CHAPTER 8

STABILITY ANALYSIS OF INDUCTION MACHINES

8.1 STABILITY ANALYSIS OF COMPLEX CONTROLLER APPLIED TO THE INDUCTION MOTOR CONTROL

The dynamics of the induction motor is represented by differential equations by using space vector concept and complex transfer function. The induction machine high dynamic performance is achieved by Field Orientation Control (FOC) given in (Blaschke 1972) and Direct Torque Control (DTC) explained by Takashashi & Noguchi (1986). The FOC allows an independent flux and torque control using the direct (d) and quadrature (q) axis components of stator current. DTC controllers generate a stator voltage vector that allows quick torque response with the smallest variation of the stator flux. Based on the induction motor model and complex transfer function, complex controller can be designed with a complex gain.

8.1.1 Direct Field Orientation Control

The direct rotor field orientation control (DFOC) allows to control the flux magnitude and torque by the stator currents. Using the complex notation on DFOC strategy where \(i_{1d}\) is the real component and \(i_{1q}\) is the imaginary component, the PI controller can be substituted by complex gain \((a + j b)\), which generates the voltage reference by using the stator current vector error as discussed by Sguarezi Filho & Ruppert (2008).

The tuning of complex gain controller requires the transfer function of closed loop controller of induction motor. The induction motor controller block diagram is as shown below.
Figure 8.1 Induction motor controller with a complex gain $K$

Where $K = a + jb$ is the complex gain and

$$G(s) = \frac{i_{1dq}}{v_{1dq}}$$

is the open loop transfer function as given below.

$$G(s) = \frac{\tilde{i}_{1dq}}{\tilde{v}_{1dq}} = \frac{(s + j\omega_1) + a_3}{(s + j\omega_1)(s + \frac{R_1}{\sigma L_1} + \frac{R_2}{\sigma L_2}) + R_1 a_3} \quad (8.1)$$

where $a_3 = \left(\frac{R_2}{\sigma L_1} \frac{jP\omega_{me}}{\sigma L_1} \right)$

The closed loop transfer function of the induction motor controller is given by

$$C(s) = \frac{\left(\frac{s + j\omega_1}{\sigma L_1} + \frac{R_2}{\sigma L_1 L_2} - \frac{jP\omega_{me}}{\sigma L_1}\right)(a + jb)}{m(s) + \frac{R_1 R_2}{\sigma L_1 L_2} - jP\omega_{me} + \frac{R_1}{\sigma L_1} + n(s)(a + jb)} \quad (8.2)$$

where $m(s) = (s + j\omega_1)\left(\frac{R_1}{\sigma L_1} + \frac{R_2}{\sigma L_2}\right)$ and

$$n(s) = \left[\frac{(s + j\omega_1)}{\sigma L_1} + \frac{R_2}{\sigma L_1 L_2} - \frac{jP\omega_{me}}{\sigma L_1}\right]$$
The subscripts 1,2 and \( m \) represent the stator, rotor and magnetization parameters respectively. \( \omega_1 \) is the synchronous speed, \( \omega_{mec} \) is the machine speed, \( R_1 \) and \( R_2 \) are the stator and rotor winding electric resistance per phase, \( L_1 \), \( L_2 \) and \( L_m \) are the stator inductance, rotor inductance and mutual inductance of stator and rotor winding respectively. \( \sigma \) is the leakage coefficient and \( P \) is the number of pair of poles of the machine.

The characteristic equation for the system is

\[
\begin{align*}
C(s) &= m(s) + \frac{R_1 R_2}{\sigma L_1 L_2} - j P \frac{\omega_{mec}}{\sigma L_1} \\
& \quad + n(s)(a + jb) \\
& = \frac{s^2}{\sigma L_1 L_m} + \frac{j}{\sigma L_1} \omega_{mec} R_1 \\
& \quad + n(s)(a + jb)
\end{align*}
\]  

(8.3)

8.1.1.1 Illustration

Consider the problem analysed by Sguarezi Filho & Ruppert (2009). For an induction motor using direct field orientation control (DFOC), the value of parameters are given as follows; Check the stability of the system for complex gain of \( K = a + jb = 75 - j 25 \).

\[
\begin{align*}
\omega_1 &= 13 \text{ rad/sec}, \\
\omega_{mec} &= 12.56 \text{ rad/sec} \\
R_1 &= 2.229 \Omega \\
R_2 &= 1.522 \Omega \\
P &= 2 \text{ (Pair of poles)} \\
L_1 &= 0.247 H \\
L_2 &= 0.2497 H \\
L_m &= 0.2384 H, \\
\sigma &= 1 - \frac{L_m^2}{L_1 L_2} = 0.08
\end{align*}
\]

By substituting all the values,

\[
\begin{align*}
m(s) &= s^2 + (189 + j13)s + (0 + j2457) \\
(n(s)(a + jb)) &= (75 + j25)s + (-262.5 - j937.5)
\end{align*}
\]
The characteristic equation for the system is
\[ C(s) = s^2 + (264 - j12)s + (425 - j1313.5) = 0 \]

Stability analysis using SPC-I

\[
\begin{array}{ccc}
+1 & -j12 & 425 \\
+264 & j1313.5 \\
\hline
-j7 & 425 \\
j17286 \\
\end{array}
\]

From the first column it is clear that both sign pairs obey SPC-I and the system is stable.

Stability analysis using SPC-II

\[
\begin{array}{ccc}
0 & 264 & -1313.5 \\
\hline
+1 & -12 & -425 \\
+264 & -1313.5 \\
\hline
-7 & -425 \\
\hline
-17285 \\
\end{array}
\]

Here also both sign pairs obey SPC-I and the system is stable.

**8.1.2 Direct Torque Control**

In DTC, the torque and stator flux become part of a complex number, where the stator flux magnitude \( \lambda_1 \) is the real component and the torque \( T \) is the imaginary component. Hence the reference signal and the error signal become complex numbers and the conventional PI controller can be substituted by a complex gain \((a + jb)\) as given by Sguarezi Filho & Ruppert 2008. The closed loop system representation is given in figure 8.2.
Figure 8.2 Direct torque control of an induction motor by PI controller with complex gain

$G(s)$ remains same for both case. The closed loop transfer function is given by

$$C(s) = \frac{(a + jb)G(s)(\sigma L_1 + j 1.5 P \lambda_1)}{1 + (a + jb)G(s)(\sigma L_1 + j 1.5 P \lambda_1)}$$

(8.4)

The characteristic equation for the closed loop system is

$$1 + (a + jb)G(s)(\sigma L_1 + j 1.5 P \lambda_1) = 0.$$  

(8.5)

This equation also will be of the form of a complex coefficient representation and stability can be analyzed by SPC-I and SPC-II by measuring the stator leakage flux ‘$\lambda_1$’.

8.2 DESIGN OF A DIRECT STATOR CURRENT PI CONTROLLER FOR A DOUBLY FED INDUCTION GENERATOR USING SIGN PAIR CRITERIA

Doubly Fed Induction Generators find a wide range of applications especially in the field of renewable energy as mentioned by Akagi & Sato (2002) and particularly in wind mills. DFIG are capable to handle a range of rotor speeds and they are able to produce power at fixed frequency; also the power electronic conversion is needed only at the lower power level which reduces losses and the cost. Most of the DFIG controllers are based on vector control and decoupling (Tapia et al 2003). A stator flux oriented reference frame is used for the decoupling of the active and reactive power of the stator
side and their independent control through the rotor currents. Another available control scheme the direct torque control is explained by Datta & Ranganathan (2001), in which switching tables are used based on rotor and stator fluxes.

But stability analysis is not done for these schemes. In the control scheme considered here as discussed by Doria Cerezo et al (2013) for the stability analysis, the stator is directly connected to the power grid and the machine is controlled through rotor voltages. The Induction Generator is represented by a model with stator voltage orientation. The linear PI control of the stator currents ensures stability for a large range of PI gain values. The 6th order characteristic polynomial with real coefficients which represents the control scheme is reduced to a cubic polynomial with complex coefficients. The reduction technique is explained by Doria Cerezo et al (2013) and Bodson & Kiselychnyk (2010).

8.2.1 Stability Analysis of DFIG with Feedback Linearising PI Current Controller

The three phase dynamic equations of a DFIG are used to get the model assuming that

i. the machine windings are identical

ii. the stator-rotor mutual inductances are sinusoidal functions of the rotor angle and

iii. the three phase system is balanced

The Blondel-Park transformation (Krause 1986) is used to decouple one of the balanced phases to get a common reference frame for all variables and also to get the state space models. Following standard convention, all two dimensional electrical signals are divided into their ‘d’ and ‘q’ components. The characteristic equation obtained from the state space model representing the linear closed loop system with the PI controller is of 6th order. The sixth
order characteristic polynomial with real coefficients can be reduced into 3\(^\text{rd}\) order polynomial with complex coefficients. The reduced state space model is as shown below Doria-Cerezo et al (2013), Bodson & Kiselychnyk (2010) and Batlle et al (2006)).

\[
A(s) \begin{pmatrix} I_s(s) \\ I_r(s) \\ V_r(s) \end{pmatrix} = \begin{pmatrix} V_s(s) \\ 0 \\ j(k_p s + k_1)I_s(s) \end{pmatrix}
\]

Where,

Stator Current \( I_s(s) = I_{sd}(s) + jI_{sq}(s) \)

Rotor Current \( I_r(s) = I_{rd}(s) + jI_{rq}(s) \)

Stator Voltage \( V_s(s) = V_{sa}(s) + jV_{sq}(s) \)

Rotor Voltage \( V_r(s) = V_{ra}(s) + jV_{rq}(s) \)

\[
A(s) = \begin{pmatrix} L_s s + R_s + j\omega_s L_s & L_{sr} s + j\omega_s L_{sr} & 0 \\ L_{sr} s & L_r s & -1 \\ j(k_p s + k_1) & 0 & s \end{pmatrix}
\]

where \( L_s \) is the stator inductance, \( R_s \) the stator resistance, \( L_r \) the Rotor inductance and \( L_{sr} \) the mutual inductance between the stator and rotor windings which are in general uncertain and time varying parameters as explained by Peresada et al (2004). The stator frequency is represented by \( \omega_s \), \( k_p \) and \( k_1 \) are the PI Controller gains. Due to thermal effects, the value of rotor resistance \( R_r \) is highly varying. By considering this effect, \( R_r \) is replaced by \( R'_r \) in the dynamic equation representing DFIG, the state model given by above equation is slightly modified as follows.

\[
A_r(s) = \begin{pmatrix} L_s s + R_s + j\omega_s L_s & L_{sr} s + j\omega_s L_{sr} & 0 \\ L_{sr} s & L_r s + R'_r & -1 \\ j(k_p s + k_1) & 0 & s \end{pmatrix}
\]

Where \( R'_r = R_r - \hat{R}_r \); \( \hat{R}_r \) is the estimated value of rotor resistance.

The characteristic equation can be obtained by finding the
determinant of $A_r(s)$ which has 3 roots that can lie anywhere in the complex plane. The complex polynomial has the form

$$detA_r(s) = a_0s^3 + (a_1 + jb_1)s^2 + (a_2 + jb_2)s + a_3 + jb_3$$  \hspace{1cm} (8.9)

Where

$$a_0 = \mu = L_sL_r - L_{sr}^2$$
$$a_1 = L_rR_s + L_sR_r'$$
$$b_1 = \omega_s \mu - k_p L_{sr}$$
$$a_2 = k_p \omega_s L_{sr} + R_sR_r'$$
$$b_2 = -k_i L_{sr} + \omega_s L_sR_r'$$
$$a_3 = k_i \omega_s L_{sr}$$
$$b_3 = 0$$

Using SPC-I, the first two rows of Routh like table are formed as given below

<table>
<thead>
<tr>
<th></th>
<th>$a_0$</th>
<th>$j b_1$</th>
<th>$a_2$</th>
<th>$j b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$j b_2$</td>
<td>$a_3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The condition for stability can be found out by finding out the first element of 3\textsuperscript{rd} row itself, which reduces computation and gives a simple procedure. By following the routh multiplication rule, the first element of 3\textsuperscript{rd} row is $j( a_1b_1 - a_0b_2 )$. As per SPC-I, the condition to be satisfied to find the marginal value for controller gain for stability is

$$a_1b_1 = a_0b_2$$  \hspace{1cm} (8.10)

By applying the stability condition ($a_1b_1 = a_0b_2$) as per Sign Pair Criteria I and II,

$$L_rR_s + L_sR_r' (\omega_s \mu - k_p L_{sr}) = \mu (-k_i L_{sr} + \omega_s L_sR_r')$$

$$L_rR_s \mu \omega_s - L_rR_s k_p L_{sr} + L_sR_r' \omega_s \mu - k_p L_{sr} L_sR_r' = -\mu k_i L_{sr} + \mu \omega_s L_sR_r'$$

$$L_rR_s \mu \omega_s - k_p L_{sr} [L_rR_s + L_sR_r'] = -\mu k_i L_{sr}$$
\[ k_1 = \frac{L_r R_s + L_s R'_r}{\mu} k_p - \frac{\omega_s L_r R_s}{L_s r} \]  

(8.11)

The above condition is in agreement with the stability requirement given by Doria Cerezo et al (2013).

With \( \bar{R}_r = 0 \), the control law is as follows

\[ k_1 = \frac{L_r R_s + L_s R'_r}{\mu} k_p - \frac{\omega_s L_r R_s}{L_s r} \]  

(8.12)

The above equality is same as the condition given by Doria Cerezo et al (2013).

8.2.2 Stability Analysis of DFIG with Direct Stator Current PI Controller

The control scheme described in the previous section offers stability for a large range of the PI gain values, but it is necessary to know some machine parameters and both the stator and the rotor currents. In this section a controller which keeps only the PI action is considered, where the rotor currents are not required and only the stator currents need to be measured. In this case a constant mechanical speed \( \omega \) is assumed for the stability analysis as discussed by Doria Cerezo et al( 2013 ) and Bodson & Kiselychnyk 2010)

The new state space model is given below, specified by Doria Cerezo et al( 2013 ), Bodson & Kiselychnyk (2010).

\[ A_{PI}(s) = \begin{pmatrix} L_s S + R_s + j \omega_s L_s & L_{sr} S + j \omega_s L_{sr} & 0 \\ L_{sr} S + j(\omega_s - \omega)L_{sr} & L_r S + R_r (\omega_s - \omega)L_r & -1 \\ j(k_p S + k_1) & 0 & S \end{pmatrix} \]  

(8.13)

Here also the characteristic equation can be obtained by finding the determinant of \( A_{PI} (s) \). The complex polynomial has the form

\[ det A_{PI}(s) = a_0 S^3 + (a_1 + j b_1) S^2 + (a_2 + j b_2) S + a_3 + j b_3 \]  

(8.14)
where
\[ a_0 = \mu \]
\[ a_1 = L_rR_s + L_sR_r \]
\[ b_1 = \mu(2\omega - \omega) - k_pL_{sy} \]
\[ a_2 = R_sR_r + \omega_s(\omega_s - \omega)\mu + k_p\omega_sL_{sy} \]
\[ b_2 = (\omega_s - \omega)L_rR_s + \omega_sL_sR_r - k_I L_{sy} \]
\[ a_3 = k_I \omega_sL_{sy} \]
\[ b_3 = 0 \]

Now applying the stability condition for marginal value of controller gain 
\[ (a_1b_1 = a_2b_2) \] as per SPC-I and SPC-II,
\[
(L_rR_s + L_sR_r)[\mu(2\omega - \omega) - k_pL_{sy}] = \mu[(\omega_s - \omega)L_rR_s + \omega_sL_sR_r - k_I L_{sy}]
\]
\[
(L_rR_s + L_sR_r)[\mu(2\omega - \omega) - (L_rR_s + L_sR_r)k_pL_{sy}]
\]
\[
= \mu[(\omega_s - \omega)L_rR_s + \omega_sL_sR_r] - \mu k_I
\]
\[
\mu[L_rR_s\omega_s + L_sR_r\omega_s - L_sR_r\omega] - k_pL_{sy}[L_rR_s + L_sR_r] = -\mu k_I L_{sy}
\]
\[
k_I = \frac{(L_rR_s + L_sR_r)k_p}{\mu} + \frac{L_sR_r\omega - \omega_s[L_rR_s + L_sR_r]}{L_{sy}}
\]  \hspace{1cm} (8.15)

This control law is in exact match with the stability condition given by

With \( \omega = 0 \), the stability for the proposed PI controller can be ensured by setting
\[
k_I < \frac{(L_rR_s + L_sR_r)}{\mu}k_p - \frac{(L_rR_s + L_sR_r)}{L_{sy}}\omega_s
\]  \hspace{1cm} (8.16)

The above equality is same as the condition given by Doria Cerezo et al (2013).
8.3 ANALYTIC CONDITIONS FOR SPONTANEOUS SELF-EXCITATION IN INDUCTION GENERATORS

Self excitation in induction generators refers to a mode of operation where voltages are generated without a connection of the machine to a voltage source or grid. The analysis of self excitation is important due to the need to protect the machines from over speeding and over voltages when accidentally disconnected. Self-excitation of induction generators is an example of nonlinear dynamic systems.

Instability of the zero equilibrium triggers a departure from the zero state and is referred as spontaneous self excitation. In such cases, the voltages grow exponentially until magnetic saturation is reached, due to small residual magnetization. The transient phenomenon of spontaneous excitation is described by Bodson & Kiselychnyk (2010) and Grantham et al (1989).

\[
A(s) = \begin{pmatrix} \frac{L_ss + R_s}{C_s} & Ms & -1 \\ Ms - jp\omega M & sL_r + R_r - j\rho\omega L_r & 0 \\ 1 & 0 & C_s + Y_L \end{pmatrix}
\] (8.17)

Where \( L_s \) is the stator inductance, \( R_s \) the stator resistance, \( L_r \) the Rotor inductance, \( R_r \) the rotor resistance, \( M \) the mutual inductance between the stator and rotor windings and \( p \) is the number of pole pairs. The generator speed is represented by \( \omega \).

Each stator winding is attached with a load as well as a capacitor \( C \) that is added to provide the required reactive power. The load is assumed to be purely resistive, with resistance \( R_L \). The capacitor \( C \) is connected in parallel with the load. \( Y_L \) is the admittance, where \( Y_L = \frac{1}{R_L} \).

The characteristic equation can be obtained by finding the determinant of \( A(s) \) which has 3 roots that can lie anywhere in the complex plane. The
complex polynomial has the following form as mentioned by Bodson & Kiselychnyk (2010) and Grantham et al (1989)

$$ \det \Lambda(s) = a_0 s^3 + (a_1 - j \omega b_1) s^2 + (a_2 - j \omega b_2) s + a_3 - j \omega b_3 = 0 $$

(8.18)

Where

$$ a_0 = C(L_s L_r - M^2) $$

$$ a_1 = Y_L(L_s L_r - M^2) + C(L_r R_s + L_s R_r) $$

$$ b_1 = a_0 $$

$$ a_2 = Y_L(L_r R_s + L_s R_r) + (C R_s R_r + L_r) $$

$$ b_2 = Y_L(L_s L_r - M^2) + C L_r R_s $$

$$ a_3 = R_r (Y_L R_s + 1) $$

$$ b_3 = L_r (Y_L R_s + 1) $$

For obtaining the analytic conditions for spontaneous excitation, with no load and zero stator resistance, the value of coefficients are as follows.

$$ a_0 = C(L_s L_r - M^2) $$

$$ a_1 = C L_s R_r $$

$$ b_1 = a_0 $$

$$ a_2 = L_r $$

$$ b_2 = 0 $$

$$ a_3 = R_r $$

$$ b_3 = L_r $$

**Stability Analysis Using SPC- I**

The Routh table is formed for the complex coefficient equation as shown below.

\[
\begin{array}{ccc}
  a_0 & -j \omega b_1 & a_2 & -j \omega b_3 \\
  a_1 & 0 & a_3 \\
\end{array}
\]
\[
\begin{array}{ccc}
-jp\omega b_1 & \frac{(a_1 a_2 - a_0 a_3)}{a_1} & -jp\omega b_3 \\
-j(a_1 a_2 - a_0 a_3) & \frac{(a_0 a_3 - a_1 b_3)}{a_0} & -jp\omega b_3 \\
-j(c^2 - (p\omega)^2 d a_0 a_1) & \frac{p\omega a_0 a_1}{p\omega a_0 a_4} & -jp\omega b_3 \\
-jp\omega b_3 & & \\
\end{array}
\]

where \( c = a_1 a_2 - a_0 a_3 \) and \( d = a_0 a_3 - a_1 b_3 \)

From the first column elements, it is clear that \( a_0 > 0, \ a_1 > 0, b_1 > 0 \) and \( b_3 > 0 \) for stability. Using the fifth element, the condition for stability can be written as

\[ c^2 - (p\omega)^2 d a_0 a_1 > 0 \] \text{ or } \[ c^2 > (p\omega)^2 d a_0 a_1 \]

And hence the analytic condition for spontaneous excitation can be written as \( c^2 < (p\omega)^2 d a_0 a_1 \).

\[
(a_1 a_2 - a_0 a_3)^2 < (p\omega)^2 (a_0 a_3 - a_1 b_3) a_0 a_1 \\
(C L_s R_r L_r) - C(L_s L_r - M^2) R_f^2 < (p\omega)^2(C(L_s L_r - M^2) R_r - C L_s R_r L_r)(C(L_s L_r - M^2) C L_s R_r)
\]

Simplifying the above equation,

\[
(C M^2 R_r)^2 < -(p\omega)^2 (CM^2 R_r) C^2 L_s R_r (L_s L_r - M^2) \\
M^2 < -(p\omega)^2 C L_s (L_s L_r - M^2) \\
M^2 (1 - (p\omega)^2 C L_s) < -(p\omega)^2 C L_s^2 L_r,
\]

If \( L_r \) the Rotor inductance is negligible,

The above equation can be written as \( M^2 (1 - (p\omega)^2 C L_s) < 0 \)

That is, \( 1 - (p\omega)^2 C L_s < 0 \)
\[(p\omega)^2 C L_s > 1\]

\[\omega > \frac{1}{p} \sqrt{\frac{1}{C L_s}}\]  

(8.19)

This result is exactly same as that of Grantham et al (1989). From the above equation, the condition for self excitation can be explained such that the electrical frequency corresponding to the mechanical speed must be greater than the natural frequency of the LC circuit composed of the stator inductance and of the capacitor.

8.4 SUMMARY

In this chapter, the stability analysis of Induction machines has been done based on the proposed algebraic criteria SPC-I and SPC-II. An induction motor with complex controller gain is analyzed which is found to be simple compared to the method given by Sguarezi Filho & Ruppert (2009). A direct stator current PI controller is designed for a Doubly Fed Induction Generator (DFIG) using SPC-I and SPC-II which is simple compared to the method explained by Doria Cerezo et al (2013). Also the condition for spontaneous self excitation is derived for an Induction Generator using the same criteria SPC-I and SPC-II. The proposed algebraic criteria are simple and direct in application compared to other schemes explained in (Bodson 2010) and Grantham et al (1989).