CHAPTER-2

Effect Of
electron beam and temperature
anisotropy on Alfven waves
2.1. INTRODUCTION

In the previous chapter, we have reviewed the basic phenomena of magnetosphere-ionosphere coupling pertaining to field-aligned currents and auroral acceleration. Among the first type of waves found to exist in plasma, Alfven waves play an important role in energy transport, in particle acceleration and heating in the earth's magnetosphere, in solar flares and the solar wind. In particular a series of spacecraft have directly detected strong Alfven wave turbulence associated with particle energization in interplanetary space such as the auroral region. The electron beam may be the cause of Alfven wave generation, which modifies the wave frequency. Therefore, in the present chapter we have examined the effect of electron beam on Alfven wave in the anisotropic plasma.

Field-aligned currents play a dominant role in the study of magnetized plasmas of magnetosphere-ionosphere coupling. In the magnetohydrodynamic description (Valid where time and spatial scales of motion are large compared to the gyroperiod and gyroradius, respectively), if perturbations of flow develop on one part of a flux tube, field-aligned currents must flow in order to communicate the charges to the entire flux tube. They are perhaps of most importance in magnetospheric physics in the study of coupling between regions where different dynamical conditions prevail but which are threaded by the same field. Under this condition Alfven wave may be generated in the plasma sheet and propagate towards the ionosphere leading the auroral processes.
In most of the theoretical work the velocity distribution functions have been assumed to be ideal Maxwellian although most turbulent heating experiments like mirror devices, allow non-Maxwellian, particularly loss-cone distribution functions. Thus, the object of the present chapter is to investigate what kind of effects the non-Maxwellian distribution may have on the current driven Alfven waves.

In the recent past particle aspect analysis has been developed for the analysis of Alfven wave (Baronia and Tiwari, 1999) that is based upon Dawson’s theory of Landau damping which was further extended by Terashima (1967), Misra and Tiwari (1979), Tiwari et al. (1985), Varma and Tiwari (1991) to the analysis of electrostatic and electromagnetic instabilities. The present chapter is attributed to investigate the effect of electron beam in anisotropic plasma on the Alfven wave for the magnetosphere-ionosphere coupling.

2.2. BASIC TRAJECTORIES

We consider plasma under static magnetic field $B_0$ (z-direction) in which collisions between particles are neglected. We shall discuss the behaviors of an Alfven wave of plane polarization in the form;

\[ k_{||} B_0, \quad k.B = 0, \quad k = (0,0,k_{||}) \quad \text{and} \quad E = (E_x,0,0) \quad ....(2.1) \]

with

\[ E_x = E_1 \cos (k_{||} z - \omega t) \quad ....(2.2) \]

\[ B_y = \frac{E_1 k_{||} c}{\omega} \cos (k_{||} z - \omega t) \quad ....(2.3) \]
where $E_x$ and $B_y$ are the electric and magnetic fields of the wave. Here, the frequency $\omega$ is assumed to be real and the amplitude $E_1$ is treated as the slowly varying function of $t$. $c$ is the velocity of light

$$
\frac{1}{E_1} \frac{dE_1}{dt} \ll \omega
$$

$B_0$ is directed along the $z$-axis and wave propagates in the direction of ambient magnetic field. The wave is assumed to start at $t = 0$ when resonant particles are not disturbed. We begin with the equation of motion for the particles as

$$
\frac{d}{dt} \mathbf{v} = \frac{1}{m} \left( E_1 \mathbf{x} (B_0 + B_y \hat{y}) \right) + \mathbf{q} \left( \mathbf{E}_1 + \frac{\mathbf{v} \times \mathbf{B}_0}{c} \right)
$$

...(2.4)

where $q$ is the charge and $m$ is the mass of the particle. $\hat{y}$ is the unit vector along the $y$ direction.

The Gaussian system of units is adopted in this chapter and interactions between particles are neglected. The electric field $E_1$ on the right hand side is considered to be a small perturbation and $v$ can be expressed as a sum of the unperturbed velocity $V$ and the perturbed velocity $u$, i.e. $v = V + u$, $u$ is determined by the following set of equations.

$$
\frac{du}{dt} + i\Omega u = \frac{q}{m} \frac{V_{||} k_{||}}{\omega} \left( 1 - \frac{k_{||} z - \omega t}{\omega} \right) E_1 \cos (k_{||} z - \omega t),
$$

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\[
\frac{du_\parallel}{dt} = \frac{q E_\parallel}{m} k_\parallel \quad \cos (\theta - \Omega t) \cos (k_\parallel z - \omega t) \quad \ldots(2.5)
\]

where \( u_+ = u_x + iu_y \) and \( \Omega = qBo / mc \)

We solve eq. (2.5) in the approximation by replacing the coordinates of the particles on the right hand side by those of free gyration, \( u_x \) and \( u_y \) are the perturbed velocities in the \( x \) and \( y \) directions respectively. \( E_1 \) and \( B_y \) are slowly varying quantities and treated as constants. Our method follows that of Terashima (1967). We start taking the trajectories of free gyration as,

\[
z = z_0 + V_\parallel t, \\
V_x(t) = V_\perp \cos (\theta - \Omega t), \\
V_y(t) = V_\perp \sin (\theta - \Omega t), \\
V_z(t) = V_\parallel = \text{constant}, \quad \ldots(2.6)
\]

where \( z_0 \) is the initial position of the particles in the \( z \) direction, \( \theta \) is initial phase of gyration. Eq. (2.5) is solved for perturbed velocities \( u(r, t) \) of charged particles in the presence of Alfvén wave as described by Baronia and Tiwari (1999) which is given as;

\[
u_x(r,t) = \frac{q V_\parallel k_\parallel}{m \cdot \omega} \Lambda_0 \\
\delta \left[ 1 - \frac{\sin (k_\parallel z - \omega t)}{a_0^2} \right] E_1 \left[ \frac{\sin (k_\parallel z - \omega t)}{a_0^2} \right] \\
\left( \frac{\sin (k_\parallel z - \omega t - \Lambda_0 t + \Omega t)}{2(\Lambda_0 - \Omega)} - \frac{\sin (k_\parallel z - \omega t - \Lambda_0 t - \Omega t)}{2(\Lambda_0 + \Omega)} \right)
\]

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\[ u_\parallel(r,t) = \frac{q}{m} \frac{V_\parallel}{\omega} k_\parallel \Lambda_0 \]

\[ \delta \frac{\cos(k_\parallel z - \omega t - \Lambda_0 t + \Omega t)}{2(\Lambda_0 - \Omega)} + \frac{\delta}{2(\Lambda_0 + \Omega)} \]

\[ u_\perp(r,t) = \frac{q}{m} \frac{V_\perp}{\omega} \frac{E_\parallel}{a_0^2} \frac{1}{\omega} \left[ \frac{\sin(k_\parallel z - \omega t + \theta - \Omega t)}{\Lambda_0 - \Omega} \right] \]

\[ + \frac{\delta}{\Lambda_0 + \Omega} \sin(k_\parallel z - \omega t + \Omega t) - \frac{\delta}{\Lambda_0 - \Omega} \sin(k_\parallel z - \omega t - \Lambda_0 t + \theta) \]

\[ \frac{\delta}{\Lambda_0 + \Omega} \sin(k_\parallel z - \omega t - \Lambda_0 t - \theta) \]  

...(2.7)

where \( \delta = 0 \) for the non-resonant particles and \( \delta = 1 \) for resonant one and

\[ \Lambda_0 = V_\parallel k_\parallel - \omega, \quad a_0^2 = \Lambda_0^2 - \Omega^2 \]  

...(2.8)

The resonant particle condition is given by Baronia and Tiwari (1999) as \( V_\parallel k_\parallel - \omega = 0 \).

These equations represent the perturbed velocities of the charged particles in the presence of Alfvén waves and have vast application in plasma heating processes, confinement devices and the space plasmas.
2.3. DENSITY PERTURBATION

To evaluate the density perturbation affected by velocity perturbation due to Alfvén waves, we consider a group of particles with the same initial condition and let its number density be

\[ n(r, t, V) = N(V) + n_1(r, t, V) \quad \ldots \quad (2.9) \]

where \( N \) is the zeroth order distribution and \( n_1 \) is perturbed density which can be derived from the equation [2.9, 2.10]

\[
\frac{dn_1}{dt} = -N(V) \nabla_z u_z \quad \ldots \quad (2.10)
\]

The expression on the right hand side is converted as function of \( t \) and initial parameters (Baronia and Tiwari, 1999) and after integration the non-resonant particle density is given as

\[
n_1(r, t, V) = \frac{q V \cdot E \cdot k_{||}^2 N(V)}{2 \omega_0} \left[ \frac{1}{\Lambda_0 - \Omega} \sin(k_{||} z - \omega t + \theta - \Omega t) + \frac{1}{\Lambda_0 + \Omega} \sin(k_{||} z - \omega t - \theta + \Omega t) \right] \quad \ldots \quad (2.11)
\]

Similarly, for the resonant particles

\[
n_1(r, t, V) = -\frac{q V \cdot E \cdot k_{||}^2 N(V)}{2 \omega_0} \left[ \frac{1}{\Lambda_0 - \Omega} \sin(k_{||} z - \omega t + \theta - \Omega t) \right] + \frac{1}{\Lambda_0 + \Omega} \sin(k_{||} z - \omega t - \Lambda_0 t + \theta) - \frac{1}{\Lambda_0 - \Omega} \sin(k_{||} z - \omega t - \Lambda_0 t - \theta) \quad \ldots \quad (2.11)
\]
\[ \frac{1}{(\Lambda_0 + \Omega)^2} \sin(k_{||}z - \omega t - \Lambda_0 t - \theta) - \frac{t}{(\Lambda_0 - \Omega)} \cos(k_{||}z - \omega t - \Lambda_0 t + \theta) \]

\[ \frac{t}{(\Lambda_0 + \Omega)} \cos(k_{||}z - \omega t - \Lambda_0 t - \theta) \]

\[ \ldots (2.12) \]

To evaluate the dispersion relation and current we can hereafter take the zeroth order distribution \( N(V) \) of the form

\[ N(V) = N_0 f_{\perp}(V_{\perp}) f_{||}(V_{||}) \]

\[ f_{\perp}(V_{\perp}) = \left[ \frac{m}{2\pi T_{\perp}} \right]^{\frac{m}{2}} \exp\left\{ - \frac{m V_{\perp}^2}{2 T_{\perp}} \right\} \]

\[ f_{||}(V_{||}) = \left[ \frac{m}{2\pi T_{||}} \right]^{1/2} \exp\left\{ - \frac{m(V_{||} - V_0)^2}{2 T_{||}} \right\} \]

where \( T_{||} \) and \( T_{\perp} \) are the parallel and perpendicular temperatures with respect to the ambient magnetic field, \( V_0 \) is beam velocity and \( N_0 \) is the background plasma density.

2.4. CURRENT

To evaluate the perturbed current per unit wavelength in the presence of Alfv\'en wave, we use the following set of equations

\[ J_{le} = \int_0^\lambda ds \int_0^\infty 2\pi V_{\perp} dV_{\perp} j_{le} dV_{||} e \left[ (N + n_i) (V + u) - NV \right]_{le} \]

and \( J_{le} = J_l - J_e \)

\[ \ldots (2.14) \]

where \( ds \) represents the length of magnetic field line elements and \( ks = k r \).
Eq. (2.14) considers current per unit wavelength not the current density per square centimeter (Terashima, 1967; Baronia and Tiwari, 1999,2000). The right hand side terms of equation (2.14) involve density perturbation $n_i$ and the velocity perturbation $u$, which are containing the oscillatory terms. If the integral $\int ds$ is not performed the evaluated current will be oscillatory. To find out the average values, the current is evaluated per unit wavelength. These currents represent the distributed currents over the entire wavelength and eq. (2.14) represents the average value of this distributed current over a wavelength of the Alfven wave.

With the help of equations (2.7), (2.11) and (2.14) we obtain the ionic current per unit wavelength as

$$J_{zi} = - \frac{3e\omega_{pe}^2 E_i^2 k_i^2 V_{Ti,i}^2}{8 m_i \omega \Omega_i^4} \quad \ldots(2.15)$$

Similarly, the electron current per unit wavelength can be written as

$$J_{ze} = \frac{3e\omega_{pe}^2 E_i^2 k_i^2 V_{Te,e}^2}{8 m_e \omega \Omega_e^4} \quad \ldots(2.16)$$

Thus the total current per unit wavelength flowing along the magnetic field in the presence of Alfven wave is given as

$$J_z = - \frac{3e k_i^2 E_i^2}{8m_e \omega} \left[ \frac{\omega_{pe}^2 V_{Te,e}^2}{\Omega_e^4} + \frac{\omega_{pe}^2 V_{Ti,i}^2}{\Omega_i^4(m_i/m_e)} \right] \quad \ldots(2.17)$$
The average values of perpendicular currents become zero in the first order. Here \( q = +e \) for ions and \(-e\) for electrons is used and \( \omega_{\text{pe}} = (4\pi N e^2 / m_e)^{1/2} \) is the plasma frequency. 
\[ V_{T_{\perp,\text{e}}} = (2 \, T_{\perp,\text{e}} / m_e)^{1/2} \] is the perpendicular component of thermal velocity where the temperature \( T \) is expressed in the unit of energy, \( \Omega_{\text{ce}} \) is the cyclotron frequency.

### 2.5. Dispersion Relation

The dispersion relation is evaluated with help of wave equation in the form

\[
\nabla(\nabla \cdot E) - \nabla^2 E = - \frac{4 \pi}{c^2} \frac{\partial J_1}{\partial t} - \frac{1}{c^2} \frac{\partial^2 E_1}{\partial t^2} \tag{2.18}
\]

where \( J_1 \) is the first order current density. In the case of plane polarized Alfven wave the wave equation becomes

\[
\frac{\partial^2 E_x}{\partial z^2} = \frac{4 \pi}{c^2} \frac{\partial J_x}{\partial t} + \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} \tag{2.19}
\]

Substituting the value of \( J_x \) in terms of perturbed velocity \( u_x \) and perturbed density \( n_1 \), the dispersion relation for Alfven wave is evaluated as

\[
\omega = k_{||} V_A \left[ 1 - \frac{(V_{T_{\perp,\text{e}}}^2 + V_0^2)}{V_A^2} \right]^{1/2} \tag{2.20}
\]

where \( V_{T_{\perp,\text{e}}} = (T_{\perp,\text{e}} / m_e)^{1/2} \) is the electron thermal velocity parallel to the magnetic field.

In the anisotropic plasma the dispersion relation can be written as
\[
\omega = k_{||} V_A \left[ 1 - \frac{\{T_{\parallel e}/T_{\perp e}\} \cdot V_{T_{\perp e}}^2 / 2 + V_0^2 \}}{V_A^2} \right]^{1/2}
\]

Here, we notice that the dispersion relation of Alfven wave is modified by thermal and beam velocities of electrons. In case both the terms are zero the dispersion relation reduces to well known form. The current driven by Alfven wave is modified through the dispersion relations. \( V_A = B_0 / (4\pi N_0 m_e)^{1/2} \) is the Alfven wave speed. In the present chapter the wave frequency \( \omega \) is considered as real and the principal part of plasma dispersion function is used and coupling of compressional mode is not considered (Baronia and Tiwari, 1999).

2.6. RESULTS AND DISCUSSION

The following plasma parameters are used to estimate the dispersion relation and associated current per unit wavelength, which may be suitable to auroral acceleration region (Tiwari and Rostoker, 1984; Baronia and Tiwari, 1991; Baronia and Tiwari, 2000).

\[
B_0 = 4300 \text{nT}, \; \Omega_i = 412 \text{s}^{-1}; \; N_0 = 5.0 \times 10^3 \text{m}^{-3}; \; E_1 = 50 \text{mV} / \text{m}; \; V_{T_{\perp e}} = 3.5 \times 10^4 \text{m/s}.
\]

Expressions for field-aligned current per unit wavelength and the dispersion relations are evaluated and results are presented in figures.

The electron beams are injected from the tail side of the magnetosphere at the substorm times constituting field-aligned currents and auroral acceleration (Tiwari and Rostoker, 1984; Baronia and Tiwari, 1991; Baronia and Tiwari, 2000). In the same
event the Alfvén waves are also observed by various rockets and satellites, therefore, the
electron beam may be the cause of Alfvén wave generation which modifies the wave
frequency and the anisotropy of plasma sheet and auroral acceleration region may affect
the field aligned-current and the wave spectrum.

The figure 2.1 shows the variation of real frequency \( \omega \) with \( k_\parallel \) for different
values of electron beam velocity \( V_0 \). It is seen that the effect of the electron beam is to
reduce the frequency when the electron beam is along the magnetic field and in the
direction of propagation vector \( k_\parallel \). It is the wave velocity, which gets modified in the
presence of beam and anisotropy in the plasma, which is a dispersionless medium for the
Alfvén wave.

Figure 2.2 predicts the variation of real frequency \( \omega \) with \( k_\parallel \) for different
values of temperature anisotropy \( (T_{\parallel e} / T_{\perp e}) \). Here we notice that the wave frequency
increases with \( k_\parallel \) but decreases with the increase of temperature anisotropy. These curves
show that the phase velocity is smaller in hot plasmas. Thus the temperature anisotropy
observed in the distant magnetotail and the auroral acceleration region also modifies the
wave spectrum.

Figure 2.3 shows the variation of field-aligned current per unit wavelength
with \( k_\parallel \) for different values of electron beam velocity. It is observed that current decreases
with increase of \( k_\parallel \) as well as \( V_0 \). The effect of electron beam parallel to magnetic field is
to decrease the parallel current by diverging it to the perpendicular current. Thus the
closer currents may be constituted towards the ionosphere and the field - aligned current
reversal may occur during the auroral acceleration processes.
Fig. 2.1 $\omega$ versus $k_{11}$ for different $V_0$
Fig. 2.2 $\omega$ versus $k_{||}$ for different $T_{||e} / T_{\perp e}$
Fig. 2.3 $J_z$ versus $k_{\|}$ for different $V_0$
Figure 2.4 demonstrates the variation of field-aligned current per unit wavelength with $k_\parallel$ for different values of temperature anisotropy. The current decreases with the increase of $k_\parallel$ as well as temperature anisotropy. It is also clear that the current increases with the phase velocity of the Alfven wave. Thus the field-aligned current is limited by the electron beam in the anisotropic magnetosphere. The theory may be applicable to the auroral ionospheric regions wherever the field-aligned currents are reported along with the particle precipitation (Tiwari and Rostoker, 1984; Baronia and Tiwari, 1991; Baronia and Tiwari, 2000). The presence of field-aligned current in auroral ionosphere can permit short wavelength instabilities to lower altitudes. Down coming electron beam along with energetic particle precipitation may reduce the Alfven wave frequencies. The study may also be useful to the experimental devices with current carrying plasmas in the presence of Alfven waves. Although, many theoretical attempts have been made to investigate the Alfven wave propagation, the single particle theory may be able to explain some of the plasma phenomena where other theories are not well suited (Tiwari et al. 1985).

2.7. SUMMARY

Particle aspect analysis applied to evaluate the dispersion relation and associated field-aligned current for Alfven waves in the auroral region. Effect of temperature anisotropy has been examined on the wave and the applicability of the finding is discussed for the magnetosphere-ionosphere coupling. The plasma in the acceleration region is supposed to be composed of two types of charged particles i.e. resonant and non-resonant particles. The non-resonant particles support the oscillatory motion of the wave whereas the resonant particles participate in the energy exchange with the wave.
Fig. 2.4 $J_z$ versus $k_{||}$ for different $T_{||e}/T_{\perp e}$