiments using a higher-dimensional electronic circuit and hysteresis (Mitsubori and Saito [217]), two resonant circuits coupled by a diode (Tamasevicius et al. [218]), and an autonomous fourth-order circuit serially connected with zener diodes (Saito [219]). Hyperchaos has also been reported from numerical simulations and experiments in coupled low dimensional circuits by Tamasevicius et al. [181,220], Lakshmanan and Murali [14], a chain of coupled Chua’s circuits by Kapitaniak and Chua [221,222], coupled logistic maps by Kaneko [223], coupled three-dimensional maps by Harrison and Lai [224] and very recently in a two dimensional map by Kapitaniak et al. [225]. Of all the circuits and systems mentioned above, the autonomous canonical Chua’s circuit introduced by Chua and Lin [152] is of considerable importance. This is because it is capable of realizing the behavior of every member of the Chua’s family (Chua and Lin [152,153]; Kyprianidis et al. [226]). It consists of two active elements, one linear negative conductor, one nonlinear resistor with symmetric piecewise linear $v$-$i$ characteristics.

In original canonical Chua's circuit, a nonlinear resistor is called Chua's diode is the unique nonlinear electric element. It plays an important role in the circuit. Due to the existence of this nonlinear element Chua’s circuit exhibits a variety of nonlinear phenomena, such as chaos, bifurcation and so on [62,153,227-229]. The characteristic of Chua's diode is described by a continuous piecewise-linear function with three segments and two non-differential break points [152,154,180,181].
However, the characteristics of nonlinear devices in practical circuits are always smooth and the implementation of piecewise-linear function requires a large amount of circuitry compared with smooth cubic function. Therefore, it is significant to investigate Chua's circuit with a smooth cubic nonlinearity from practical viewpoint [184]. Hartley proposed to replace the piecewise-linear nonlinearity in Chua’s circuit with a smooth cubic nonlinearity.

In the present report the behavior of a fourth-order autonomous hyperchaotic Canonical Van der Pol Duffing oscillator circuit has been studied. This circuit consists of two active elements, one linear negative conductance and smooth cubic nonlinearity exhibiting a symmetrical piecewise-linear \(v-i\) characteristic. Two inductances \((L_1, L_2)\) two capacitances \((C_1, C_2)\) and one locally active resistor \((R)\) is also included in the circuit, serve as the control parameters.

Hyperchaos is defined as a chaotic attractor with more than one positive Lyapunov exponents, i.e., its dynamics expand in more than one direction [5]. In other words, the dynamics expand not only small line segments, but also small area elements, there giving rise to a ‘thick’ chaotic attractor. Most hyperchaotic and bifurcation effects cited in the literature have been observed in electric circuits. They include the period-doubling route to chaos, the intermittency route to chaos, the quasiperiodicity route to chaos and of course the crisis [154,213,218,230]. This popularity is attributed to the advantages which electric circuits offer to experimental hyperchaos studies, such complicated hyperchaotic wave forms are expected to be utilized for realization of several hyperchaotic applications such a chaos communication system with robustness against various interferences including multi-user
The plan of the paper is as follows. In section 5.2, we present the details of realization of the proposed autonomous circuit. The results of the observations from the laboratory experimental simulation and the conformation through analytical calculation and numerical simulation on the dynamics of the circuit are presented in section 5.3. Finally, in section 5.4, we summarize and conclude the results and indicate further direction.

### 5.2. CIRCUIT DESCRIPTION AND EXPERIMENTAL RESULTS

The fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit we have studied is presented in Fig. 5.1. It consists of two active elements, one linear negative conductance ($G_j$) using current feedback op-amp and smooth cubic nonlinearity with an odd symmetric piecewise-linear $v-i$ characteristic [184]. This fourth-order circuit is based on a third-order autonomous piecewise-linear circuit introduced by Chua and Lin, capable to realize every member of the canonical Chua’s circuit family [152]. Applying Kirchoff’s laws, the set of four first-orders coupled autonomous differential equations as given below:

\[
\begin{align*}
C_1 \frac{dV_1}{dt} &= G_1V_1 - i_{t_1} - f(V_1 - V_2) \\
C_2 \frac{dV_2}{dt} &= i_{t_1} - i_{t_2} + f(V_1 - V_2) \\
L_1 \frac{di_{t_1}}{dt} &= V_1 - V_2 - i_{t_1}R \\
L_2 \frac{di_{t_2}}{dt} &= V_2
\end{align*}
\]  

... (5.1)
Fig. 5.1: Fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit.

While $V_1$ and $V_2$ are the voltages across the Capacitors $C_1$ and $C_2$, $i_{L_1}$ and $i_{L_2}$ denote the currents through the inductances $L_1$ and $L_2$ respectively, the term $f(V_1-V_2)$ representing the characteristic of the smooth cubic nonlinearity can be expressed mathematically:

$$f(V_1-V_2) = a(V_1-V_2) + b(V_1-V_2)^3$$  \hspace{1cm} ... (5.2)

For our present experimental study we have chosen the following typical values of the circuit in Fig. 5.1. Were $L_1 = 350 \, mH$, $L_2 = 100 \, mH$, $C_1 = 10 \, nF$, $C_2 = 100 \, nF$ and $G_1 = -0.14706 \, mS$. Here the variable resistor ‘$R$’ is assumed to be the control parameter. By decreasing the value of ‘$R$’ from 16,000 $\Omega$ to 6,500 $\Omega$, the circuit behavior of Fig. 5.1 is found to transit from a period-doubling route to chaos and then to hyperchaotic attractor through border collision bifurcation behavior followed by period-doubling windows and boundary crisis etc., [186,221,231,232].

The hyperchaotic attractors of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 5.2. Experimental time series were registered using an experimental storage oscilloscope for discrete values of $C_1$ and $C_2$ are shown if Fig. 5.3.
Fig. 5.2: Experimental results of the projections of hyperchaotic attractor onto different planes.
The distribution of power in a signal $x(t)$ is the most commonly quantified by means of the power density spectrum or simply power spectrum. It is the magnitude-square of the Fourier transforms of the signal $x(t)$. It can detect the presence of hyperchaos when the spectrum is broad-banded. The power spectrum corresponding to the voltage $V_1(t)$ waveform across the capacitor $C_1$ for the hyperchaotic regime is shown in Fig. 5.4 which resembles broad-band spectrum noise.

Fig. 5.3: Experimental results of the hyperchaotic time series.

Fig. 5.4: Experimental results of the projections of hyperchaotic power spectrum.
5.3. NUMERICAL CONFIRMATION

The hyperchaotic dynamics of circuit as shown in Fig. 5.1 is studied by numerical integration of the normalized differential equations [232]. For a convenient numerical analysis of the experimental system given by Eq. (5.1), we rescale the parameters as

\[ x_1 = \sqrt{bR} V_1, \ x_2 = \sqrt{bR} V_2, \ x_3 = \sqrt{bR^3} i_x, \ x_4 = \sqrt{bR^3} i_x, \ \tau = \frac{t}{RC_2}, \]

\[ \beta_1 = \frac{C_1 R^2}{L_1}, \ \beta_2 = \frac{C_2 R^2}{L_2}, \ \alpha = aR, \ \gamma = C_1 R, \ \upsilon_1 = \frac{C_2}{C_1}, \ \upsilon_2 = \frac{C_2}{C_2} \]

and then redefine \( \tau \) as \( t \).

Eq. (5.1) and Eq. (5.2) reduce to the following set of normalized equations of the fourth-order Canonical Van der Pol Duffing oscillator circuit as given below:

\[
\begin{align*}
\dot{x}_1 &= \upsilon_1 \left( \gamma x_1 - x_3 - \alpha (x_1 - x_2) - (x_1 - x_2)^3 \right) \\
\dot{x}_2 &= \upsilon_2 \left( x_3 - x_4 + \alpha (x_1 - x_2) + (x_1 - x_2)^3 \right) \\
\dot{x}_3 &= \beta_1 \left( x_1 - x_2 - x_3 \right) \\
\dot{x}_4 &= \beta_2 \left( x_2 \right)
\end{align*}
\]  

... (5.3)

The dynamics of Eq. (5.3) now depends upon the parameters \( \upsilon_1, \ \upsilon_2, \ \gamma, \ \alpha, \ \beta_1, \)

and \( \beta_2 \). The experimental results have been verified by numerical simulation of the normalized Eq. (5.3) using the standard fourth-order Runge-Kutta method for a specific choice of system parameters employed in the experimental simulation results. That is, in the actual experimental set up the resistor ‘\( R \)’ is varied from \( R = 16,000 \ \Omega \) downward to \( 6,500 \ \Omega \). Therefore in the numerical simulation, we study the corresponding Eq. (5.3) for in the range \( R = 16,000 \ \Omega \) to \( 6,500 \ \Omega \). From our numerical investigations, we find that for the value of ‘\( R \)’ above \( 16,000 \ \Omega \) periodic limit cycle motions is obtained. When the value of ‘\( R \)’ is decreased to lower than \( 16,000 \ \Omega \)
particularly in the range $R = (16,000 \ \Omega$ to $6,500 \ \Omega)$ the system displays a period-doubling route to chaos and then to hyperchaos through boundary condition [180]. These numerical results of the hyperchaotic attractor of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 5.5. It is gratifying to note that the numerical results agree qualitatively very well with that of the experimental results. Fig. 5.6 shows the numerical chaotic time series was registered using a discrete value of ‘$R$’ serving as the control parameter.
Fig. 5.5: Numerical results of the projections of hyperchaotic attractor onto different planes.
5.3.1. One parameter Bifurcation diagram and Lyapunov exponents

The main features of the fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit can be summarized in the one parameter bifurcation diagram drawn in the \((R-x)\) plane (Fig. 5.7 (a)). Note that \(x\) is the rescaled variable in Eq. (5.3), \(x = V_2/V\). This bifurcation diagram clearly indicates that in the region \(R = (19,000 \, \Omega, 10,000 \, \Omega)\) the system undergoes period-doubling bifurcation sequence to chaos, shows periodic windows through hyperchaotic region are observed in Fig. 5.1.

The Lyapunov exponent’s \(\lambda_1, \lambda_2, \lambda_3, \lambda_4\) were obtained using the Wolf algorithm. For periodic orbits \(\lambda_1 = 0, \lambda_2, \lambda_3, \lambda_4 < 0\), for quasi-periodic orbits \(\lambda_1 = \lambda_2 = 0, \lambda_3, \lambda_4 < 0\), while for chaotic attractor \(\lambda_1 > 0, \lambda_2 = 0, \lambda_3, \lambda_4 < 0\) and for hyperchaotic attractor \(\lambda_1 > \lambda_2 > 0, \lambda_3 = 0, \lambda_4 < 0\). The Lyapunov spectrum in the \((R-\lambda_1, \lambda_2)\) plane, that is the first two maximal Lyapunov exponents versus fixed range of the control parameter as \(R\) is decreased, is shown in Fig. 5.7 (b). This correlate to the bifurcation diagram, Fig. 5.7 (a). In the range \((16,000 \, \Omega \text{ to } 6,500 \, \Omega)\) the system exhibits periodic windows with no positive Lyapunov exponent. When \(R\) is decreased further in the
range (16,000 \( \Omega \) to 6,500 \( \Omega \)), the system becomes chaotic with a single positive Lyapunov exponent (\( \lambda_1 \)). The chaotic nature is also characterized by a single positive Lyapunov exponent (\( \lambda_4 \)). It is quite fascinating to look at the window region in the range (16,000 \( \Omega \) to 6,500 \( \Omega \)), which corresponds to an entirely different dynamical behavior. It has been observed that for \( R > 6,500 \Omega \) the attractors of the system are in any one of the smooth regions of the piecewise segments. Correspondingly, the attractors exhibit one of the generic types of bifurcations, namely period-doubling, saddle-node, or hopf-bifurcations. A section of the bifurcation diagram and the Lyapunov spectrum for the range (16,000 \( \Omega \) to 6,500 \( \Omega \)) are shown in Figs. 5.7(a) and 5.7(b), respectively, for clarity. The Lyapunov spectrum in the \( (R-\lambda_1, \lambda_2) \) plane, that is the first two maximal Lyapunov exponents versus fixed range of the control parameter as \( R \) is decreased, the hyper chaotic Lyapunov exponents shown in Fig. 5.7(b) for \( R = 6,962 \Omega \) the Lyapunov exponents are \( \lambda_1 = 0.7058396, \lambda_2 = 5.51873E-004, \lambda_3 = -22.0062038 \) and \( \lambda_4 = -32.2491702 \). The numerical computational results of system (Eq. (5.3)) are shown in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
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<tr>
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</tr>
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<td>Hyperchaos</td>
</tr>
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Fig. 5.7(a): For the normalized Eq. (5.3): One parameter bifurcation diagram in the \((R - x)\) plane at fixed range of control parameter of \(R = (19,000 \ \Omega \text{ to } 10,000 \ \Omega)\).

Fig. 5.7(b): For the normalized Eq. (5.3): Two largest Lyapunov exponents versus \(R\) for two trajectories in the \((R - \lambda_1, \lambda_2)\) plane.
5.4. MULTI - SIMULATION RESULTS

In recent years, circuit simulators such as multi-simulation have been used for the simulation of the chaotic circuits. Multi-simulation of the simple-4D hyperchaotic Canonical Van der Pol-Duffing oscillator circuit with a diode pairs (Fig. 5.1) were carried out using \( C_1 = 10 \, nF \), \( C_2 = 100 \, nF \), \( L_1 = 350 \, mH \), \( L_2 = 100 \, mH \), \( R = 10 \, K\Omega \), \( R_L = 6 \, \Omega \), and \( R_N = -6,800 \, \Omega \). For the implementation of the non-linear element \( G_N \), a negative impedance \((-R_N)\) is connected in parallel with a diode pairs (e.g. 1N4148, \( B_p = 0.65 \, V \), \( R_D = 25 \, \Omega \) at 5 mA). Typical phase portraits of the system corresponding to different regimes are shown in Fig. 5.8. There is good agreement between the numerical simulation Fig. 5.5 and the multi-simulation results.
Fig. 5.8: Simulation results of the projections of hyperchaotic attractor onto different planes.
5.5. CONCLUSIONS

We have presented a simple-4D hyperchaotic Canonical Van der Pol Duffing oscillator circuit which has symmetrical piecewise-linear elements. We can confirm hyperchaotic attractor on computer simulation or circuit experiments. The attractive feature of this circuit is the presence of hyperchaotic attractor over a range of parameter values, which might be useful for applications in controlling of hyperchaos, synchronization and in secure communication system.