KINETIC ALFVEN WAVE IN AN INHOMOGENEOUS PLASMA IN THE PRESENCE OF GENERAL LOSS-CONE DISTRIBUTION FUNCTION AND INHOMOGENEOUS ELECTRIC FIELD
- PARTICLE ASPECT ANALYSIS
5.1 INTRODUCTION

In the fourth chapter we have investigated kinetic Alfven wave in the presence of inhomogeneous electric field applied perpendicular to ambient magnetic field in an anisotropic inhomogeneous magnetoplasma. In the present chapter we have investigated the dispersion relation, current and growth rate of the kinetic Alfven with loss-cone distribution function in the presence of inhomogeneous electric field applied perpendicular to ambient magnetic field is an anisotropic inhomogeneous magnetoplasma.

Theoretical studies of both high latitude ULF phenomena and the wave at equatorial to middle latitudes involve kinetic Alfven waves. These waves are considered in a number of modes as an agent of magnetosphere-ionosphere coupling. (Marchenko et. al. 1996) Among the first type of wave found to exist in plasmas, Alfven wave play an important role in energy transport, in particle acceleration and heating in the earth's magnetosphere, in solar flares and the solar wind. In particular a series of spacecraft have directly detected strong Alfven wave turbulence associated with particle energization in interplanetary space such as the auroral region (Dubinin et. al. 1988, Louarn et. al. 1994, Wahlund et. al. 1994, Volwerk et. al.1996, Huang et. al. 1997).

Several workers have observed that kinetic Alfven wave can propagate in a plasma with $\beta > \frac{m_e}{m_i}$ (the electron-to-ion mass ratio). A low $\beta$ plasma supports low frequency kinetic Alfven waves and
electrostatic waves. Kinetic Alfvén waves are found to have a two dimensional structure with electrostatic and magnetic components with phase velocity (parallel to the external magnetic field $\mathbf{B}_o$) much smaller than the electron thermal velocity. Furthermore, kinetic Alfvén waves are observed to propagate everywhere uniformly in a magnetized hot plasma (Stix 1981, Kalita and Bhatta 1997, Hasegawa 1977, Hasegawa and Mima 1978).

In the magnetotail lobe which has been identified as a region of open magnetic field lines (Fennell et. al. 1975, Yeager and Frank 1976), the plasma velocity distribution characteristics are qualitatively similar to those of the plasma sheet boundary layer. (Huang et. al. 1987) In this region, energetic ions with tailward directed velocities and anisotropic ion distribution signatures are often observed (Lui and Krimigis 1983).

The starting point for the model analyzed below is the well known fact that in an auroral system the electric field perpendicular to magnetic field reverses direction across a very narrow latitude range which is usually close to the edges of the auroral oval (Frank and Gurnett 1971, Carlson and Kelley 1977). Frank and Gurnett have predicted that these field reversals occur on field lines along which inverted-V precipitation has been observed. The field reversals are related to field aligned currents (Akasofu 1977). The loss rate in the presence of loss-cone has been discussed in mirrors and also in stellarators.
Recently Itoh et al. (1989) derived an approximate form which describes the loss rate in stellarators under the influence of a loss-cone in the collisionless limit.

In this chapter, particle aspect theories have been adopted to investigate the effect of anisotropy and steepness of loss-cone on drift kinetic Alfvén wave in the presence of an inhomogeneous electric field in a low β plasma. These are based on Dawson's (1961) theory of Landau damping and further extended by several workers (Panday and Sharan 1994, Misra and Tiwari 1980, Tiwari and Varma 1993, Varma and Tiwari 1994) to the analysis of electrostatic and electromagnetic instabilities. The whole plasma is considered to consist of resonant and non-resonant particles. Non-resonant particles support the oscillatory motion of the waves while the resonant particles participate in the energy exchange with the wave. The trajectories of particles are then evaluated within the frame work of linear theory. Using these particle trajectories in the presence of drift kinetic Alfvén wave, the dispersion relation, current and growth rate have been obtained by energy conservation method. Effect of steepness of loss-cone distribution is discussed on the dispersion relation, current and growth rate of the instability. The results are derived for auroral acceleration region which are relevant to laboratory plasma also.

The wave propagating obliquely to the magnetic field in a plane normal to density gradient and applied electric field has been
considered in an anisotropic plasma. The ambient magnetic field is directed along Z-axis and density gradient and perpendicular electric field are in the Y-direction. The wave propagating in the X-Z plane has been considered.

5.2 BASIC ASSUMPTION AND PARTICLE TRAJECTORY

The particle trajectories are evaluated in the frame work of linear theory. We are interested in the wave satisfying the criteria.

\[ V_{T_i} \ll \frac{\omega}{K} \ll V_{Te} ; \omega \ll \Omega_i, \Omega_e ; K_{\perp e}^2 \ll K_{\parallel i}^2 \ll 1 \]  \hspace{1cm} (5.1)

where \( V_{T_i} \) and \( V_{Te} \) are the thermal velocities of the ions and electrons along the magnetic field, \( \Omega_{i,e} \) and \( \rho_{i,e} \) are the gyrofrequencies and mean groradii of the two species, \( K_{\perp} \) and \( K_{\parallel} \) are the components of the wave vector \( \mathbf{k} \) across and along the static magnetic field \( \mathbf{B}_0 \), \( \omega \) represents the wave frequency. The wave is assumed to be of the form.

\[ \phi = \phi_1 \cos (K_{\perp} x + K_{\parallel} z - \omega t) \]

and \[ \Psi = \Psi_1 \cos (K_{\perp} x + K_{\parallel} z - \omega t) \]  \hspace{1cm} (5.2)

with electrostatic limit

\[ \mathbf{E}_1 = \mathbf{E}_\perp + \mathbf{E}_\parallel \]

\[ \mathbf{E}_\perp = - \nabla_\perp \phi, \mathbf{E}_\parallel = - \nabla_\parallel \Psi \]  \hspace{1cm} (5.3)

where \( \phi_1 \) and \( \Psi_1 \) are assumed to be a slowly varying function of time and \( \phi \) and \( \Psi \) are the electrostatic potential of the wave.

The inhomogeneous impressed electric field \( \mathbf{E}(y) \) has the form
\[
\bar{E}(y) = E_0 \left(1 - \frac{y^2}{a^2}\right)
\]

where "a" is thought to be comparable to the mean ion gyroradius but much larger than the Debye length. When \(y^2/a^2 \ll 1\), \(\bar{E}(y)\) reduces to a constant uniform field. This form of variation may permit the reversal of electric field which may be conformal to the observed values of the electric fields (Mozer and Lucht 1974) in the particular plasmas.

The equation of motion of a particle is

\[
m \frac{dv}{dt} = q \left( E + \frac{1}{c} v \times B_0 \right)
\]

(5.4)

where the collisions between particles are neglected. \(\bar{E} = \bar{E} + \bar{E}(y)\) is the total electric field consisting of perturbed wave field and the impressed electric field. Velocity \(v\) can be expressed as the sum of the unperturbed velocity \(\bar{v}\) and the perturbed one \(u\) that is \(v = \bar{v} + u\). Thus perturbed velocity \(u\) is determined by the following set of equations.

\[
\frac{du}{dt} + i \bar{\Omega} u = - \frac{q}{m} \left[ \phi_1 K_{\perp} + \frac{v_{\parallel} K_{\parallel} K_{\perp}}{\omega} \right] \sin (K_{\perp} x + K_{\parallel} z - \omega t)
\]

(5.5)

\[
\frac{d\bar{u}}{dt} = - \frac{q}{m} \left[ \Psi_1 + \frac{v_{\parallel} K_{\parallel} K_{\perp}}{\omega} \right] \cos (\bar{\theta} - \bar{\Omega} t) \sin (K_{\perp} x + K_{\parallel} z - \omega t)
\]

where \(\bar{u} = \bar{u}_x + i \bar{u}_y\) and \(\bar{\Omega} = QB_0 / mc\), \(\bar{u}_x\) and \(\bar{u}_y\) are the perturbed velocities in the x and y direction respectively. Equation (5.5) is solved under the approximation of free gyration treating \(\phi_1\) and \(\Psi_1\) as constants. This process is similar to that of the linearization of the
Vlasov equation as wave amplitude is independent of \( t \). The charged particle trajectories in the presence of inhomogeneous electric field and constant magnetic field are same as Terishima (1974).

\[
x(t) = x_o + \Delta + \frac{\gamma_s}{\Omega} \left(1 - \frac{3}{4} \cdot \frac{\tilde{E}'(y)}{\Omega^2}\right) [\sin(\Theta - \Omega t) - \sin \Theta]
\]

\[
y(t) = y_o - \frac{\gamma_s}{\Omega} \left(1 + \frac{1}{4} \cdot \frac{\tilde{E}'(y)}{\Omega^2}\right) [\cos(\Theta - \Omega t) - \cos \Theta]
\]  
(5.6)

\[
z(t) = z_o + \gamma_s t
\]

where \( \Theta \) is the phase of \( V_\perp \) at \( t = 0 \) and \( r_o \equiv (x_o, y_o, z_o) \) is the initial position of particles at \( t = 0 \).

\[
\tilde{E}(y) = \frac{q}{m} \cdot E(y)
\]

and

\[
\Delta = -\frac{\tilde{E}(y)t}{\Omega} \left[1 + \frac{E'(y)}{E(y)} \cdot \frac{1}{4} \cdot \left(\frac{\gamma_s}{\Omega}\right)^2 + \ldots\ldots\right]
\]  
(5.7)

The second term in the parenthesis on the right hand side of equation (5.7) is the finite gyroradius correction and \( d\Delta / dt \) represents the drift velocity due to impressed electric field. By substituting (5.6) into equation (5.5) we get the oscillatory solution of \( u(t) \) for the non-resonant particles. For resonant particles we take into account the initial condition \( u(t = 0) = 0 \) as the basic assumption.

The perturbed velocities \( u(r, t) \) for non-resonant and resonant particles have been evaluated as (Terashima 1967)
\[
\begin{align*}
\mathbf{u}_x(r,t) &= -\frac{q}{m} \left[ \phi_1 K_{\perp} - \frac{v_{\parallel} K_{\parallel}}{\omega} K_{\perp} (\phi_1 - \Psi_1) \right] \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_1(\mu) \\
&\quad \left[ -\frac{\eta_n}{\Lambda} \cos \phi_{nl} - \frac{\Lambda}{2} \frac{\eta_{n+1}}{\eta_{n-1}} \cos(\phi_{nl} - \gamma_{n-1}t) - \frac{\Lambda}{2} \frac{\eta_{n-1}}{\eta_{n+1}} \cos(\phi_{nl} - \gamma_{n+1}t) \right] \\
\mathbf{u}_y(r,t) &= -\frac{q}{m} \left[ \phi_1 K_{\perp} - \frac{v_{\parallel} K_{\parallel}}{\omega} K_{\perp} (\phi_1 - \Psi_1) \right] \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_1(\mu) \\
&\quad \left[ -\frac{\Omega}{\Lambda} \sin \phi_{nl} - \frac{\Lambda}{2} \frac{\eta_{n+1}}{\eta_{n-1}} \sin(\phi_{nl} - \gamma_{n-1}t) + \frac{\Lambda}{2} \frac{\eta_{n-1}}{\eta_{n+1}} \sin(\phi_{nl} - \gamma_{n+1}t) \right] \\
\mathbf{u}_z(r,t) &= -\frac{q}{m} \left[ K_{\parallel} J_1 + \frac{v_{\perp} K_{\perp}}{\omega} K_{\parallel} (\phi_1 - \Psi_1) \right] \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_1(\mu) \\
&\quad \left[ \frac{1}{\eta_n} \cos \phi_{nl} - \Lambda \cos(\phi_{nl} - \gamma_{n}t) \right]
\end{align*}
\]

\[\text{where}\]

\[\Lambda = 0 \text{ for the non-resonant particles}\]

\[\Lambda = 1 \text{ for resonant particles}\]

\[\eta_n = n \Omega + V_{\parallel} K_{\parallel} + K_{\perp} \Delta - \omega\]
\[ \tilde{\Delta} = - \left( \frac{E(y)}{\Omega} \right) \left[ 1 + \frac{E''(y)}{E(y)} \cdot \frac{1}{4} \cdot \left( \frac{V}{\Omega} \right)^2 \right] + \ldots \ldots \] 

\[ A^2_n = \gamma^2_n - \Omega^2 ; \]

\[ \mu = \frac{K \cdot V}{\Omega} \left( 1 + \frac{3}{4} \cdot \frac{E'(y)}{\Omega^2} \right) \]  \[ (5.9) \]

Also use was made of the identity.

\[ \exp \left[ -i \mu \sin (\hat{\theta} - \Omega t) \right] = \sum_{-\infty}^{+\infty} J_n (\mu) \exp \left[ -i n (\hat{\theta} - \Omega t) \right] \]

and \( \cos \phi \cdot \exp \left[ -i \mu \sin \phi \right] = \frac{n}{\mu} \sum_{-\infty}^{+\infty} J_n (\mu) \exp (in \phi) \)

\( J_n (\mu) \) represents the Bessel's function of order \( n \).

and \( \phi_{nl} = K_\perp y + K_\parallel z - \omega t + (n - 1)(\Omega t - \hat{\theta}) \)

**5.3 DENSITY VARIATION**

In order to find out the density perturbation associated with the velocity perturbation \( \overline{u(r,t)} \) let us consider a group of particles with the same initial conditions and the number density be of the form.

\[ n(\overline{r,t,V}) = N(\overline{y,V}) + n_1(\overline{r,t,V}) \]  \[ (5.10) \]

where \( N \) is the zeroth order distribution and \( n_1 \) is the perturbed density which is determined by the equation (Terashima 1967)

\[ \frac{\partial n_1}{\partial t} = -N(\overline{V}) (\nabla_\perp \cdot u_x + \nabla_\parallel u_z) \]  \[ (5.11) \]

Expressing the right right hand side of equation (5.11) as the function
of "t" and after integrating we obtain the perturbed density for the non-resonant particles as

\[ n_1(r,t) = N(V) \cdot \frac{q}{m} \cdot \frac{K}{A} \left[ \phi_1 - \frac{V}{\omega} \frac{K}{\Omega} (\phi_1 - \Psi_1) \right] \cos \phi \]

\[ \frac{2J_o(\mu)J_1(\mu)K \Omega}{\Omega^2} \cdot \frac{3}{4} \cdot \frac{E''(y)}{\Omega^2} + \frac{J_o(\mu)2K}{\Omega(\eta^2 - \Omega^2)} \]

\[ \frac{N(V)K^2}{\eta^2} \cdot \frac{q}{m} \Psi_1 \cos \phi \cdot \sum_{-\infty}^{+\infty} J_o(\mu) \]

\[ \approx N(V) \sum_{-\infty}^{+\infty} J_o(\mu) \frac{q}{m} \left\{ \frac{K^2}{\Lambda^2} \left[ \phi_1 - \frac{V}{\omega} \frac{K}{\Omega} (\phi_1 - \Psi_1) \right] + \frac{K^2}{\eta^2} \Psi_1 \right\} \cos \phi \quad (5.12) \]

and the perturbed density for the resonant particles under the assumption that \( n_1(t = 0) = 0 \) we get.

\[ n_1(r,t) = N(V) \sum_{-\omega}^{+\omega} J_o(\mu) \frac{q}{m} \left\{ K^2 \left[ \phi_1 - \frac{V}{\omega} \frac{K}{\Omega} (\phi_1 - \Psi_1) \right] \right\} \]

\[ \left[ \frac{1}{\Lambda^2} \cos \phi - \frac{1}{2\Omega(\eta-\Omega)} \cos(\phi - (\eta+\Omega)t) + \frac{1}{2\Omega(\eta+\Omega)} \cos(\phi - (\eta-\Omega)t) \right] + \]

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\[ \Psi_1 \frac{K^2}{\eta^2} \left[ \cos \phi + \eta t \sin (\phi - \eta t) - \cos (\phi - \eta t) \right] \]  

(5.13)

where \( A^2 = \eta^2 - \Omega^2 \).

Similarly the equation for perturbed density in the inhomogeneous plasma can be written as (Terashima 1967).

\[ \frac{dn_1}{dt} = - (\overline{\mathbf{V} \cdot \mathbf{u}}) N - u_y \frac{dN}{dy} \]  

(5.14)

With the help of the above equation the density variation for non-resonant particles is

\[ n_1(r,t) = N(V) \sum_{-\infty}^{+\infty} J_o(\mu) \frac{q}{m} \left\{ [\phi_1 - \frac{V_{||}}{\omega} K_{||}] (\phi - \Psi_1) \right\} \]  

(5.15)

and for resonant particles

\[ n_1(r,t) = N(V) \sum_{-\infty}^{+\infty} J_o(\mu) \frac{q}{m} \left\{ [\phi_1 - \frac{V_{||}}{\omega} K_{||}] (\phi - \Psi_1) \right\} \]  

\[ \left[ \frac{K^2}{A^2} - \frac{\Omega^2}{\eta} \frac{\mathbf{d} \cdot \mathbf{K}}{T_{\perp}} \right] \cos \phi - \frac{1}{2\Omega (\eta - \Omega)} \cos (\phi - (\eta + \Omega) t) \]  

\[ \left[ K_{\perp}^2 - \frac{\mathbf{d} \cdot \mathbf{K}}{T_{\perp}} m \Omega \right] + \frac{1}{2\Omega (\eta + \Omega)} \cos (\phi - (\eta - \Omega) t) \left[ K_{\perp}^2 - \frac{\mathbf{d} \cdot \mathbf{K}}{T_{\perp}} m \Omega \right] \]
\[
\Psi_{1} \frac{K_{||}^2}{\eta^2} \left[ \cos \phi + \eta t \sin(\phi - \eta t) - \cos(\phi - \eta t) \right]
\]

(5.16)

Where \( v_d \) is the pressure drift defined as

\[
v_d = \frac{T_{\perp}}{m \Omega} \cdot \frac{1}{N} \cdot \frac{\partial N}{\partial y}
\]

and \( T_{\perp} \) is perpendicular temperature. To calculate the current dispersion relation and growth rate we use the general distribution function of the form (Tiwari and Restoker 1984).

\[
N(V) = \frac{N \Omega}{\pi^{3/2} V_{T\perp}^{2(J+1)} V_{T\parallel}^{J!}} \exp \left[ - \frac{v_{\perp}^2}{V_{T\perp}^2} - \frac{v_{\parallel}^2}{V_{T\parallel}^2} \right]
\]

\[
f(V_{\perp}) = \frac{V_{\perp}^{2J}}{\pi V_{T\perp}^{2(J+1)} J!} \exp \left[ - \frac{v_{\perp}^2}{V_{T\perp}^2} \right]
\]

\[
f(V_{\parallel}) = \frac{1}{\pi^{1/2} V_{T\parallel}^{1/2}} \exp \left[ - \frac{v_{\parallel}^2}{V_{T\parallel}^2} \right]
\]

(5.17)

where \( J = 0, 1 \ldots \) is the distribution index, \( \varepsilon \) is the small parameter of the order of inverse density gradient scale length.

\[
V_{T\parallel}^2 = 2T_{\parallel} / m, \quad V_{T\perp}^2 = (J+1)^{-1} (2T_{\perp} / m)
\]

for \( J = 0 \), \( N(V) \) reduces to bi-Maxwellian velocity distribution function.
5.4 CURRENT DENSITY

To evaluate the perturbed current density we use the following set of equations.

\[
\bar{J}_{\text{ie}} = \int_0^\infty ds \int_0^{2\pi} dV_\perp \int_{-\infty}^{+\infty} dV_\parallel e \left[ \left( N + n_1 \right) \left( V + u \right) - NV \right]
\]

and \( \bar{J} = \bar{J}_{\text{ie}} - \bar{J}_e \) \hspace{1cm} (5.18)

with the help of equation (5.8), (5.14), (5.17) and (5.18) we obtain the current density in X and Z direction as.

\[
J_x = \frac{K_\perp K_\parallel^2 \lambda \Psi_1}{8\pi} \left[ \frac{\omega^2}{m e} \right] \left[ \frac{\phi_1 - \Psi_1}{\omega} \right] \left[ 1 - \frac{2(\omega - \omega_{iE})^2}{K_\parallel^2 V_\parallel^2 T_\parallel^2} \right] + \frac{2(\omega - \omega_{iE})}{K_\parallel^2 V_\parallel^2 T_\parallel^2} \Psi_1 \right] - \frac{\omega^2}{m_i \Omega_i} \frac{\phi_1}{(\omega - \omega_{iE})} \left( 1 - \frac{2(\omega - \omega_{iE})^2}{K_\parallel^2 V_\parallel^2 T_\parallel^2} \right) \left( \Psi_1 \right)
\]

\[
J_z = \frac{K_\parallel \Psi_i e \lambda}{8\pi} \left[ \frac{\omega^2}{m e} \right] \left[ \frac{K_\perp^2}{\Omega_i^2} \right] \left[ \frac{\phi_1 - \Psi_1}{\omega} \right] + \frac{2(\omega - \omega_{iE})}{K_\parallel^2 V_\parallel^2 T_\parallel^2} \Psi_1 \right] - \frac{8(\omega - \omega_{iE})\Psi_1}{K_\parallel^2 V_\parallel^4 T_\parallel^4}
\]

\[
- \frac{\omega^2}{m_i} \left[ \frac{K_\perp^2}{\Omega_i^2} \right] \frac{\phi_1}{(\omega - \omega_{iE})} - \frac{4 \Psi_1}{K_\parallel V_\parallel^3 T_\parallel^3} + \frac{4 \Psi_1}{V_\parallel^2 T_\parallel^4 (\omega - \omega_{iE})} \left[ 1 - \frac{2(\omega - \omega_{iE})^2}{K_\parallel^2 \rho_1^2 (J+1)} \right]
\]

\[
(5.20)
\]
where

\[ \omega_{iE} = \omega_E (1 - \delta), \quad \delta = \frac{\rho_{i}^2}{2a_i^2}, \quad \omega_{E} = -\frac{K_L E_o}{B_o} \]

Here we notice that the ion gyroradius correction has been included in the expression of \( \omega_{iE} \). The electron gyroradius correction is not taken into account.

5.5 DISPERSION RELATION

We would apply the change neutrality condition at \( x = 0 \) to obtain the dispersion relation.

\[ \tilde{n}_i \big|_{x=0} = \tilde{n}_e \big|_{x=0} \quad (5.21) \]

where \( n_{i,e} \) are the integrated perturbed densities for the non-resonant particles that is

\[ \tilde{n}_{i,e} = \int_0^\infty 2\pi V_{\perp} dV_{\perp} \int_{-\infty}^\infty \Psi \left( n_{i,e} \right) (r,t) \quad (5.22) \]

with the help of equation (5.12) and (5.17) we obtain

\[ \tilde{n}_i = \frac{\omega_{pi}}{4\pi q} \left\{ -\frac{K_L^2 \phi}{\Omega_{i}^2} + \frac{K_L^2}{(\omega - \omega_{iE})^2} \right\} (1 - \frac{1}{2} K_L^2 \rho_{i}^2 (J+1)) \]

\[ \tilde{n}_e = \frac{\omega_{pe}}{4\pi q V_{T_{le}}^2} \Psi \quad (5.23) \]

and we get the relation between \( \phi \) and \( \Psi \) using equation (5.21) and (5.23).
\[
\phi = - \frac{\Omega_i^2}{K_{\perp}} \left[ \frac{\omega_{pe}^2}{\omega_{iE}^2 V_{Te} (1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1))} - \frac{K_{\parallel}^2}{(\omega - \omega_{iE})^2} \right] \psi
\]  
(5.24)

Using perturbed ion and electron densities \( \tilde{n}_i \) and \( \tilde{n}_e \) and Ampere's law in the parallel direction we obtain the relation as

\[
\frac{\partial}{\partial z} \nabla_\perp^2 (\phi - \psi) = \frac{4\pi}{c^2} \frac{\partial J_z}{\partial t}
\]  
(5.25)

with the help of equation (5.24) and equation (5.25) we obtain the dispersion relation for kinetic Alfvén wave of the form.

\[
\left(1 - \frac{(\omega - \omega_{iE})^2}{K_{\parallel}^2 C_s^2 (1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1))}\right) \left(1 - \frac{\omega^2 (1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1))}{K_{\parallel}^2 V_A^2}\right)
\]

\[
\frac{\omega_i^2 E (1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1))}{K_{\parallel}^2 V_A^2} - \frac{\omega_{iE}^2 (\omega - \omega_{iE})^2}{c^2 \Omega_i^2 K_{\parallel}^2} \frac{K_{\parallel}^2 T_{\parallel i}^2}{\rho_i^2 m_i}
\]

\[
\left(- \frac{\omega_{pe}^2 \Omega_i^2}{K_{\perp}^2 \omega_{iE}^2 V_{Te} (1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1))} + \frac{K_{\parallel}^2 \Omega_i^2}{(\omega - \omega_{iE})^2} - 1\right) \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right)
\]  

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\[
\frac{\omega^2}{\omega_{\text{pe}}^2} \frac{v_{\text{Tie}}^2}{v_{\text{ei}}^2} = \frac{K_{\perp i}^2}{\Omega_i^2 K_{\parallel i}^2}
\]

where \( C_s^2 = \frac{\omega_{\text{pe}}^2}{\omega_{\text{pi}}^2} \) is the ion acoustic speed and \( v_A^2 = \frac{C_{\Omega i}^2}{\omega_{\text{pi}}^2} \) is the Alfvén speed.

Similarly we get the dispersion relation for drift kinetic Alfvén wave as.

\[
(1 - \frac{(\omega - \omega_{\text{iE}})^2}{K_{\parallel}^2 C_s^2 (1 - \frac{1}{2} K_{\perp i}^2 \omega_{\text{iE}}^2 (J+1))}) (1 - \frac{\omega_{\text{Dd}}^2}{K_{\parallel}^2 v_A^2} (1 - \frac{1}{2} K_{\perp i}^2 \omega_{\text{iE}}^2 (J+1))) + \\
\frac{\omega_{\text{iE}}^2}{K_{\parallel}^2 v_A^2} \left(1 - \frac{1}{2} K_{\perp i}^2 \omega_{\text{iE}}^2 (J+1)) \right) \frac{D_{\text{d}}}{D_{\text{iE}}} = \frac{K_{\perp i}^2}{\Omega_i^2} \frac{\omega_{\text{Dd}}^2}{K_{\parallel}^2 v_A^2} -
\]

\[
\frac{\omega_{\text{pe}}^2}{\omega_{\text{pi}}^2 v_{\text{Tie}}^2} (1 - \frac{1}{2} K_{\perp i}^2 \omega_{\text{iE}}^2 (J+1)) - \frac{K_{\parallel}^2}{\omega_{\text{pi}}^2 (\omega - \omega_{\text{iE}})^2} \left( \frac{\omega_{\text{pi}}^2 \omega_{\text{iE}}}{\omega_{\text{iE}}^2} \right) \frac{D_{\text{iE}}}{D_{\text{d}}} = \frac{K_{\perp i}^2}{\Omega_i^2} \frac{\omega_{\text{pe}}^2 (\omega - \omega_{\text{iE}})^2}{K_{\parallel}^2 v_{\text{Tie}}^2} m_i
\]
\[
\frac{V}{T_{\perp i}} \frac{K}{m_i \omega_i^2 (\omega - \omega_i)} \left( 1 - \frac{1}{2} \frac{K_{\perp i}^2}{K_{\parallel}^2} (J+1) \right) \left( 1 - \frac{1}{2} \frac{K_{\perp i}^2}{K_{\parallel}^2} (J+1) \right)
\]

(5.26)

where
\[
D_d = \left( 1 - \frac{V_i^d K_{\perp i}}{T_{\perp i}} \right) - \frac{\Omega_i^2}{K_{\perp i}^2} \frac{m_i}{(\omega - \omega_i)} \right)^{-1}
\]

5.6 ENERGY BALANCE AND GROWTH RATE

The wave energy density \( W_w \) per unit wavelength is the sum of the pure field energy and the changes in energy of the non-resonant particles that is

\[
W_w = \frac{\lambda K^2 \Psi^2}{8\pi} + W_i + W_e
\]

where \( W_{i,e} \) is defined by.

\[
W_{i,e} = \int_0^\infty ds \int_0^\infty \frac{\lambda \omega}{2\pi} \frac{\omega}{dV} \int_0^\infty \frac{m_{i,e}}{2} \left[ (N + n_1)(\nu + u)^2 - \bar{N} \bar{V}^2 \right]_{i,e}
\]

We have evaluated equation (5.27) with the help of equation (5.8), (5.13) and (5.17) with \( \Lambda = 0 \) by substituting.

\[
K_S = r^2; \quad \lambda = \frac{2\pi}{K}; \quad \omega^2 = \frac{4\pi N_{i,e}^2}{m_{i,e}}
\]

Under the basic assumption we get the energy associated with the ion and electron components of non-resonant particles as
\[ W_i = \frac{\lambda K_i^2 \Psi_{i1}^2}{16\pi} \frac{\omega^2}{(\omega - \omega_{ie})^2} (1 - K_{i\perp}^2 \rho_i^2 (J+1))(1 - 4 \frac{T_{\parallel i}}{T_{\perp i}} \frac{K_{\perp i} V_i}{(\omega - \omega_{ie})}) \]

\[ W_e = \frac{\lambda K_{\parallel e}^2 \Psi_{e1}^2}{16\pi} \frac{\omega^2}{\omega_{pe}} K_{\parallel e}^2 \left( \frac{T_{\parallel e}}{m_e} \right) \]

(5.28)

The oscillatory motion of the non-resonant electrons contains the major part of the energy (Misra and Tiwari 1979).

Hence

\[ W \approx W_e \approx \frac{\lambda K_{\parallel e}^2 \Psi_{e1}^2}{16\pi} \frac{\omega^2}{\omega_{pe}} K_{\parallel e}^2 \left( \frac{T_{\parallel e}}{m_e} \right) \]

(5.29)

We calculate the resonance energy of the electrons per unit wavelength.

The main contribution is given by the parallel components of velocity.

Expanding the integral around \( V_{\parallel} = \omega^*/K_{\parallel} \) we find the expression for energy as (Misra and Tiwari 1979).

\[ W_r = -\pi \frac{\lambda \Psi_{11}^2}{8\pi} \frac{\omega^2}{\omega_{pe}} \frac{K_{\perp} V_{\perp}^e}{T_{\perp e} \left( \frac{m_e}{K_{\parallel}} \right)} f_{\parallel e} \left( \frac{\omega_e}{K_{\parallel}} \right) f_{\parallel e} \left( \frac{\omega_e}{K_{\parallel}} \right) \frac{\omega^*}{\omega^*} \]

(5.30)

Where

\[ \int_{-\infty}^{\infty} 2\pi V_{\perp} dV_\perp f_\perp (V_{\perp}) \int_0^4 J_0^4 (\mu) = 1 - (J + 1) K_{\perp i}^2 \rho_i^2 \]

Using the law of energy conservation

\[ \frac{d}{dt} (W + W_r) = 0 \]
The growth rate is derived as.

\[
\gamma = \frac{1}{\phi} \frac{d\phi}{dt} = \pi \omega^* \frac{T_{\| e}}{m_e} \left\{ \frac{K_{\perp e}^2}{K_{\| \perp e} / m_e} \right\} f_{\| e}(\omega_e^* / K_{\|}) + f'_{\| e}(\omega_e^* / K_{\|}) \]

(5.31)

Substitution for \( f_{\| e}(\omega_e^* / K_{\|}) \) and \( f'_{\| e}(\omega_e^* / K_{\|}) \) the expression transforms to.

\[
\gamma = \gamma \frac{\omega_e^*}{e} \left[ \frac{T_{\| e}}{T_{\perp e}} - 1 \right] \exp \left[ - \frac{\omega_e^*}{e} \right] \]  (5.32)

In equation (5.31) and (5.32) it should be noted that the growth rate is affected by the dispersion relation and different electric fields and the dispersion relation produces different results for different values of distribution index \( J \). Hence the distribution index modifies the growth rate as well as dispersion relation.

5.7 RESULT AND DISCUSSION

In the present analysis the expressions for the dispersion relation, current and growth rate have been derived in the presence of variable electric field including the effect of the steepness of loss-cone. The following parameters have been used to evaluate the dispersion relation, current and growth rate in auroral acceleration region (Wong et. al. 1985).

\[ B_0 = 4300 \text{ nT, } \Omega_i = 412 \text{ Sec}^{-1}, \quad K_{\|} = 1 \times 10^{-10} \text{ m/Sec, } T_{\| e} / T_{\perp e} = 2, \frac{V_e}{V_d} = \]
200 m/Sec, \( \frac{\omega_p^2}{\Omega_i^2} = 100 \), \( T_{||}/T_{\perp} = 4 \).

The dispersion relation (equ. 5.26) has been solved numerically using Newton Raphson's method and the values of \( \omega \) are plotted versus \( K_{\perp} \rho_i \) for different values of distribution index \( J \) in figure 5.1 and 5.2 for the different electric fields. It is noted that wave frequency \( \omega \) decreases and frequency band reduces in width for higher values of \( J \). Thus lower frequencies may be possible in narrow emission band of \( K_{\perp} \) which may be due to the decrease of ion drift velocity by the averaging of wave field over the Larmor orbit in the presence of steep loss-cone distribution function. Effect of electric field on the frequency \( \omega \) is also seen in the form of Doppler shift. The increasing value of \( \omega \) with respect to \( K_{\perp} \rho_i \) is due to finite Larmor radius effect which also is the finding of the model.

Figure 5.3 and 5.4 show the variation of growth rate \( \gamma/\omega \) with \( K_{\perp} \rho_i \) for different values of \( J \), and for different values of electric field \( E_o \). The graph exhibits the increase of the growth rate for the higher values of \( J \) as well \( K_{\perp} \rho_i \). It is also clear that the growth rate decreases with applied electric field. The shifting of the maximum growth rate towards the lower values of \( K_{\perp} \rho_i \) with higher value of \( J \) exhibits that wave emissions of shorter wave number are possible for higher \( J \). The value of \( \omega_{iE_o} \) increases with \( E_o \) and the frequency is shifted towards lower values and hence the growth rate decreases with \( E_o \). On the other hand the effect of loss-cone distribution index \( J \) is remarkable on
$\mathbf{E}_0 = -150 \, \mu \text{V/m}$
$B_0 = 4300 \, \text{nT}$
$KT_{\text{He}} = 100 \, \text{eV}$
$KT_{\text{He}} = 10 \, \text{KeV}$
$K_{\text{He}} = 1.0 \times 10^{-10} \, \text{m}$
$rac{T_{\text{He}}}{T_{\text{Le}}} = 4$
$rac{\omega^2 \pi i}{\mathcal{N}_i} = 100$

Fig. 5.1 Frequency ($\omega$) versus perpendicular wave number ($K_{\perp} \gamma_i$) for different distribution index $J$. 
Fig. 5.2 Frequency (ω) versus perpendicular wave number (K⊥S_i) for different distribution index J.

- E_0 = 200 μV/m
- B_0 = 4300 nT
- K_{T_{ei}} = 100 eV
- K_{T_{He}} = 10 KeV
- \( K_n = 1.0 \times 10^{-10} \text{ m}^{-1} \)
- \( \frac{T_{He}}{T_{le}} = 4 \)
- \( \frac{\omega^{2d_i}}{\sqrt{\lambda_{2,i}^2}} = 100 \)
Fig. 5.3 Growth rate ($\sqrt{\gamma}/\omega$) versus perpendicular wave number ($K_p \rho_i$) for different distribution index $J$. 

- $E_0 = -150 \mu V/m$
- $B_0 = 4300 nT$
- $K T_{\parallel i} = 100 eV$
- $K T_{\parallel e} = 10 KeV$
- $K_{\parallel i} = 1.0 \times 10^{-10} m$
- $T_{\parallel e}/T_{\perp e} = 4$
- $\omega_{pi}^2/\Omega_i^2 = 100$
Fig. 54 Growth rate ($\gamma / \omega$) versus perpendicular wave number ($k_{\perp} \rho_i$) for different distribution index $J$.

- $E_0 = -200 \, \text{mV/m}$
- $B_0 = 4300 \, \text{nT}$
- $K_{Ti} = 100 \, \text{eV}$
- $K_{Te} = 10 \, \text{KeV}$
- $K_{H} = 1.0 \times 10^{-6} \, \text{m}$

$T_{He} \quad T_{Te}$

$\frac{\omega_{pi}^2}{\omega_{ci}^2} = 100$
the growth rate because of correction due to ion gyroradius.

Figure 5.5 and 5.6 are plotted for current in Z direction $J_z$ versus $K_\perp \rho_i$ for different $J$ and electric field $E_o$. It is found that $J_z$ decreases rapidly with higher values of $K_\perp \rho_i$ and become negative. $J_z$ also decreases with higher value of distribution index $J$ but the effect of electric field $E_o$ on $J_z$ is negligible because of the direction of electric field is perpendicular to the magnetic field. For the higher value of $K_\perp$ the ionic current in more effective, which becomes negative. Thus the distribution index has directly effected the ionic current.

Figure 5.7 and 5.8 predict the variation in current in x direction $J_x$ with $K_\perp \rho_i$ for different distribution index $J$ and electric field $E_o$. It is noticed that the perpendicular current is increased with $E_o$ and the steepness of loss-cone. From the figure it is clear that perpendicular current reverses the direction due to steepness of loss-cone distribution index $J$. The peak value of perpendicular current corresponds to the direction reversal of parallel current. This may be due to mirroring of charged particles as the particles are reflected back at the mirroring point. The hot particles may be the agents for generating the field aligned and perpendicular currents. Because the thermal energy associated with particles within the middle magnetosphere regions dominants the bulk flow energies it is the current associated with the hot particles that are thought to generate currents. Thus we may conclude that field aligned and perpendicular currents associated
Fig. 5.5 Parallel current ($J_z$) versus perpendicular wave number ($K_\perp \eta_i$) for different distribution index $J$.
Fig. 5.6 Parallel current ($J_z$) versus perpendicular wave number ($K_{\perp} \varphi_1$) for different distribution index $J$. 

- $E_0 = -200 \mu V/m$
- $B_0 = 4300 nT$
- $K_{Te} = 100 eV$
- $K_{Te} = 10 KeV$
- $K_{He} = 1.0 \times 10^{-10} m^{-1}$
- $\frac{T_{He}}{T_{Le}} = 4$
- $\frac{\omega_{pi}}{\omega_{ci}} = 100$
- $J = 1$
- $J = 2$
- $J = 4$
Fig. 5.7 Perpendicular current \((J_X)\) versus perpendicular wave number \((K_\perp \rho_i)\) for different distribution index \(J\).
Fig. 5.8 Perpendicular current \( J_x \) versus perpendicular wave number \( K_{||} \rho_i \) for different distribution index \( J \)

- \( E_0 = -200 \text{ mV/m} \)
- \( B_0 = 4300 \text{ nT} \)
- \( K T_{||i} = 100 \text{ eV} \)
- \( K T_{\perp e} = 10 \text{ keV} \)
- \( K_{||} = 1.0 \times 10^{-10} \text{ m}^{-1} \)
- \( \frac{T_{\perp e}}{T_{\perp i}} = 4 \)
- \( \frac{\omega_{pe}^2}{\sqrt{\omega_i^2}} = 100 \)
with kinetic Alfvén wave in the magnetosphere-ionosphere coupling system depend upon perpendicular wave length and ion gyroradius by the direction and magnitude. The wave may be generated by pressure gradient on magnetic flux tube in the distant magnetosphere and propagates towards the ionosphere constituting field-aligned and closure current. The field aligned currents may reverse its direction while propagating towards the ionosphere as the magnetic flux tube converges and the distribution index becomes higher. The frequency of the wave is determined by the perpendicular wave number and the ion gyroradius. The wave characteristics are controlled by perpendicular electric field in the system.

The theory may be applicable to other space plasmas as well as laboratory plasmas, sharp density gradients appear in a variety of geophysical processes in the near space region of the earth for example in auroral acceleration region and plasma sheet boundary layer. In Tokamak reactors for heating and confinement purposes the large value of radial electric field and its gradient have been associated with kinetic Alfvén wave. In most of the theoretical work the velocity distribution function have been assumed to be ideal Maxwellian although most turbulent heating experiments have been done in mirror like devices which in general allow non Maxwellian, particularly loss-cone distribution function (Wong et. al. 1985, Hirose 1976). The theory may be applicable to such hot particle mirror experiments. Single particle
theories may be able to explain some of the plasma phenomena where other
theories are not well suited.

5.8 SUMMARY:

Dispersion relation, current and growth rate of the kinetic Alfven wave with general loss-cone distribution function in a low $\beta$
inhomogeneous plasma in the presence of an inhomogeneous electric field
applied perpendicular to the magnetic field have been obtained by
investigating the trajectories of the charged particles. The whole
plasma is considered to consist of resonant and non-resonant particles.
It is assumed that non-resonant particles support the oscillatory motion
of kinetic Alfven wave while the resonant particles participate in the
energy exchange with the wave. The effects of steepness of loss-cone
distribution and inhomogeneity of electric field are discussed on the
dispersion relation, current and the growth rate of the instability. The
results are interpreted for the space plasma parameters appropriate to
the auroral acceleration region of the earth’s magnetoplasma.