CHAPTER - 5

PROPAGATION CHARACTERISTICS OF STRIP-CLAD FIBERS

5.1 INTRODUCTION

The metal clad optical fibers are used as sensors. A novel variation of the metal clad fibers by helical clad fibers was reported by Singh and Singh (194) and Singh et. al (195). Where they have predicted the dependence of the propagation characteristics of the helical clad fiber on the pitch angle. The present simplified analysis involving more appropriate boundary conditions shows no such dependence on the pitch angle as well on other parameters live core radius and operating wavelength more appropriate analysis involving metal helix as cladding as well as using tape helix as the metal cladding also show no such dependence.

When helix pitch angle is increased, the helix becomes more and more aligned towards z-axis. When the pitch angle is made $90^0$, the helix becomes parallel to the z-axis. We assume a fiber core which is cladded by such fine paralleled strips on its boundary. The situation is shown is Fig. 5.1. No such design has been reported in literature so far. The present chapter deals with the propagation characteristics of such a strip clad optical fiber. First the simplified analysis is presented assuming the core-cladding boundary as a dielectric. In the second part the wires are assumed to be conducting along the strips.
Fig. 5.1 STRIP CLAD FIBERS
PART A. DIELECTRIC CLADDING

5.2 THEORETICAL ANALYSIS AND CHARACTERISTIC EQUATION

The structure here considered is divided into two region (i) \( r < a \) corresponding to the core and (ii) \( r > a \) corresponding to the cladding assumed to be infinitely extended.

Maxwell's equations together with the constitutive relations lead to the wave equation for \( \mathbf{E} \) and \( \mathbf{H} \). The field components \( E_z \) and \( H_z \) satisfy the scalar wave equation which is written in the cylindrical co-ordinate system. Taking the Z-axis as the direction of propagation of monochromatic sinusoidal electromagnetic. Wave, the general field expressions are given by

\[
E_z = E(r, \phi) e^{j(\omega t - \beta z)} \quad \text{--------- (5.1)}
\]

\[
H_z = H(r, \phi) e^{j(\omega t - \beta z)} \quad \text{--------- (5.2)}
\]

Where \( \beta \) is the propagation constant.

Assuming a harmonic \([\exp(j \nu \phi)]\) dependence for the fields a Bessel equation for the radial part of the wave equation is obtained. For fields remaining continuos at \( r = 0 \), the following solution for \( r < a \) is chosen ref. (184-185).

\[
E_{z1} = A J_\nu (k_1 r) e^{j(\omega t - \beta z + \nu \phi)} \quad \text{--------- (5.3)}
\]

\[
H_{z1} = B J_\nu (k_1 r) e^{j(\omega t - \beta z + \nu \phi)} \quad \text{--------- (5.4)}
\]
Where \( k_1^2 = \omega^2 \mu_0 \varepsilon_1 - \beta^2 \) and \( \varepsilon_1 = \varepsilon_0 \varepsilon_{1r} \) is the permittivity of the core. For \( r > a \) the boundary condition at infinite demand a decaying solution and hence.

\[
E_{z2} = C \ K_v (k_2 r) e^{j(\omega r + \beta z + \psi)} \tag{5.5}
\]

\[
H_{z2} = D \ K_v (k_2 r) e^{j(\omega r - \beta z + \psi)} \tag{5.6}
\]

Where \( k^2 = \omega^2 \mu_0 \varepsilon_2 - \beta^2 \) and \( \varepsilon_2 = \varepsilon_0 \varepsilon_{2r} \) is the permittivity of the cladding region.

A, B, C, D are unknown constants. \( J_v (k_1 r) \) denotes the Bessel’s function of the first kind and \( K_v (k_2 r) \) is the modified Bessel function of the second kind and \( \nu \) is the order of the respective function. The quantities \( \varepsilon_{1r} \) and \( \varepsilon_{2r} \) are the relative permittivities of the core and the cladding respectively.

Now we use Maxwell’s equations for the remaining field components.

(1) for the core region \( r < a \) we have

\[
E_{\phi 1} = - \frac{j}{k_1^2} \frac{\beta}{r} \left[ A j v (k_1 r) - \mu \omega B k_1 J' (k_1 r) \right] e^{j(\omega t - \beta z + \psi)} \tag{5.7}
\]

\[
H_{\phi 1} = - \frac{j}{k^2} \frac{\beta}{r} \left[ B j v (k_1 r) + \omega \varepsilon_1 k_1 A J' (k_1 r) \right] e^{j(\omega t - \beta z + \psi)} \tag{5.8}
\]

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Again for cladding region \( r > a \) we have

\[
E_{\phi_2} = -\frac{j}{k_2^2} \left[ \frac{-C}{r} C j \nu K_\nu(k_2 r) - \mu \omega D k_2 K'_\nu(k_2 r) \right] e^{j(\omega t - \beta z + \phi)} - (5.9)
\]

\[
H_{\phi_2} = -\frac{j}{k_2^2} \left[ \frac{-D}{r} j \nu K_\nu(k_2 r) + \omega \in k_2 K'_\nu(k_2 r) \right] e^{j(\omega t - \beta z + \phi)} - (5.10)
\]

5.3 BOUNDARY CONDITION

The electric field along the \( z \) and \( \phi \) are continuous directions. When written in the terms of the field quantities they are

\[
\begin{align*}
E_{z_1} &= E_{z_2} & (i) \\
E_{\phi_1} &= E_{\phi_2} & (ii)
\end{align*}
\]

The magnetic field is also continuous along \( z \) and \( \phi \) continuous. When written in terms of field quantities. They are

\[
\begin{align*}
H_{z_1} &= H_{z_2} & (iii) \\
H_{\phi_1} &= H_{\phi_2} & (iv)
\end{align*}
\]

Substituting the calculated field components in these boundary conditions at \( r = a \) we have four equation in terms of four unknown coefficient A, B, C. and D.

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First boundary condition gives

\[ A \ J_{\nu} (k_1 r) e^{j(\omega t - \beta z + \nu \phi)} = C \ K_{\nu} (k_2 r) e^{j(\omega t - \beta z + \nu \phi)} \]

\[ \Rightarrow \ A \ J_{\nu} (k_1 r) e^{j(\omega t - \beta z + \nu \phi)} - C \ K_{\nu} (k_2 r) e^{j(\omega t - \beta z + \nu \phi)} \quad \text{(5.11)} \]

Second boundary condition gives

\[ - \frac{j}{k_1^2} \left\{ \frac{\beta}{r} A j \nu \ J_{\nu} (k_1 r) - \mu \omega B k_1 J_{\nu} (k_1 r) \right\} e^{j(\omega t - \beta z + \nu \phi)} \]

\[ = - \frac{j}{k_2^2} \left\{ \frac{\beta}{r} C j \nu \ K_{\nu} (k_2 r) - \mu \omega D k_2 K_{\nu} (k_2 r) \right\} e^{j(\omega t - \beta z + \nu \phi)} \]

\[ \Rightarrow \ + \ \frac{\nu \beta}{k_1^2 r} A \ J_{\nu} (k_1 r) + \frac{\mu j \omega}{k_1} B J_{\nu} (k_1 r) \]

\[ - \frac{\nu \beta}{k_2^2 r} C \ K_{\nu} (k_2 r) - \frac{j \mu \omega}{k_2} D K_{\nu} (k_2 r) = 0 \quad \text{(5.12)} \]

Third boundary condition gives

\[ B \ J_{\nu} (k_1 r) e^{j(\omega t - \beta z + \nu \phi)} = D \ K_{\nu} (k_2 r) e^{j(\omega t - \beta z + \nu \phi)} \]

\[ \Rightarrow \ B \ J_{\nu} (k_1 r) - D \ K_{\nu} (k_2 r) = 0 \quad \text{(5.13)} \]
Fourth boundary condition gives

\[
\frac{j}{k_1^2} \frac{\beta}{r} \left[ -B j \nu \ J_\nu (k_1 r) + \omega \in_1 A k_1 J'_\nu (k_1 r) \right] e^{j(\alpha t - \beta z + \nu \phi)}
\]

\[
= -\frac{j}{k_2^2} \left[ -D j \nu \ K_\nu (k_2 r) + \omega \in_2 C k_2 K'_\nu (k_2 r) \right] e^{j(\alpha t - \beta z + \nu \phi)}
\]

\[
\Rightarrow \frac{\nu \beta}{k_1^2 r} J_\nu (k_1 r) - \frac{j \omega \in_1}{k_1} A J'_\nu (k_1 r)
\]

\[
-\frac{\nu \beta}{k_2^2 r} D K_\nu (k_2 r) + \frac{j \omega \in_2}{k_2} C K'_\nu (k_2 r) = 0
\]

\[\text{(5.14)}\]

Eliminating the four unknown constants \(A, B, C, \) and \(D,\) the following characteristics determinant equation is obtained.

\[
\begin{vmatrix}
J_\nu (k_1 r) & 0 & -K_\nu (k_2 r) & 0 \\
\frac{\nu \beta}{k_1^2 r} & \frac{\mu j \omega}{k_1} J_\nu (k_1 r) & -\frac{\nu \beta}{k_2^2 r} K_\nu (k_2 r) & -\frac{j \mu \omega}{k_2} K'_\nu (k_2 r) \\
0 & \frac{\nu \beta}{k_1} J_\nu (k_1 r) & 0 & -K_\nu (k_2 r) \\
-j \omega \in_1 & \frac{\nu \beta}{k_1^2 r} J'_\nu (k_1 r) & \frac{\nu \beta}{k_2} K_\nu (k_2 r) & -\frac{\nu \beta}{k_2^2 r} K'_\nu (k_2 r)
\end{vmatrix} = 0
\]

\[\text{(5.15)}\]

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Now expanding the above equation 5.15, we get

\[
J_v \left( k_1 r \right) \frac{\mu_j \omega}{k_1} + J'_v \left( k_1 r \right)[0 \pm K_v \left( k_2 r \right) \frac{j \omega}{k_2}]
\]

\[
+ J_v \left( k_1 r \right) \frac{\nu \beta}{k_2^2 r} K_v \left( k_2 r \right) \left[ -J_v \left( k_1 r \right) \frac{\nu \beta}{k_2^2 r} K_v \left( k_2 r \right) + K_v \left( k_2 r \right) \frac{\nu \beta}{k_1^2 r} J_v \left( k_1 r \right) \right]
\]

\[
- J_v \left( k_1 r \right) \frac{j \mu \omega}{k_2} K'_v \left( k_2 r \right) \left[ J_v \left( k_1 r \right) \frac{j \omega}{k_2} - 0 \right]
\]

\[
- K_v \left( k_2 r \right) \frac{\nu \beta}{k_1^2 r} J_v \left( k_1 r \right) \left[ -J_v \left( k_1 r \right) \frac{\nu \beta}{k_2^2 r} K_v \left( k_2 r \right) + K_v \left( k_2 r \right) \frac{\nu \beta}{k_1^2 r} J_v \left( k_1 r \right) \right]
\]

\[
+ K_v \left( k_2 r \right) \frac{j \omega}{k_1} J'_v \left( k_1 r \right) \left[ - \frac{j \mu \omega}{k_1} J'_v \left( k_1 r \right) K_v \left( k_2 r \right) + \frac{j \mu \omega}{k_2} K'_v \left( k_2 r \right) J_v \left( k_1 r \right) \right]
\]

\[
= 0
\]

Further simplification gives

\[
J_v \left( k_1 r \right) J'_v \left( k_1 r \right) K_v \left( k_2 r \right) K'_v \left( k_2 r \right) \frac{\mu_j \omega}{k_1} \frac{j \omega}{k_2}
\]

\[
- J^2_v \left( k_1 r \right) K^2_v \left( k_2 r \right) \frac{\nu \beta}{k_2^2 r} \frac{\nu \beta}{k_2^2 r} + K^2_v \left( k_2 r \right) J^2_v \left( k_1 r \right) \frac{\nu \beta}{k_1^2 r} \frac{\nu \beta}{k_2^2 r}
\]

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\[- J^2 \nu (k_1 r) K^2 \nu (k_2 r) \frac{j \mu \omega}{k_2} \frac{j \omega \epsilon_2}{k_2} \]

\[+ K \nu (k_2 r) \frac{\nu \beta}{k_1^2 r} J \nu (k_1 r) J \nu (k_1 r) \frac{\nu \beta}{k_2^2 r} K \nu (k_2 r) - K \nu (k_2 r) \frac{\nu \beta}{k_1^2 r} J \nu (k_1 r) \]

\[+ K \nu (k_2 r) \frac{\nu \beta}{k_1^2 r} J \nu (k_1 r) J \nu (k_1 r) \frac{\nu \beta}{k_2^2 r} K \nu (k_2 r) J \nu (k_1 r) = 0 \]

\[\Rightarrow J \nu (k_1 r) J' \nu (k_1 r) K \nu (k_2 r) K' \nu (k_2 r) \frac{j^2 \omega^2 \mu \epsilon_2}{k_1 k_2} - J^2 \nu (k_1 r) K^2 \nu (k_2 r) \frac{j^2 \mu \omega^2 \epsilon_2}{k_2^2} \]

\[- J^2 \nu (k_1 r) K^2 \nu (k_2 r) \frac{\nu \beta}{k_1^2 r} \frac{\nu \beta}{k_2^2 r} + K^2 \nu (k_2 r) J^2 \nu (k_1 r) \frac{\nu \beta}{k_2^2 r} \frac{\nu \beta}{k_2^2 r} \]

\[+ K^2 \nu (k_2 r) J^2 \nu (k_1 r) \frac{\nu \beta}{k_1^2 r} \frac{\nu \beta}{k_2^2 r} - K^2 \nu (k_2 r) J^2 \nu (k_1 r) \frac{\nu \beta}{k_2^2 r} \frac{\nu \beta}{k_2^2 r} \]

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\[ - K_{\nu}^2 (k_2 r) J_\nu^2 (k_1 r) \frac{j^2 \omega^2 \epsilon_1 \mu}{k_1^2} + K_{\nu}' (k_2 r) K_{\nu} (k_2 r) J_{\nu}' (k_1 r) J_{\nu} (k_1 r) \frac{j^2 \mu \omega^2 \epsilon_1}{k_1 k_2} = 0 \]  \hspace{1cm} \text{----- (5.16)}

This is general characteristic equation. For special case \( \nu = 0 \) Eqn (5.16) is reduced as following using Bessel's identities \( J_0' = -J_1 \) and \( K_0' = -K_1 \),

\[ J_0 (k_1 r) J_1 (k_1 r) K_0 (k_2 r) K_1 (k_2 r) \left[ \frac{\mu \omega^2 \epsilon_1}{k_1 k_2} + \frac{\omega^2 \mu \epsilon_2}{k_1 k_2} \right] \]

\[ - J_0^2 (k_1 r) K_1^2 (k_2 r) \frac{\mu \omega^2 \epsilon_2}{k_2^2} - K_0^2 (k_2 r) J_1^2 (k_1 r) \frac{\omega^2 \epsilon_1 \mu}{k_1^2} = 0 \]  \hspace{1cm} \text{----- (5.17)}

#### 5.4 CUT-OFF CONDITION

For the case when \( \nu = 0 \), the cut off condition can be considered taking the limit \( k_2 \to 0 \), one can thus obtain from the characteristics Eqn. (5.17)

\[ J_0 (k_2 r) = 0 \]  \hspace{1cm} \text{----- (5.18)}

The cut-off parameter \( k_c \) is thus given by the roots of the equation (5.18).
PART-B - METAL CLADDING

5.5 THEORETICAL CONSIDERATION AND CHARACTERISTIC EQUATION

The structure has been divided into two regions.

(i) \( r < a \) (core)
(ii) \( r > a \) (cladding)

Maxwell's equations along with the constitutive relations lead to the wave equations for \( E \) and \( H \). The time harmonic field components \( E_z \) and \( H_z \) satisfy the scalar wave equation, which is written in the cylindrical co-ordinate system. Assuming monochromatic sinusoidal electromagnetic waves propagating in the \( Z \)-direction.

The general field expression are given by

\[
E_z = E(r, \phi) e^{j(\omega t - \beta z)} \quad \text{----- (5.19)}
\]

\[
H_z = H(r, \phi) e^{j(\omega t - \beta z)} \quad \text{----- (5.20)}
\]

Where \( \beta \) is the propagation constant. Assuming the harmonic \( [\exp(j \omega \phi)] \) dependence for the field. A Bessel Equation for the radial part of the wave equation is obtained. For field remaining continuous at \( r = 0 \) the following solution for \( r < a \) is

\[
E_{z1} = A J_\nu(k_1 r) e^{j(\omega t - \beta z + \psi)} \quad \text{----- (5.21)}
\]

\[
H_{z1} = B J_\nu(k_1 r) e^{j(\omega t - \beta z + \psi)} \quad \text{----- (5.22)}
\]
Where \( k_1^2 = \omega^2 \mu_0 \epsilon_1 - \beta^2 \) and \( \epsilon_1 = \epsilon_0 \epsilon_{1r} \) is the permittivity.

For the core region \( r < a \), the other field components \( E_\phi \) and \( H_\phi \) are calculated as

\[
E_{\phi 1} = - j \frac{\beta}{k_1^2} \left[ \frac{\omega B k_1 J_\nu(k_1 r)}{J_\nu(k_1 r)} \right] e^{j(\omega t - \beta z + \phi)} \tag{5.23}
\]

\[
H_{\phi 1} = - j \frac{\beta}{k_1^2} \left[ \frac{\omega A k_1 J'_\nu(k_1 r)}{J_\nu(k_1 r)} \right] e^{j(\omega t - \beta z + \phi)} \tag{5.24}
\]

for \( r > a \) the boundary condition at infinity demands a decaying solution and hence.

\[
E_{z 2} = C \ K_\nu(k_2 r) \ e^{j(\omega t - \beta z + \phi)} \tag{5.25}
\]

\[
H_{z 2} = D \ K_\nu(k_2 r) \ e^{j(\omega t - \beta z + \phi)} \tag{5.26}
\]

Where \( k_2^2 = \beta^2 - \omega^2 \mu_0 \epsilon_2 \) and \( \epsilon_2 = \epsilon_0 \epsilon_{2r} \) is the permittivity of the cladding region, \( \epsilon_{2r} = (n + ix)^2 \) being the relative complex permittivities of the metal cladding also \( n \) the refractive index and \( A, B, C \) and \( D \) are unknown constant the extension co-efficient. \( J_\nu(k_1 r) \) denotes the Bessel function of the first kind and \( K_\nu(k_1 r) \) is the modified Bessel function of second kind and \( \nu \) in the order of the respective function. The quantities \( \epsilon_1 r \) and \( \epsilon_2 r \) are the relative permittivites of the core and cladding respectively.
Now we use Maxwell's equation for the remaining field components. For the cladding region \( r > a \) we have

\[
E_{\phi_2} = - \frac{j}{k_2^2} \frac{\gamma}{r} \left[ C \, j \, v \, K_v (k_2 r) - \mu \, \omega \, D k_2 \, K'_v (k_2 r) \right] e^{j(\omega t - \gamma r + \psi)} \quad \text{---- (5.27)}
\]

\[
H_{\phi_2} = - \frac{j}{k_2^2} \frac{\gamma}{r} \left[ D \, j \, v \, K_v (k_2 r) + \omega \, \epsilon \, C \, k_2 \, K'_v (k_2 r) \right] e^{j(\omega t - \gamma r + \psi)} \quad \text{---- (5.28)}
\]

In the above Eqn. (5.21 to 5.28), the quantities A, B, C and D are unknown constant and \( J_v (k_1 r) \) denotes the Bessel's function of first kind \( v \) and \( K_v (k_2 r) \) is modified Bessel's function of the second kind being the order of the respective function. Although the fields inside a helix are expressed usually in terms of modified Bessel's function \( I_v (k_1 r) \) here we have chosen \( J_v (k_1 r) \) function on arguments, which have already been explained in the previous chapter.

### 5.6 BOUNDARY CONDITION

The boundary condition on the core-cladding interface remains the same as in case of dielectric (section 5.3) but the electric field along the direction of wire is assumed to be zero.

Where written in the terms of field quantities they are

\[
E_{z_1} = 0 \quad \text{------(i)}
\]

\[
E_{\phi_1} = E_{\phi_2} \quad \text{------(ii)}
\]

\[
H_{z_1} = H_{z_2} \quad \text{------(iii)}
\]

\[
H_{\phi_1} = H_{\phi_2} \quad \text{------(iv)}
\]
Substituting the calculated field components in these boundary condition at \( r = a \) we have four equation in terms of four unknown coefficient A, B, C, and D.

First boundary condition gives

\[
A \ J_\nu \left( k_1 r \right) e^{j(\omega t - \beta z + \nu \phi)} = 0
\]

\[\Rightarrow \ A \ J_\nu \left( k_1 r \right) = 0 \quad \text{--- (5.29)}\]

Secondary boundary condition gives

\[
- \left[ \frac{j}{k_1^2} \right] \left\{ A j \nu \ J_\nu \left( k_1 r \right) - \mu_\omega B k_1 J'_\nu \left( k_1 r \right) \right\} \ e^{-j\beta z}
\]

\[
- \left[ \frac{j}{k_2^2} \right] \left\{ C j \nu \ K_\nu \left( k_2 r \right) - \mu_\omega D k_2 K'_\nu \left( k_2 r \right) \right\} \ e^{-j\gamma z}
\]

\[\Rightarrow \ + \ \frac{\nu \beta}{k_1^2 r} \ A \ J_\nu \left( k_1 r \right) e^{-j\beta z} + \frac{\mu \omega j}{k_1} \ B \ J'_\nu \left( k_1 r \right) e^{-j\beta z}
\]

\[
- \ \frac{\nu \gamma}{k_2^2 r} \ C \ K_\nu \left( k_2 r \right) e^{-j\gamma z} - \frac{j \mu \omega}{k_2} \ D \ K'_\nu \left( k_2 r \right) e^{-j\gamma z} = 0 \quad \text{--- (5.30)}
\]

Third boundary condition gives

\[
B \ J_\nu \left( k_1 r \right) e^{-j\beta z} = D \ K_\nu \left( k_2 r \right) e^{-j\gamma z}
\]

\[\Rightarrow \ B \ J_\nu \left( k_1 r \right) e^{-j\beta z} - D \ K_\nu \left( k_2 r \right) e^{-j\gamma z} = 0 \quad \text{--- (5.31)}
\]

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Fourth boundary condition gives

\[
\frac{j}{k_1^2} \frac{\beta}{r} \left[ -B J_\nu (k_1 r) + \omega \in_1 A k_1 J'_\nu (k_1 r) \right] e^{j(\omega t - \beta z + \phi)}
\]

\[
= -\frac{j}{k_2^2} \frac{\gamma}{r} \left[ -D J_\nu (k_2 r) + \omega \in_2 C k_2 K'_\nu (k_2 r) \right] e^{j(\omega t - \gamma z + \phi)}
\]

\[
\therefore \frac{\nu \beta}{k_1^2} B J_\nu (k_1 r) e^{-j\beta z} - \frac{j \omega \in_1}{k_1} A J'_\nu (k_1 r) e^{-j\beta z}
\]

\[
-\frac{z \gamma}{k_2^2} D K_\nu (k_2 r) e^{-j\gamma z} + \frac{j \omega \in_2}{k_2} C K'_\nu (k_2 r) e^{-j\gamma z} = 0 \quad \text{(5.32)}
\]

Eliminating these coefficients the following characteristics equation for the lowest order mode is obtained is

\[
\begin{vmatrix}
J_\nu (k_1 r) & 0 & 0 & 0 \\
\frac{\nu \beta}{k_1^2} J_\nu (k_1 r) e^{-j\beta z} & \frac{\mu \omega}{k_1} J_\nu (k_1 r) e^{-j\beta z} & \frac{\nu \gamma}{k_2^2} K_\nu (k_2 r) e^{-j\gamma z} & \frac{j \mu \omega}{k_2} K'_\nu (k_2 r) e^{-j\gamma z} \\
0 & J_\nu (k_1 r) e^{-j\beta z} & 0 & -K_\nu (k_2 r) e^{-j\gamma z} \\
\frac{j \omega \in_1}{k_1} J_\nu (k_1 r) e^{-j\beta z} & \frac{j \omega \in_2}{k_2^2} K'_\nu (k_2 r) e^{-j\gamma z} & z \gamma K_\nu (k_2 r) e^{-j\gamma z} & -K_\nu (k_2 r) e^{-j\gamma z}
\end{vmatrix} = 0 \quad \text{(5.33)}
\]

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Now expanding the above equation (5.33), we get:

\[
J_\nu(k_1 r) \quad \frac{\mu j \omega}{k_1} \quad J_\nu'(k_1 r) \quad e^{-j\beta z} \left[ 0 + K_\nu(k_2 r) e^{-j\gamma z} \right] + \frac{j \omega e_2}{k_2} \quad K'_\nu(k_2 r) \quad e^{-j\gamma z}
\]

\[
+ \frac{\nu \gamma}{k_2^2 r} K_\nu(k_2 r) e^{-j\gamma z} - \frac{z \gamma}{k_2^2 r} K_\nu(k_2 r) e^{-j\beta z} + K_\nu(k_2 r) e^{-j\gamma z} \frac{\nu \beta}{k_1^2 r} - J_\nu(k_1 r) e^{-j\beta z}
\]

\[
- \frac{j \omega \mu}{k_2} K'_\nu(k_2 r) e^{-j\gamma z} \left[ J_\nu(k_1 r) e^{-j\beta z} - K'_\nu(k_2 r) e^{-j\gamma z} - 0 \right] = 0 \quad \text{-------(5.34)}
\]

Further simplification gives,

\[
\frac{J_\nu(k_1 r) \quad J_\nu'(k_1 r) \quad K_\nu(k_2 r) \quad K'_\nu(k_2 r)}{k_1} \quad \frac{\mu j \omega}{k_1} \quad \frac{j \omega e_2}{k_2} \quad \frac{\nu \gamma}{k_2^2 r} K_\nu(k_2 r) \quad \frac{\nu \gamma \nu \beta}{k_2^2 r} K_\nu(k_2 r) \quad \frac{\nu \gamma \nu \beta}{k_2^2 r} K_\nu(k_2 r)
\]

\[
- \frac{j \omega \mu}{k_2} \frac{j \omega e_2}{k_2} = 0 \quad \text{-------(5.35)}
\]

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This is the general characteristics equation for special case \( v = 0 \) and using Bessel's function identities \( K_0' = -K_1 \) and \( J_0' = -J_1 \), the Eqn (5.35) is reduced as

\[
- J_0 (k_1 r) J_1 (k_1 r) K_0 (k_2 r) K_1 (k_2 r) \frac{\mu \omega^2 \varepsilon_2}{k_1 k_2} + J_0^2 (k_1 r) K_2 (k_2 r) \frac{\omega^2 \mu \varepsilon_2}{k_2^2} = 0 \quad ------ (5.36)
\]

Or

\[
J_0 (U) J_1 (U) K_0 (\sqrt{V^2 - U^2}) K_1 (\sqrt{V^2 - U^2}) \frac{4 \pi^2 \mu \varepsilon_2, a^2}{\lambda^2 U (\sqrt{V^2 - U^2})} + J_0^2 (U) K_2 (\sqrt{V^2 - U^2}) \frac{4 \pi^2 \mu \varepsilon_2, a^2}{\lambda^2 (V^2 - U^2)} = 0 \quad ------ (5.37)
\]

Where \( U = k_1 a \) and \( W = k_2 a \)

For getting numerical estimates of the dispersion relation, we have to describe in some way the parameters of the metallic cladding in the optical frequency band. As has been shown in chapter - 3, the metal in optical frequency band may be considered practically as dielectric. The permittivity of the quartz and silver are given by Eqns. (3.20 and 3.21) respectively.
5.7 CUT-OFF CONDITION

For the case when \( v = 0 \), the cut off condition can be considered taking the limit \( k_2 \to 0 \). One can, thus, obtain from the characteristics Eqn. (5.36)

\[
J_0 (k_c r) \quad J_1 (k_c r) = 0
\]

The cut-off parameter \( k_c \) is thus given by the roots of the equation (5.38).

5.8 RESULTS AND DISCUSSION

The equations (5.17) and (5.36) or (5.37) present the propagation characteristics of the dielectric and metal strip clad fibers respectively. The mode cut off parameter is given by equation (5.18) for a dielectric strip which is similar to that for a step-index fibers. The mode cut off parameter for the metal strip clad fiber is given by equation (5.38) which involve \( J_0 \) and \( J_1 \) as product. Hence, the TE, TM as well as hybrid modes can propagate in such a fibers.

The numerical computation of the dispersion characteristic for dielectric strip-clad fiber is given in Table 5.1 and plotted in Fig. 5.2. As expected, no dependence on operating wavelength and core parameter was obtained, hence only one table and one graph are given. It is observed that only TE and TM mode propagate in such fibers.

The computed values of the dispersion characteristics for the metal strip clad fibers is given in Table 5.2 and plotted in Fig. 5.3. It is observed that a metal strip clad fiber allows hybrid mode along with the TE and TM modes.

The computed value of the propagation constant \( b \) are plotted in Fig. 5.4 - 5.5 for the dielectric and metal clad strip fibers respectively. The propagation constant in both the cases shows normal behaviour. The fractional power in both the cases are plotted in Fig. 5.6 - 5.7 and is observed to have normal behaviour.
### Table – 5.1. Normalized frequency V Vs Core Parameter U

Core radius = 5, 25, 50 μm Operating wavelength = 0.86, 1.33, 1.55 μm

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Fig. 5.2- Normalized Frequency Vs Core Parameter

$a = 5, 25, 50 \mu m$
$\psi = 0^0, 15^0, 30^0, 45^0$
$\lambda = 0.86, 1.33, 1.55 \mu m$
### Table 5.2. Normalized frequency $V$ Vs Core Parameter $U$

Core radius = 5, 25, 50 µm Operating wavelength = 0.86, 1.33, 1.55 µm

Pitch angle = $0^\circ$ to $45^\circ$

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Fig. 5.3- Normalized Frequency Vs Core Parameter

\[a = 5, 25, 50 \, \mu m\]
\[\psi = 0^0, 15^0, 30^0, 45^0\]
\[\lambda = 0.86, 1.33, 1.55 \mu m\]
Fig. 5.4- Normalized Frequency Vs Propagation Constant

\[ a = 5, 25, 50 \, \mu m \]
\[ \psi = 0^0, 15^0, 30^0, 45^0 \]
\[ \lambda = 0.86, 1.33, 1.55 \mu m \]
Fig. 5.5- Normalized Frequency Vs Propagation Constant

\[ a = 5, 25, 50 \, \mu m \]
\[ \psi = 0^0, 15^0, 30^0, 45^0 \]
\[ \lambda = 0.86, 1.33, 1.55 \, \mu m \]
Fig. 5.6- Normalized Frequency Vs Fractional Power

- $a = 5, 25, 50 \mu m$
- $\psi = 0^0, 15^0, 30^0, 45^0$
- $\lambda = 0.86, 1.33, 1.55\mu m$

Diagram showing the relationship between $n$ and $v$ with the specified parameters.
Fig. 5.7 - Normalized Frequency Vs Fractional Power

\[ a = 5, 25, 50 \, \mu m \]
\[ \psi = 0^0, 15^0, 30^0, 45^0 \]
\[ \lambda = 0.86, 1.33, 1.55 \mu m \]