KINETIC ALFVEN WAVE IN THE PRESENCE OF GENERAL LOSS-CONE DISTRIBUTION
FUNCTION IN INHOMOGENEOUS MAGNETOPLASMA - PARTICLE ASPECT ANALYSIS
3.1 INTRODUCTION

In the previous chapter we have investigated the dispersion relation, growth rate and field aligned currents for the auroral acceleration region by means of kinetic Alfven waves. The main purpose of this chapter is to study the special case of drift kinetic Alfven wave instability associated with loss-cone distribution function. The present chapter emphasises the investigation of the effect of loss-cone distribution function on electromagnetic drift kinetic Alfven waves and the steepness of distribution index on the dispersion relation and growth rate of the instability.

Among the first type of waves found to exist in plasmas, kinetic Alfven waves play an important role in energy transport, in particle acceleration and heating, in driving field-aligned currents and in explaining ultra-low frequency (ULF) emissions in the Earth's magnetosphere (Leonovich 1995, Klimushkin 1997, Huang et. al. 1997). Theoretical studies of both the high latitude ULF phenomena and the waves at equatorial to middle latitudes involve kinetic Alfven waves (KAW). These waves are considered in a number of models as an agent of magnetosphere-ionosphere coupling.

Field-aligned currents play a dominant role in the study of magnetized plasmas of magnetosphere-ionosphere coupling. In the magnetohydrodynamic description (valid where time and spatial scales of motion are large compared to the gyroperiod and gyroradius
respectively), if perturbations of flow develop on one part of a flux tube, field-aligned currents must flow in order to communicate the changes to the entire flux tube. They are perhaps of most importance in magnetospheric physics in the study of coupling between regions where different dynamical conditions prevail but which are threaded by the same field (Southwood and Kivelson 1991). Under the situation kinetic Alfven waves may be generated by density inhomogeneity of plasma sheet and propagate to the ionosphere.

Magnetohydrodynamic approach of Alfven waves in the magnetosphere has been almost restricted to using simplified models assuming cold ions. However, the finite Larmor radius of warm ions in the equatorial magnetosphere can determine the perpendicular wavelength of kinetic Alfven waves. Inclusion of finite ion Larmor radius effects may provide the necessary dispersion which limits the short wavelength transverse scale of MHD models (Marchenko et. al. 1996).

In the past propagation of kinetic Alfven waves in an inhomogeneous plasma has been studied in a number of investigations (Mikhailovskii 1967, Hasegawa and Chen 1975, Hasegawa and Mima 1978, Moghaddam et.al.1989) has predicted that the drift wave instability, in a finite beta plasma, excites the Alfven waves. Since the drift wave has a perpendicular wavelength comparable to ion gyroradius, the excited Alfven wave is actually the kinetic Alfven wave, which was first introduced by Hasegawa and Chen (1975) in connection with plasma
heating. In the context of magnetosphere-ionosphere coupling kinetic Alfven waves have been studied in both homogeneous and inhomogeneous magnetic fields in various investigations using basically, a magnetohydrodynamic approach (Klimushkin 1997, Goertz 1984, Leonovich and Mazur 1989,1995). Leonovich and Mazur (1989,1995) have studied kinetic Alfven waves in a two dimensional inhomogeneous plasma in a curved magnetic field.

In most of the theoretical work carried out so far for the analysis of kinetic Alfven waves the velocity distribution functions have been assumed either as ideal Maxwellian or bi-Maxwellian ignoring the steepness of the loss-cone feature. Plasmas in mirror like devices and in the auroral region with curved and converging magnetic field lines, may considerably depart from a Maxwellian distribution and become anisotropic, provided there is a relatively low degree of plasma collisionality, and in general permit steep loss-cone distribution functions (Tiwari and Varma 1993, Varma and Tiwari 1992,Gaelzer et. al. 1997). Our purpose in this paper is to investigate the effect of steepness of the loss-cone distribution on kinetic Alfven waves in the inhomogeneous magnetospheric plasma. An alternative model usually called as particle aspect analysis (Tiwari and Varma 1993,1991,Varma and Tiwari 1992) is applied for the first time and offers an advantage over the magnetohydrodymanmic approach in dealing with the finite Larmor radius effect and temperature anisotropy in the inhomogeneous magnetoplasma.
The basic assumptions are those of earlier work on drift waves (Tiwari and Varma 1993) in which the plasma has been considered to consist of resonant and non-resonant particles and the wave growth was discussed by energy conservation method. We have considered a kinetic Alfvén wave propagating obliquely to the constant magnetic field (z-direction), and two different potentials in the x-z plane for the evaluation of the charged particle trajectory. The direction of density gradient is along the y-axis.

The organization of the chapter is as follows. In section-II we have evaluated charged particle perturbed velocities in the presence of kinetic Alfvén wave fields. Density perturbation in the presence of KAW is evaluated in section-III and section IV considers the dispersion relation. In section V we have evaluated the expressions for current densities. Expressions for energy balance and growth rate are derived in section VI. Section VII deals with results and discussion.

3.2 BASIC TRAJECTORIES

In the present model we have developed the particle aspect analysis for electromagnetic kinetic Alfvén waves in low $\beta \ (\beta = 2\mu_o P/B_o^2)$ inhomogeneous plasma with general loss-cone distribution function. In an anisotropic plasma the wave is considered to propagate in a plane perpendicular to the density gradient i.e. in the x-z plane. In the mathematical analysis we follow the procedure considered in ref. (Tiwari and Varma 1993, 1991, Varma and Tiwari 1992, Terashima 1967).
The kinetic Alfvén wave is assumed to start at $t=0$ when the resonant particles are undisturbed. We are interested in the kinetic Alfvén wave which satisfy the condition:

$$V_{T_{i}||} << \frac{\omega}{K_{||}}, \quad \omega << \bar{\Omega}_{i}, \bar{\Omega}_{e}; \quad K_{\perp i}^{2} \rho_{i}^{2} << K_{\perp e}^{2} \rho_{e}^{2} < 1 \quad (3.1)$$

where $V_{T_{i}||}$ and $V_{T_{e}||}$ are the mean velocities of ions and electrons along the magnetic field, $\Omega_{i,e}$ are gyration frequencies and $\rho_{i,e}$ the mean gyroradii of the respective species.

In the present chapter we have extended particle aspect analysis for electromagnetic perturbations. We have adopted two potential representation which is commonly used to express electromagnetic perturbations in low-$\beta$ plasmas (Hasgawa and Chen 1975, 1976, Hasgawa and Mima 1978, Kadomtsev 1965). The idea is to decouple the compressional Alfvén mode by assuming $(\nabla \times E)_{\parallel} = -\frac{\delta B_{z}}{\delta t} = 0$, that is taking into account only the effect of the field line bending. This enables one to use a scalar potential $\phi$ to express the perpendicular components of the wave electric field,

$$E_{\perp} = -\nabla_{\perp} \phi$$

Because the wave is electromagnetic, however, we have to use $E_{\parallel}$ which is not equal to $-\frac{\delta \phi}{\delta z}$. For $E_{\parallel}$ we hence use a different potential, $\Psi$;

$$E_{\parallel} = -\nabla_{\parallel} \Psi,$$

and $\phi \neq \Psi$. The potentials $\phi$ and $\Psi$ must satisfy suitable field equations.
We begin by considering the wave electric field $E$ of the form

$$ E = E_\perp + E_\parallel $$

$$ \phi = \phi_0 \cos (K_\perp x + K_\parallel z - \omega t) $$

$$ \Psi = \Psi_0 \cos (K_\perp x + K_\parallel z - \omega t) \quad (3.2) $$

where $\phi_0$ and $\Psi_0$ are assumed to be a slowly varying function of time $t$, and $\omega$ is the wave frequency. The equation of motion of particle is

$$ m \frac{d\vec{v}}{dt} = q \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B}_0 \right) \quad (3.3) $$

where the collisions between particles are neglected. $\vec{v}$ can be expressed as a sum of the unperturbed velocity $\vec{v}$ and the perturbed velocity $\vec{u}$, i.e. $\vec{v} = \vec{v} + \vec{u}$, $\vec{u}$ is determined by the following set of equations

$$ \frac{du_x}{dt} + i \Omega u_x = -\frac{q}{m} \left[ \phi_0 K_\perp - \frac{V_\parallel K_\parallel}{\omega} (\phi_0 - \Psi_0) \right] \sin (K_\perp x + K_\parallel z - \omega t) $$

$$ \frac{du_y}{dt} = \frac{q}{m} \left[ K_\parallel \Psi_0 + \frac{V_\parallel K_\perp}{\omega} (\phi_0 - \Psi_0) \cos (\dot{\phi} - \Omega t) \right] \sin (K_\perp x + K_\parallel z - \omega t) \quad (3.4) $$

where $u_x = u_x + iu_y$, $\dot{\phi}$ is the initial phase of velocity and $\Omega = q B_0 / mc$

$u_x$ and $u_y$ are the perturbed velocities in the $x$ and $y$ directions respectively. Equation(3.4) is solved by replacing the coordinates of charged particles to that of free gyration (Tiwari and Varma 1993) which provides perturbed velocity $\vec{u}(t)$. The perturbed velocity $\vec{u}(t)$ can be further transformed to $\vec{u}(\vec{r},t)$ by the use of trajectories of free gyration once again (Tiwari and Varma 1993). For the resonant particles it is necessary to take into account the initial condition $\vec{u}(\vec{r},t=0) = 0$
inferred from the assumption stated above. Thus

\[ u_x(\mathbf{r}, t) = - \frac{q}{m} \left[ \phi_1 K_{1\perp} - \frac{V_{\parallel}}{\omega} K_{1\parallel} \right] \left( \phi_1 - \Psi_1 \right) \sum_{n} \frac{J_n(\mu)}{\sum_{1} J_1(\mu)} \]

\[ \frac{\Lambda_n}{2} \cos \xi_{n1} - \frac{\delta}{2} \Lambda_{n+1} \cos(\xi_{n1} - \Lambda_{n-1} t) - \frac{\delta}{2} \Lambda_{n-1} \cos(\xi_{n1} - \Lambda_{n+1} t) \]

\[ u_y(\mathbf{r}, t) = - \frac{q}{m} \left[ \phi_1 K_{1\perp} - \frac{V_{\parallel}}{\omega} K_{1\parallel} \right] \left( \phi_1 - \Psi_1 \right) \sum_{n} \frac{J_n(\mu)}{\sum_{1} J_1(\mu)} \]

\[ \frac{\Omega_n}{2} \sin \xi_{n1} - \frac{\delta}{2} \Lambda_{n+1} \sin(\xi_{n1} - \Lambda_{n-1} t) + \frac{\delta}{2} \Lambda_{n-1} \sin(\xi_{n1} - \Lambda_{n+1} t) \]

\[ u_z(\mathbf{r}, t) = - \frac{q}{m} \left[ K_{\parallel} \Psi_1 + \frac{V_{\parallel}}{\omega} K_{1\parallel} \right] \left( \phi_1 - \Psi_1 \right) \sum_{n} \frac{J_n(\mu)}{\sum_{1} J_1(\mu)} \]

\[ \frac{1}{\Lambda_n} \left[ \cos \xi_{n1} - \delta \cos(\xi_{n1} - \Lambda t) \right] \]

(3.5)

where \( \delta = 0 \) for the non-resonant particles and \( \delta = 1 \) for resonant one and

\[ \Lambda_n = n \Omega + V_{\parallel} \frac{K_{\parallel}}{\omega} - \omega; \quad \mu = K_{1\perp} \frac{V_{\perp}}{\Omega} \]

\[ \xi_{n1} = (1 - n)(\Theta - \Omega t) + K_{\perp} x + K_{\parallel} z - \omega t \]

(3.6)

\[ a_n^2 = \Lambda_n^2 - \Omega^2; \]
Also use was made of
\[
\exp[-i \mu \sin(\Theta - \Omega t)] = \sum_{-\infty}^{+\infty} J_n(\mu) \exp[-i n(\Theta - \Omega t)]
\]

\[
\cos \phi \cdot \exp(-i \mu \sin \phi) = \frac{1}{\mu} \sum_{-\infty}^{+\infty} J_n(\mu) \exp(i n \phi)
\]

Integration of equation (3.5) gives the perturbed coordinates of the particles \(x, y, z\) which in addition to trajectories of free gyration exhibits the true path of the particles. In view of the approximations introduced in the beginning, the dominant contribution comes from the term \(n=0\). The resonant criterion is given by \(k_1 V - \omega = 0\). Therefore, this resonance condition means that the electrons see the wave independent of \(t\) in the particles frame. The particles satisfying the above conditions are called resonant. \(J_n(\mu)\) and \(J_1(\mu)\) are Bessel's functions which arise from the different periodical variation of charged particle trajectories. The terms represented by Bessel's functions show the reduction of the field intensities due to finite gyroradius effect.

3.3 DENSITY PERTURBATION

In order to find out the density perturbation associated with the velocity perturbation, \(\vec{u}(\vec{r}, t)\), we consider the equation (Tiwari and Varma 1993, Terashima 1967)

\[
\frac{dn_1}{dt} = - \tilde{(\vec{\nabla} \cdot \vec{u})} N - u_y \frac{dN}{dy} \tag{3.7}
\]

where \(N(V)\) represents the zeroth order distribution function. Expressing the right hand side of the equation (3.7) as a function of \(t\) (Tiwari and
Varma 1993) and after integration we obtain the perturbed density for the non-resonant and resonant particles in the presence of kinetic Alfvén wave for the inhomogeneous plasma as,

\[ n_1(\mathbf{r}, t) = N(\mathbf{V}) \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_1(\mu) \frac{q}{m} \left[ (\phi_1 - \frac{V_\parallel K_\parallel}{\omega}) (\phi_1 - \Psi_1) \right] \]

\[ \frac{k_\perp^2}{2a_n} - \Omega_T^2 \frac{V_\parallel K_\parallel}{nQ_2 T_\perp} + \frac{k_\parallel^2}{\Lambda_n} \left[ \frac{V_\parallel K_\parallel}{n} (\phi_1 - \Psi_1) \right] \cos \xi_{n1} \]

(3.8)

and for resonant particles

\[ n_1(\mathbf{r}, t) = N(\mathbf{V}) \sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_1(\mu) \frac{q}{m} \left[ (\phi_1 - \frac{V_\parallel K_\parallel}{\omega}) (\phi_1 - \Psi_1) \right] \]

\[ \left[ \frac{k_\perp^2}{2a_n} - \Omega_T^2 \frac{V_\parallel K_\parallel}{nQ_2 T_\perp} \right] \cos \xi_{n1} - \frac{1}{2\Omega_T} \Lambda_{n+1} \cos(\xi_{n1} - \Lambda_{n+1} t) \]

\[ K_\perp \frac{V_\parallel K_\parallel}{T_\perp} m \Omega_T + \frac{1}{2\Omega_T} \Lambda_{n-1} \cos(\xi_{n1} - \Lambda_{n-1} t) \left[ K_\parallel^2 \frac{V_\parallel K_\parallel}{T_\perp} m \Omega_T \right] \]

\[ \left[ \frac{\Psi_1}{\mu} + \frac{K_\perp V_\parallel}{\mu \omega} (\phi_1 - \Psi_1) \right] \frac{k_\parallel^2}{\Lambda_n^2} \left[ \cos \xi_{n1} + \Lambda t \sin(\xi_{n1} - \Lambda t) \right] - \cos(\xi_{n1} - \Lambda t) \right] \}

(3.9)

where \( V_d \) is the diamagnetic drift velocity which is defined by
\[ V_d = \frac{T}{m \Omega} \cdot \frac{1}{N} \cdot \frac{\partial N}{\partial y} \]

where \( V_d = 0 \), for the homogeneous plasma, and \( T_\perp \) is the perpendicular temperature. To calculate the dispersion relation and growth rate we use the general loss-cone distribution function of the form (Tiwari and Varma 1993, Gaelzer et. al. 1997, Wong et. al. 1985, Dory et. al. 1965)

\[
N(y, V) = \frac{N_0 V_\perp^{2J}}{\pi^{3/2} V_\perp^2 (J+1) V_\parallel^{2J} J!} \exp \left[ -\frac{V_\perp^2}{V_\perp^2} - \frac{V_\parallel^2}{V_\parallel^2} \right]
\]

\[
f_\perp(V_\perp) = \frac{V_\perp^{2J}}{\pi V_\perp^{2(J+1)} J!} \exp \left[ -\frac{V_\perp^2}{V_\perp^2} \right]
\]

\[
f_\parallel(V_\parallel) = \frac{1}{\pi^{1/2} V_\parallel} \exp \left[ -\frac{V_\parallel^2}{V_\parallel^2} \right]
\]

(3.10)

where \( \epsilon \) is a small parameter of the order of inverse of density gradient scale length, \( J \) is an integer known as "the loss-cone index" (Gaelzer et. al. 1997). In the case of \( J=0 \) this represents a bi-Maxwellian distribution and for \( J=\infty \) this reduces to the Dirac Delta function (Tiwari and Varma 1993). \( \frac{V_\parallel^2}{V_\parallel} = \frac{2T_\parallel}{m} \) and \( \frac{V_\perp^2}{V_\perp} = (J+1)^{-1} \frac{2T_\perp}{m} \) are the squares of parallel and perpendicular thermal velocities with respect to the external magnetic field. Index \( J \) characterizes the width of the
loss-cone. Moreover, $f_{\perp}(V_{\perp})$ is peaked about $J^{1/2} V_{T\perp}$ and has a half width of $\Delta V_{\perp} \sim J^{-1/2} V_{T\perp}$ (Varma and Tiwari 1992). In the present chapter we introduce further developments in the description of the KAW for inhomogeneous plasma and evaluate a generalized dispersion relation useful for the plasma particles described by nonrelativistic loss-cone distribution of Dory-Guest-Harris type (Dory et. al. 1965).

3.4 DISPERSION RELATION

To evaluate the dispersion relation we calculate the integrated perturbed density for non-resonant particles as

$$\tilde{n}_{i,e} = \int_0^\infty 2\pi V_{\perp} \, dV_{\perp} \int_{-\infty}^\infty dV_{\parallel} \, n_{i,e}(\vec{V},t)$$  \hspace{1cm} (3.11)

with the help of equation (3.8) and (3.10) we find for inhomogeneous plasma

$$\tilde{n}_i = \frac{N_{e0}}{m_i} \left[ -\frac{K_{\perp}^2 \Phi}{\Omega_i^2} + \frac{K_{\parallel}^2 \Psi}{2\omega} + \frac{V_i K_{\perp} \rho_i}{T_{\perp i} \omega} \right] (1 - \frac{1}{2} \frac{K_{\perp}^2 \rho_i^2}{\Omega_i^2} (J+1))$$  \hspace{1cm} (3.12a)

$$\tilde{n}_e = \frac{\omega^2}{\omega_{pe}} \frac{\Psi}{4\pi e V_{T\parallel e}^2}$$  \hspace{1cm} (3.12b)

It is observed that the essential feature of the kinetic Alfven wave is retained even in this ideal case. For Maxwell's equation we use the quasi-neutrality condition,

$$\tilde{n}_i = \tilde{n}_e$$
and we get the relation between $\phi$ and $\Psi$ as,

$$
\phi = -\frac{2}{K_\parallel} \left[ \frac{\Omega_i^2}{\omega_p e} \frac{2}{\omega_{pe}} \frac{V_i^2}{T_i} (1 - \frac{1}{\omega} K_\perp \varphi_i^2(J+1)) - \frac{K_\parallel^2}{\omega^2} \right].
$$

\[ (1 - \frac{V_i}{T_i} K_\parallel) \frac{\Omega_i^2}{\omega} \frac{m_i}{K_\perp} \]^{-1} \Psi \quad (3.13)

Using perturbed ion and electron densities $\tilde{n}_i$ and $\tilde{n}_e$ and Ampere's law in the parallel direction, we obtain the equation,

$$
\frac{\partial}{\partial z} \nabla_{\perp}^2 (\phi - \Psi) = \frac{4\pi}{C_i^2} \frac{\partial}{\partial t} J_z \quad (3.14)
$$

where,

$$
J_z = e \int_{-\infty}^{\omega} 2\pi V_{\perp} dV_{\perp} \int_{-\infty}^{+\infty} dV_{\parallel} [(N(\tilde{V}) u_{z}(\tilde{r},t) + V_{\parallel} n_1(\tilde{r},t) - (N(\tilde{V}) u_{z}(\tilde{r},t) + V_{\parallel} n_1(\tilde{r},t))]_{e}
$$

and $J_z$ is the current density which is contributed by first order perturbations of density and velocity. With the help of equation (3.13) and equation (3.14) we obtain the dispersion relation for kinetic Alfvén wave in inhomogeneous plasma as,

$$
(1 - \frac{\omega^2}{K_\parallel^2 C_i^2} (1 - \frac{1}{\omega} K_\perp \varphi_i^2(J+1))) (1 - \frac{\omega^2}{K_\parallel^2 V_A})
$$
\[ D_d = \frac{K_{\perp}^2 \omega_i^2}{K_{\parallel}^2 \Omega_i^2} \left( \frac{\omega_p^2}{\omega_{pe}} \right) \left( \frac{\omega_{pe}^2}{\omega_{pi}^2} \right) \left( 1 - \frac{1}{2} K_{\perp}^4 \rho_i^2 (J+1) \right) - \frac{K_{\parallel}^2}{\omega_i^2}. \]

\[
\left( \frac{\omega_{pi}^2}{\omega_i^2} \right) \frac{\omega_{pi}^2}{C^2 \Omega_i^2 K_{\parallel}^2 m_i T_{\perp i}} - \frac{\omega_{pi}^2}{\omega_i^2} \frac{\omega_{pi}^2}{C^2 \Omega_i^2 K_{\parallel}^4 K_{\perp}^4} \left( 1 - \frac{1}{2} K_{\perp}^4 \rho_i^2 (J+1) \right). \]

(3.15)

where

\[ D_d = \left( 1 - \frac{V_{d i}^2}{\Omega_i^2} - \frac{m_i}{K_{\perp}^2} \right)^{-1} \]

where \( C_s^2 = \frac{\omega_{pi}^2 V_{T i}^2}{\omega_{pe}^2} \) is the ion acoustic speed and \( V_A^2 = \frac{C_s^2 \Omega_i^2}{\omega_{pi}^2} \)

is the Alfven speed. The dispersion relation of kinetic Alfven wave reduces to that derived by Hasegawa and Chen (1976) under the approximation, \( V_{d i}^2 = 0, J = 0 \) and \( I_0(\lambda_i) e^{-\lambda_i} \sim 1 - \lambda_i \) (Terashima 1967), for \( \lambda_i = \frac{1}{2} K_{\perp}^2 \rho_i^2 < 1 \) as we have applied. \( I_0(\lambda_i) \) is the modified Bessel function.

3.5 CURRENT DENSITY

Since the average value of current vanishes which is contributed by first order perturbations of density and velocity due to their periodical
variations, we evaluate the average current which is the second order perturbation. To evaluate the perturbed current density we use the following set of equations.

\[
\overline{J}_{ie} = \int_0^\infty ds \int_0^{2\pi} V_\perp dV_\perp \int_{-\infty}^{+\infty} dV_\parallel e \left[ (N + n_1)(\overline{V} + \overline{u}) - \overline{N} \right]
\]

and

\[
\overline{J} = \overline{J}_i - \overline{J}_e
\]  \hspace{1cm} (3.16)

with the help of equations (3.5) and (3.8) and (3.10) we obtain

\[
\overline{J}_{xe} = \frac{Ne_3 K_\perp}{2} \frac{2}{m_e} \left[ \frac{\Psi_1 (\phi_1 - \Psi_1)}{2 \omega} - \frac{2 \omega m_e}{K_\parallel} \frac{1}{\Omega_1} \right] \]  \hspace{1cm} (3.17)

\[
\overline{J}_{xi} = -\frac{Ne_3 K_\perp}{2} \frac{2}{m_i} \left[ \frac{\phi_1 \Psi_1}{\omega} \right] (1 - \frac{2}{K_\perp} \frac{2}{\rho_i} (J+1)) \]  \hspace{1cm} (3.18)

Similarly for the current density in Z direction

\[
\overline{J}_{ze} = -\frac{Ne_3 K_\parallel}{2} \frac{2}{m_e} \left[ -\frac{\psi_1}{\Omega_e} \right] - \frac{2 \omega m_e}{K_\parallel} \frac{1}{\Omega_1} + \frac{\Psi_1}{3} \frac{8 \omega}{K_\parallel V_\parallel} \frac{2}{K_\parallel V_\parallel e} \]  \hspace{1cm} (3.19)

\[
\overline{J}_{zi} = -\frac{Ne_3 K_\parallel}{2} \frac{2}{m_i} \left[ \frac{\phi_1}{\omega} - \frac{4 \Psi_1}{K_\parallel V_\parallel i} \right] (1 - \frac{2}{K_\perp} \frac{2}{\rho_i} (J+1)) \]  \hspace{1cm} (3.20)

Substituting equation (3.17) and (3.18) into equation (3.16) we obtain
\[ J_x = \frac{K \lambda \Psi_1^2}{8\pi} \left[ \frac{\omega_{pe}^2}{\omega} \left[ \left( \frac{\phi - \Psi_1}{\omega} \right) + \frac{2}{K_{\parallel} V_{T\parallel e}} \Psi_1 \right] - \frac{\omega_{pi}^2}{m_i \Omega_i^2} \left( 1 - K_{\perp}^2 \phi_i^2 (J+1) \right) \right] \]  

(3.21)

Similarly,

\[ J_z = \frac{K_{\parallel} \Psi_1 e}{8\pi} \left[ \frac{\omega_{pe}^2}{\omega} \left[ \frac{K_{\perp}^2}{\omega} \left( \frac{\phi - \Psi_1}{\omega} \right) + \frac{2}{K_{\parallel} V_{T\parallel e}} \Psi_1 \right] - \frac{8}{K_{\parallel}^2 V_{T\parallel e}^4} \right] \]

\[ - \frac{\omega_{pi}^2}{m_i} \left[ \frac{K_{\perp}^2}{\omega} \frac{\phi_i}{\Omega_i^2} - \frac{4 \Psi_1}{K_{\parallel} V_{T\parallel i}^3} \right] \left( 1 - K_{\perp}^2 \phi_i^2 (J+1) \right) \]  

(3.22)

In the evaluation of current densities it was assumed that the field-aligned and perpendicular currents are due to electromagnetic kinetic Alfvén wave and the contribution due to diamagnetic drift was neglected.

3.6 ENERGY BALANCE AND GROWTH RATE

The oscillatory motion of non-resonant electrons carries the major part of energy (Tiwari and Varma 1993, Terashima 1967). The wave energy density per unit wavelength \( W \) is the sum of pure field energy and the
changes in energy of the non-resonant particles \( W_{i,e} \). It is observed that the wave energy is contained in the form of the oscillatory motion of the non-resonant electrons (Tiwari and Varma 1993, Terashima 1967). Thus

\[
W \approx W_{e} \approx \frac{\lambda \psi_{i}^{2}}{16\pi} \frac{\omega}{pe} \frac{2}{K_{||}^{2}} \left( \frac{T_{\| e}}{m_{e}} \right) \tag{3.23}
\]

Now we calculate the resonance energy \( W_{r} \) of the electrons per unit wave length, that is

\[
W_{r} = \int_{0}^{\infty} dS \int_{0}^{\infty} \frac{dV_{\|}}{2\pi} \psi_{i}^{2} \frac{1}{(\omega/K_{||}) + \Delta V} \left( \frac{1}{2} N m_{e} u_{z}^{2} + n_{l} m_{e} u_{z} \right) dV_{\|} \quad \tag{3.24}
\]

With the help of equations (3.5), (3.9), (3.10) and (3.24) expanding the integrand around \( V_{\|} = \omega/K_{||} \) and following the procedure as discussed in ref (Tiwari and Varma 1993, Terashima 1967), in the limiting case of \( K_{\perp e} \ll 1 \) we obtain

\[
W_{r} = \frac{\lambda}{8\pi} \frac{\psi_{i}^{2}}{m_{e}} \frac{\omega}{pe} \frac{2}{K_{||}^{2}} \left( \frac{\omega}{K_{||}} d(T_{\| e}/T_{\perp e}) \right) \left[ \frac{\omega}{K_{||}^{2}} \frac{m_{e}}{K_{||}^{2} (2T_{\| e}/m_{e})^{1/2}} \right] \exp \left( -\frac{\omega^{2}}{2T_{\parallel e} K_{||}^{2}} \right) \tag{3.25}
\]
Where, \( K S = \bar{K} \cdot \bar{r} \) and \( \bar{K} = 2\pi / \lambda \) and 
\[
\omega_{p1,e}^2 = 4\pi N_0 e^2/m_{i,e}
\]

Using the law of conservation of energy, we calculate the growth rate of the drift kinetic Alfvén wave by,

\[
\frac{d}{dt} (W_w + W_r) = 0 \tag{3.26}
\]

With the help of equation (3.23) and (3.25) we have found the growth rate of the drift kinetic Alfvén wave as,

\[
\gamma/\omega = \sqrt{\pi} \frac{\omega}{K_{||} V_{Te}} \left[ \frac{T_{e||}}{T_{e\perp}} - 1 \right] \exp \left[ -\frac{2}{K_{||} V_{Te}^2} \right] \tag{3.27}
\]

where, 
\[
V_{\perp,e}^2 = \left( \frac{2T_{\perp,e}}{m_e} \right); \quad V_d^e \text{ represents electron diamagnetic drift velocity and the value of } \omega \text{ for drift kinetic Alfvén wave has to be substituted. It is noted that the kinetic Alfvén wave can be excited only when } \frac{T_{e||}}{T_{e\perp}} k_{\perp} V_d^e > \omega. \text{ Thus the kinetic Alfvén wave is excited as the usual drift wave.}

3.7 RESULTS AND DISCUSSION

In the numerical evaluation of the growth rate, current and dispersion relation, we have used the following parameters for the auroral acceleration region (Tiwari and Rostoker 1984) \( B_0 = 4300 \text{ nT}, \Omega_i = 412 \text{ s}^{-1}, K_{Ti} = 100 \text{ eV}, K_{Te} = 10 \text{ KeV}, V_d^e = 50 \text{ cm/Sec}, \frac{\omega_{p1,i}^2}{\Omega_i^2} = 100. \)

Figure 3.1 shows the relation between wave frequency \( \omega \) (rad/s) vs \( k_{\perp,i} \) for \( T_{e||}/T_{e\perp} = 4 \) and different values of distribution index \( J \). We have
Fig 31: Frequency versus perpendicular wave number for different distribution index

- $B_0 = 430\,\text{nT}$
- $k_{\parallel i} = 100\,\text{eV}$
- $k_{\perp e} = 10\,\text{keV}$
- $k_{\perp i} = 10 \times 10^{-13} \,\text{cm}^{-1}$
- $T_i / T_e = 4$
- $v_{ei} = 50\,\text{cm/s}$
- $\omega_{pi} / \Omega_p = 100$
arbitrarily selected $T_{e|}/T_{\perp} = 4$. However the choice $T_{e|} > T_{\perp}$ may exhibit the inhibited streaming of plasma particles along the magnetic field lines. Here it is noticed that frequency $\omega$ increases with $k_{\perp i}$ and the distribution index $J$ is more effective towards the higher perpendicular wave numbers. Furthermore, it is observed that the wave frequency $\omega$ is decreased at the higher values of $k_{\perp i}$ and the distribution index $J$ which may be due to the decrease of ion drift velocity by the averaging of the wave field over the Larmor orbit in the presence of steep loss-cone distribution functions. The steepness of loss-cone distribution index appears through the ion gyroradius effects which actually determines the wave frequency. The electron Larmor radius effect does not contribute to the wave frequency significantly because $k_{\perp e} \ll 1$ so that it is not affected by loss-cone distribution index $J$. Thus a significant effect may appear in the wave frequency by finite Larmor radius effect in which addition to loss-cone distribution functions can be incorporated through the particle aspect analysis.

Figure 3.2 shows the variation of growth rate $\gamma/\omega$ with $k_{\perp i}$ for different values of $J$. It is seen that a higher distribution index enhances the growth rate and permits a low frequency for emission. For higher $k_{\perp i}$ there exists no frequency band for increasing values of $J$. Thus, mirror like configurations with steep distribution index may be unstable for the electromagnetic drift kinetic Alfvén wave emission. The enhancement of growth rates towards lower $k_{\perp i}$ is also clear from the
FIG 3.2 GROWTH RATE VERSUS PERPENDICULAR WAVE NUMBER FOR DIFFERENT DISTRIBUTION INDEX J

- $B_0 = 430 \text{ nT}$
- $K_{\text{ihi}} = 100 \text{ eV}$
- $K_{\text{hi}} = 10 \text{ KeV}$
- $K_{\parallel} = 1.0 \times 10^{-13} \text{ cm}$
- $\frac{T_{\parallel}}{T_\perp} = 4$
- $V_0 = 50 \text{ cm/sec}$, $\frac{\omega_{pi}}{\Omega_{ci}} = 100$
figure. The increase of $J$ narrows down the emission band and the wave can be generated for the lower values of $k_{\perp i}$. The steep loss-cone structures are analogous to mirror like devices with higher mirror ratio which may accelerate the charged particles along the magnetic field by extracting energy of the particles moving perpendicular to the magnetic field. Thus more energetic particles may be available to provide energy to the wave by wave-particle interaction. Variation of growth rate with respect to $k_{\perp i}$ for different values of $V_e^d$ has been shown in figure 3.3. The effect of diamagnetic drift on the kinetic Alfvén wave is observed. It is noted that growth rate $\gamma/\omega$ increases with $V_e^d$ as well as $k_{\perp i}$ which suggests that the of kinetic Alfvén wave is excited due to plasma density inhomogeneity. Equation (3.27) shows that perpendicular gradient of plasma density acts as a source of KAW along the field lines. It follows that when convection changes in the equatorial plane of the magnetosphere in a manner that produces an east-west density gradient, Alfvénic disturbances will be setup which propagate to the ionosphere. Alfvénic oscillations will not be excited only if the changes are much slower than Alfvénic wave decay time within the system. While reaching the ionosphere, the amplitudes of the waves may grow on converging field lines due to higher values of distribution index $J$. The waves can bounce between the ionosphere and the magnetosphere due to ionospheric conductivity.

Figure 3.4 shows the decrease of $J_z$ with the increase of $k_{\perp i}$ as well as
$B_0 = 4300 \text{ nT}$
$K \theta_{th1} = 100 \text{ eV}$
$K \theta_{th2} = 10 \text{ keV}$
$K_R = 1.0 \times 10^{-13} \text{ cm}^{-1}$

$\frac{T_{th1}}{T_{th2}} = 4$
$J = 0$

$\omega_{Bi}^2 \frac{p_i}{m_i} = 100$

$V_{o}^d = 60$
$V_{o}^d = 50$
$V_{o}^d = 40$

FIG33: GROWTH RATE VERSUS PERPENDICULAR WAVE NUMBER FOR DIFFERENT DRIFT VELOCITY
$J_z \times 10^5$ (amp/cm²)

$J = 0$
$J = 2$
$J = 4$

$B_0 = 4300$ nT
$K_{T_{iii}} = 100$ eV
$K_{T_{||}} = 10$ keV
$K_u = 1.0 \times 10^{-3}$ cm⁻¹
$T_{\|} = 4$
$\frac{T_{\perp}}{T_{\|}}$
$v_{ce} = 50$ cm/sec, $\omega_{ce} = 100$

$FIG 34$: CURRENT IN Z DIRECTION VERSUS PERPENDICULAR WAVE NUMBER FOR DIFFERENT DISTRIBUTION INDEX
distribution index $J$ whereas figure (3.5) exhibits increase of $J_x$ with $k\rho_i$ and $J$. Both the figures predict the current exchange between the $x$ and $z$ directions. Thus we may predict that field aligned currents can be generated by drift kinetic Alfvén wave which couples to the perpendicular current as well as potential drop along the auroral field lines in the acceleration region. The reversal of field aligned current with perpendicular wave number at higher $J$ values is also noticed in figure 3.4 which may be due to the coupling of potential drops along and perpendicular to the magnetic fields. Thus perpendicular and parallel electric fields coupled by drift kinetic Alfvén wave also have their effect on the current pattern. Streaming particles along the converging magnetic field of ionospheric side may encounter the mirroring force, and therefore, slow down and be reflected back. As a consequence the current decreases and reverses its direction at higher values of distribution index $J$ as evidenced by fig. 3.4. The increase in perpendicular current is also due to the increase in perpendicular velocity due to the mirroring force. In this situation of the field-aligned current reversal, either the wave is dissipated or bounces back to establish a standing wave pattern. Field-aligned currents are the critical feature of magnetosphere-ionosphere coupling. In this work we have attributed their origin to plasma density inhomogeneity of the magnetosphere. The subauroral region-II field-aligned current system is commonly attributed to pressure driven current sources (Southwood and
FIG 3.5 CURRENT IN X-DIRECTION VERSUS PERPENDICULAR WAVE NUMBER FOR DIFFERENT DISTRIBUTION INDEX

- $J_x \times 10^{-15} \text{ (amp/cm)}$
- $J = 0$
- $J = 2$
- $J = 4$

- $B_0 = 4300 \text{ nT}$
- $T_{\text{ini}} = 100 \text{ eV}$
- $K_{\text{Tini}} = 10 \text{ KeV}$
- $K_{\text{ni}} = 1.0 \times 10^{-13} \text{ cm}^{-1}$
- $v_{\text{th}} = 50 \text{ cm/sec}$
- $\frac{\omega_{\text{pi}}}{\omega_{\text{ci}}} = 100$
Kivelson 1991). When changes in the magnetospheric pressure take place suddenly (for example, when a magnetospheric substorm expansion takes place) the plasma density changes will lead to a new field-aligned current distribution which will transiently be carried out to the ionosphere along the field by KAW that will bounce back and forth on closed field lines. Well known Pi-2 geomagnetic pulsations commonly associated with the onset of substorms are the signatures of the pressure driven changes of the field-aligned current system.

Destabilizing effects due to the steep loss-cone on different instabilities have been also reported in various papers (Tiwari and Varma 1993, Varma and Tiwari 1992, Gaezler et. al. 1997). Sharp density gradients appear in a variety of geophysical processes in the near space region of the Earth, for example the equatorial spread-F and the ring current inner edge acceleration region. The equilibrium dipolar magnetic field of the earth is curved in the meridional plane and may introduce loss-cone effects in the particle distribution function. The index J measures the steepness of the loss-cone feature. Thus the behavior studied for the drift kinetic Alfven wave may be of importance in the electromagnetic emission around the auroral acceleration region. The sharp density gradients may appear owing to the particle precipitation in the auroral zone (Tiwari and Rostoker 1984). Energetic particles may create a temperature anisotropy at the substorm times which may be also the cause of drift kinetic Alfven wave emissions. The kinetic Alfven
wave instability has been analyzed by a number of workers for uniform fields and a Maxwellian plasma. Their results are utilized to explain observed electromagnetic emission and current at the auroral acceleration region. However, the loss-cone distribution function which may be more realistic for the auroral acceleration region, modifies the wave spectrum.

In the analysis we have considered a loss-cone distribution and its influence on the kinetic Alfven wave instability. The loss-cone distribution may destabilize Bernstein mode (Hasegawa 1975) which may have higher growth rate. However, in the present analysis we have concentrated upon kinetic Alfven wave excited by density inhomogeneity and the detailed treatment of high frequency ($\omega > \Omega_i$) electrostatic Bernstein wave instability may be the matter of further investigation.

3.8 SUMMARY

Particle aspect analysis is extended for kinetic Alfven waves in an inhomogeneous magnetoplasma in the presence of general loss-cone distribution function. Effect of finite Larmor radius is incorporated in the finite temperature anisotropic plasma. Expressions for field-aligned current, perpendicular current (to B), dispersion relation, particle energy and growth rate are derived and effects of steepness of loss-cone distribution and plasma density inhomogeneity are discussed. The treatment of the kinetic Alfven wave instability is based on the assumption that the plasma consists of resonant and non-resonant
particles. It is assumed that resonant particles support the oscillatory nature of the wave and hence the excitation of the wave is dealt with by the wave particle energy exchange method. The applicability of the investigation is discussed for auroral acceleration phenomena.