CHAPTER V

NEW ASPECTS AND SUGGESTIONS ABOUT NON-DESTRUCTIVE OPTICAL MEANS FOR DETERMINING THE REFRACTIVE INDEX OF LENS' MATERIAL

Present chapter consists of a general formula which is common, and can be extended to derive new formulas for lens' index in terms of new parameters. The focal length of the test lens depends upon the refractive index of the liquid inside the glass cell. It briefs all the methods described in preceding chapters.

It also reflects and explore the possibilities of new formulas and method for determining refractive indices of lenses.
NEW ASPECTS AND SUGGESTIONS ABOUT NON-DESTRUCTIVE OPTICAL MEANS FOR DETERMINING THE REFRACTIVE INDEX OF LENS' MATERIAL

This chapter communicates the non-destructive techniques reported in this thesis for measuring the refractive indices. These different methods are based on the use of

1. Shearing interferometric planoparallel plate.
2. Coherent optical processing configuration using Fabry-Perot etalon.
3. Diffraction produced by double slit.

Basically, these devices are very common features of an optical laboratory. The main contribution discussed in this work is to establish new formulas of lens' index involving the property of planoparallel glass plate, Fabry-Perot etalon and double slit.

The focal length of a lens is related with its refractive index. Therefore, the lens formula for a test lens immersed in liquid inside a glass cell is reproduced as
\[ \frac{1}{f} = (n-n_L)(c_1-c_2) + (n-n_L)^2 \cdot \frac{t}{c_1c_2/n} \quad \ldots \quad (5.1) \]

This general formula consists of various constructional parameters such as radii of surfaces, refractive indices, thickness of the lens, lens' focal length etc. Hopkins (1953, 1955, 1957), Rosenhauer and Rosenbruch (1967), Rosenhauer (1956, 1957) Glaze brook (1950) have discussed them in detail.

In the last decade particularly in the past few years, nondestructive techniques have gained importance. The attainable accuracy of these methods is \( \pm 1 \times 10^{-4} \) for the refractive index. This accuracy is normally sufficient for calculating the image quality from design data.

If the higher accuracy is demanded for refractive index measurement, an adequately sized prism with a refracting angle of about 60° must be made from the material to be measured. With the sides large enough and extremely plane surfaces (radius of curvature larger than 60 km.), in a precision goniometer, the refractive index can be measured to better than \( 10^{-6} \), Rosenbruch and Stenger (1980).
However, the measurement of the refractive index of optical glasses up to the accuracy of the order of $10^{-6}$ is a difficult task. Moreover, these techniques of spectrometers and refractometers producing high accuracies can not be applied if the specimen is a lens. Under some circumstances, Smith (1966) and Kingslake (1978), the characterization of an unknown constructed lens is required. One of the difficult parameters to be determined in case of a lens is its refractive index. The problem arises because the lens already has a given physical structure (radii of curvatures, thickness, shape of the surfaces) and hence, the usual techniques for measuring the refractive index can not be applied.

5.1 **PARAXIAL APPROXIMATION** :

In view of the above aspects Rodriguez et al. (1981) have derived a formula for lens' index and also performed an experiment to measure the refractive index of a constructed lens. Using the meridional ray tracing Kingslake (1978), the mathematical analysis for deducing lens' index relation has been adopted by Comejo-Rodriguez (1981) as follows:
\[ n \sin i = n' \sin i' \quad \ldots \ (5.2) \]
\[ u + i = u' + i' \quad \ldots \ (5.3) \]
\[ y = \frac{\sin (u+i)}{c} \]

These equations are valid for a refracting surface under paraxial approximation.

Now these relations can be modified for a lens (i.e. two refracting surfaces separated by a thickness \( t \)), as follows:

\[ \tan u'_1 = \frac{y_1-y_2}{t-(x_1-x_2)} \quad \ldots \ (5.5) \]

where all these parameters are mentioned in fig. (5.1).

If the incident ray is parallel to optical axis (i.e. \( u_1 = 0 \)), Equations (5.2) and (5.4) produce

\[ n' = \frac{y_1c_1}{\sin i'_1} \quad \ldots \ (5.6) \]

Equation (5-3) can be given

\[ i'_1 = i - u_1 \quad \ldots \ (5.7) \]

Hence,

\[ \sin i'_1 = \sin (i-u_1) \quad \ldots \ (5.8) \]
$i_1$ & $i_2$ = Angles of the rays with respect to the normal to the surface
$u_1$, $u_1'$ & $u_2'$ = Angles of the rays with respect to the optical axis
$t$ = Thickness of the lens
$n'$ = Refractive index of the lens

$r_1$ & $r_2$ = Radii of curvatures
$y_1$ & $y_2$ = Heights on each surfaces
$x_1$ & $x_2$ = Sagittae corresponding to the heights $y_1$ & $y_2$

FIG. 5.1 - PARAMETERS INVOLVED IN THE REFRACTION OF A RAY IN ONE AND TWO SURFACES.
Thus, the refractive index of lens' material is given by

\[
n' = \frac{y_1 c_1}{\sin^{-1}(y_1 c_1) - \tan^{-1}\left(\frac{y_1 - y_2}{t(x_1 - x_2)}\right)}
\]

\[\text{.... (5.9)}\]

Now all the necessary parameters like \( t, c_1, c_2, y_1 \) and \( y_2 \) (heights of the rays on each refracting surface) are measured. The sagittae \( x_1 \) and \( x_2 \) corresponding to the heights \( y_1 \) and \( y_2 \) are also calculated.

This lens' index relation (5.9) has several parameters and hence an error analysis just calculating the corresponding errors, is essential. Now, substituting \( i' \) in terms of \( i_1 \) and \( u'_1 = u_2 \) equation (5.6) becomes,

\[
n' = \sin i_1 / (\sin i_1 \cos u_2 - \cos^2 i_1 \sin u_2)
\]

\[\text{.... (5.10)}\]

Now eliminating \( \cos u_2 \) and \( \sin u_2 \), the equation (5.10) reduce to

\[
n' = \frac{y_1 c_1 \sqrt{(y_1 - y_2)^2 + [t-(x_1 - x_2)]^2}}{y_1 c_1 [t-(x_1 - x_2)] + (y_1 - y_2) \sqrt{1 - y_1^2 c_1^2}}
\]

\[\text{.... (5.11)}\]
5.1.1 ERROR ANALYSIS:

The errors due to each of the variables $r_1$, $y_1$, $y_2$ and $t$ can be determined by taking partial differentiation of equation (5.11). These errors can be written as.

$$
\Delta n'_{r_1} = n^2[A(1- \frac{1}{\cos i_1}) (\sin u_2 + B) + \frac{\sin u_2}{r_1 \cos i_1}] \Delta r_1
$$

...... (5.12)

$$
\Delta n'_{r_2} = -n^2A \left[1- \frac{1}{\sqrt{1-(y_2/r_2)^2}} \right] (\sin u_2 + B) \Delta r_2
$$

...... (5.13)

$$
\Delta n'_{y_1} = n^2 [A \tan i_1 (\sin u_2 + B)
+ \frac{\sin u_2 \cos^2 u_2}{(y_1 - y_2)} (1- \frac{1}{\tan i_1}) - \frac{\sin u_2}{r_1 \cos i_1}] \Delta y_1
$$

...... (5.14)

$$
\Delta n'_{y_2} = -n^2 \frac{\sin u_2}{(y_1 - y_2)} (\sin u_2 + B)
\left[ \frac{\sin u_2}{\sqrt{(y_2/r_2)^2-1}} + \cos u_2 \right] \Delta y_2
$$

...... (5.15)
\[ \Delta n'_t = -n^2 A (\sin u_2 + B) \Delta t \quad \ldots (5.16) \]

where,

\[ A = \frac{\sin u_2}{(y_1 - y_2)} \text{ and } B = \frac{\cos u_2}{\tan i_1} \quad \ldots (5.17) \]

It has been found that it is not easy to define the meridional plane. Consequently, the measurement of the height, \( y_1 \) and \( y_2 \) produce wrong result for equation (5.9) which has been derived for the meridional plane. They have observed that the value of lens' index is \( n' = 1.527 \pm .005 \).

It is evident that despite the fact of simple and easy equations, the experimental work is very tedious and challenging task. As a result of its cumbersome nature, it is not advisable to use it frequently. Otherwise also it gives accuracy of the order of \( \pm 0.005 \).

The present thesis reports non-destructive non-miscible liquid immersion techniques which have far reaching impacts on the refractive index measurements. The liquid immersion technique has various salient features. It is very interesting because the focal length of the test lens is the only parameter which is
always used to derive new formulas for the refractive index of the lens.

5.2 **GENERAL FORMULA** :

The lens formula (5.1) is valid when the test lens is immersed in liquid inside the glass cell. It can also be written as

\[
\frac{1}{f} = \left( t \cdot c_1 c_2 / n \right) n_L^2 + \left( c_2 - c_1 - 2t \cdot c_1 c_2 \right) n_L + \left( c_1 - c_2 + t \cdot c_1 c_2 \right) n
\]

\[\ldots (5.17)\]

\[
\frac{1}{f} = A_2 \ n_L^2 + A_1 \ n_L + A_0 \quad \ldots (5.18)
\]

Here \( \frac{1}{f} \) Vs \( n_L \) is a second order polynomial.

In this case, we construct the least squares polynomial approximation (Phillips and Taylor 1973) of degree two.

\[
\frac{1}{f} = A_2 \ n_L^2 + A_1 \ n_L + A_0 \quad \ldots (5.19)
\]

where,

\[
A_2 = t \ c_1 c_2 / n \quad (5.20)
\]

\[
A_1 = -\left[ 2tc_1 c_2 + c_1 - c_2 \right] \quad (5.21)
\]

\[
A_0 = [tc_1 c_2 + c_1 - c_2] n \quad (5.22)
\]
Now equations (5.19), (5.20) and (5.22) can be rewritten as

\[ A_2 = t c_1 c_2 \quad (5.23) \]

\[ A_1 = -[2tc_1 c_2 + c_1 - c_2] \quad (5.24) \]

\[ \frac{A_0}{n} = [t c_1 c_2 + c_1 - c_2] \quad (5.25) \]

So, by adding equations (5.23), (5.24) and (5.25) we get,

\[ 0 = A_2 n + A_1 + A_0/n \]

or,

\[ 0 = A_2 n^2 + A_1 n + A_0 \quad \ldots \quad (5.26) \]

The roots of this equation (5.26) gives us the value of the refractive index of the lens.

5.3 **REFRACTIVE INDEX OF A LIQUID:**

The relation (5.19) can be used to plot \((1/f)\) Vs \((n_L)\). Thus, the experimental points and least-squares straight line approximation will predict the refractive indices of unknown liquids provided the focal length of the test lens immersed in that liquid is determined.
5.4 GENERAL FORMULA FOR LENS' INDEX:

The lens formula (5.1) is applicable for a test lens immersed in a liquid of refractive index $n_L$. This formula can be written as -

5.4.1 FOR PLANO-CONVEX LENS

$$\frac{1}{f} = (n-n_L) c_1$$ \hspace{1cm} (5.27)

Here $c_2 = 0$ for a plano-convex lens.

5.4.2 FOR A THIN LENS:

The value of factor $(n-n_L)^2 t c_1 c_2/n$ is very small and hence, it can be neglected. So, equation (5.1) reduces to

$$1/f = (n-n_L) (c_1 - c_2)$$ \hspace{1cm} (5.28)

The main difficulty faced by workers was the measurement of $c_1$ and $c_2$. This problem is solved by a new formulation of equations. As it has been already discussed, in non-miscible non-destructive technique with independent liquids in glass cell (for instance, if $i^{th}$ and $j^{th}$ liquids) the equations (5.27) and (5.28) can be expressed as
\[
\frac{1}{f_i} = (n-n_i) K_1 \quad \ldots (5.29a)
\]
\[
\frac{1}{f_j} = (n-n_j) K_1 \quad \ldots (5.29b)
\]
\[
\frac{1}{f_i} = (n-n_i) K_2 \quad \ldots (5.30a)
\]
\[
\frac{1}{f_j} = (n-n_j) K_2 \quad \ldots (5.30b)
\]

where,

\[K_1 = c_1 \text{ for plano-convex lens}\]
\[K_2 = c_1 - c_2 \text{ for a thin lens.}\]

The constant \(K_1\) & \(K_2\) can be eliminated by using (5.29a & 5.29b) and (5.30a & 5.30b) respectively which gives -

\[
\frac{f_j}{f_i} = \frac{n-n_i}{(n-n_j)} \quad \ldots (5.31)
\]

On solving equation (5.31), we have

\[
n = \frac{(n_j f_j - n_i f_i)}{(f_j - f_i)} \quad \ldots (5.32)
\]

It is the general relation for lens' index.

The significance of the present contributions detailed in this thesis lies in the use of equation (5.32). The methods reported are stated as follows:
(1) **SHEARING INTERFEROMETRIC TECHNIQUE:**

As mentioned in Chapter II, the factor 
\[ (f_j - f_i) = x_{ij} \]

is of concern. Thus, the lens' index formula (5.32) can directly be used.

(2) **F.P. ETALON TECHNIQUE:**

It has been established in Chapter III that the focal length and the diameters of interference rings are dependent. So, using equations (3.17) and (3.18), \( f_i \) and \( f_j \) can be written as.

\[
\frac{1}{f_i} = \frac{\text{constant}}{(D_{m+1}^2 - D_m^2)^{\frac{1}{2}}} \quad \quad \text{(5.33)}
\]

\[
\frac{1}{f_j} = \frac{\text{constant}}{(D_{m+1}^2 - D_m^2)^{\frac{1}{2}}} \quad \quad \text{(5.34)}
\]

Hence, by substituting these values in equation (5.32) we get

\[
n = \frac{n_j(D_{m+1}^2 - D_m^2)^{\frac{1}{2}} - n_i(D_{m+1}^2 - D_m^2)^{\frac{1}{2}}}{(D_{m+1}^2 - D_m^2)^{\frac{1}{2}} - (D_{m+1}^2 - D_m^2)^{\frac{1}{2}}}
\]

3. **DOUBLE SLIT DIFFRACTION:**

The separation between two successive diffraction
dots \( w \) is related with focal length \( f \) of the test lens as

\[
\frac{1}{f} = \frac{\lambda}{2d} \frac{1}{w} \quad \text{...(5.35)}
\]

Therefore, by using equations (5.35) and (5.32) for \( i^{th} \) and \( j^{th} \) liquids we obtain.

\[
n = \frac{n_j w_j - n_i w_i}{w_j - w_j} \quad \text{(5.36)}
\]

Therefore, we conclude that the new concept of non-miscible immersion technique provides new formulas for the refractive index of a lens.

Thus, this chapter includes all the possible formulas for lens' index where focal length depends on new parameters depending on the optical devices.