CHAPTER 2

STATEMENT OF THE PROBLEM
From the critical examination of the literature reviewed in Chapter 1 and from the oscillograms reported by some earlier workers in this laboratory and elsewhere concerning oscillations, it appears that the electrical oscillations observed in a gas discharge at low pressures might be relaxation oscillations. Since the methods and circuits adopted by these workers do not seemingly consist of any relaxation circuit as such, as a part of the overall circuit it was thought that probably it is the discharge tube itself which gives rise to the oscillations. In the sequel it can be assumed that it must be the tube capacity which originates the oscillations.

From this point of view, a preliminary study was undertaken. Oscillations were observed and under the conditions when the oscillations are present, a part of the voltage across the tube was fed to the oscilloscope and a few oscillograms of the voltage were taken. Two of these oscillograms are shown in Fig. 2.1. These oscillograms show the voltage rise with time, which seem to be exponential as in the case of capacity charging. Charging equation of a capacity was therefore derived to ascertain whether the voltage rise observed in Fig. 2.1 is due to capacity
Fig. 2.1 Oscillograms showing the voltage growth across the discharge tube.

(a) $P = 2.0$ Torr, $I = 100$ µA (without 0-V level).
(b) $P = 5.0$ Torr, $I = 600$ µA (0-V level also photographed).
charging.

The voltage excursion of Fig. 2.1 is typical and is always above the zero level. A typical voltage oscillogram along with the zero level is shown in Fig. 2.1(b). The voltage, thus rises from a positive voltage, say \( V_1 \) to a higher voltage, say \( V_2 \).

2.1 Derivation of the Charging Equation Pertaining to the Conditions of Figure 2.1

If a capacitor \( C \) is connected to a voltage source through a resistor \( R \) and it charges from an initial voltage \( V_1 \) to a final voltage \( V_2 \) by a current \( I \), then the charging is represented by the equation

\[
IR + \frac{1}{C} \int I \, dt + V_1 = V_2.
\]

Differentiating and dividing by \( R \) gives

\[
\frac{dI}{dt} + \frac{I}{RC} = 0,
\]

which has a solution

\[
I = A e^{t/RC}
\]

From Eq. (2.1) at \( t = 0 \) and \( I = I(0) \)

\[
I(0) = \frac{V_2 - V_1}{R}
\]
From Eq. (2.3) at \( t = 0 \) and \( I = I(0) \)

\[ I(0) = A \quad \ldots \quad (2.5) \]

From Eqs. (2.4) and (2.5)

\[ A = \frac{V_2 - V_1}{R} \]

so that

\[ I = \frac{V_2 - V_1}{R} e^{-t/RC} \quad \ldots \quad (2.6) \]

The voltage across the resistor is

\[ VR = (V_2 - V_1) e^{-t/RC} \quad \ldots \quad (2.7) \]

and the voltage across the capacitor at any instant \( t \) (Fig. 2.2) is given by

\[ V' = V_1 + \frac{1}{C} \int_0^t I \, dt \]

Using Eq. (2.1)

\[ V' = V_1 + V_2 - V_1 - IR \]

\[ = V_2 - (V_2 - V_1) e^{-t/RC} \]

from Eq. (2.7).

Thus,

\[ \frac{V_2 - V'}{V_2 - V_1} = e^{-t/RC} \]

or

\[ \frac{V_2 - V_1 - (V' - V_1)}{V_2 - V_1} = e^{-t/RC} \]

Replacing, \( V_2 - V_1 \) by \( V_C \) and \( V' - V_1 \) by \( V \), gives

\[ \frac{V_C - V}{V_C} = e^{-t/RC} \]
FIG: 2.2 - EXPONENTIAL GROWTH OF VOLTAGE WITH TIME.
which can be written as
\[ \log \left( \frac{V_c}{V_c - V} \right) = \frac{t}{RC} \] ... (2.8)

Eq. (2.8) gives a straight line if plotted between
\[ \log \left( \frac{V_c}{V_c - V} \right) \text{ and } t. \]

To verify whether the voltage rise shown in Fig. 2.1 is exponential, a trace of Fig. 2.1(a) was made, which is given in Fig. 2.3. From the trace, the values of \( t, V, \frac{V_c}{V_c - V} \) and \( \log \frac{V_c}{V_c - V} \) are tabulated in Table 2.1 and \( \log \frac{V_c}{V_c - V} \) versus \( t \) are plotted in Fig. 2.3, which gives a straight line. It thus confirms that the voltage rise of Fig. 2.1 is exponential and represents the charging of capacity.

2.2 The Model

Since the voltage developed across the tube is exponential it must represent the charging of capacity. The capacity concerned with here is the internal capacity of the discharge tube itself in the active state.

The model can, therefore be presented by showing the discharge tube along with its equivalent capacity \( C_e \). Fig. 2.4, and may be elaborated as follows:
### Table 2.1

Time \( t \), Voltage \( V \), \( \frac{V_c}{V_c-V} \) and \( \log \frac{V_c}{V_c-V} \) Derived from the trace given in Fig. 2.3.

<table>
<thead>
<tr>
<th>Time ( t ) (arbitrary unit)</th>
<th>( V ) (arbitrary unit)</th>
<th>( \frac{V_c}{V_c-V} )</th>
<th>Log ( \frac{V_c}{V_c-V} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.2</td>
<td>1.429</td>
<td>0.1551</td>
</tr>
<tr>
<td>5</td>
<td>2.3</td>
<td>2.353</td>
<td>0.3722</td>
</tr>
<tr>
<td>10</td>
<td>3.3</td>
<td>5.714</td>
<td>0.7569</td>
</tr>
<tr>
<td>15</td>
<td>3.7</td>
<td>13.333</td>
<td>1.1249</td>
</tr>
<tr>
<td>20</td>
<td>4.0 (( \approx V_c ))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG: 2.3 - RELATION BETWEEN $\log \frac{V_c}{V_c - V}$ AND TIME (T)
FIG: 2.4 - DISCHARGE TUBE AND ITS EQUIVALENT CAPACITY.
A discharge tube requires a certain minimum current $I_{\text{min}}$ for the breakdown. At the low applied voltage, the current is also low and the breakdown does not occur, but this current charges the equivalent capacity. As the applied voltage is gradually increased the equivalent capacity charges to successively higher voltages, and when the voltage across it attains a certain value the discharge occurs. The occurrence of the breakdown is due to the fact that at that instant, the voltage across the equivalent capacity is just sufficient to provide the required $I_{\text{min}}$ for the breakdown. At the instant of breakdown, the equivalent capacity discharges to a certain lower voltage at which the gaseous breakdown ceases to exist. It so happens because at the lower voltage the equivalent capacity fails to supply the required $I_{\text{min}}$ for sustaining the breakdown. Hereafter, the equivalent capacity again starts charging up to the same higher voltage as attained previously, followed again by the next discharge. Thus, charging and discharging of the equivalent capacity occur in succession. During every discharge, the equivalent capacity produces a current pulse through the discharge tube. These current pulses are, therefore, also produced in succession in accordance with the voltage across the equivalent capacity and are called oscillations.
It is therefore, hoped that the equivalent capacity model can not only explain the oscillations but possibly other observations also pertaining to them; and it is from this point of view that the present studies have been conducted.
REFERENCES


