CHAPTER - IV

USE OF DIFFRACTION EFFECTS FOR EVALUATING THE
REFRACTIVE INDICES OF LIQUIDS AND LENSES

This chapter is devoted to the study of the technique based on diffraction effect for evaluating the refractive indices of liquids and lenses. The diffraction phenomenon of light is rarely used for the measurements of refractive indices.

The Fourier transform (F.T.) property of the lens has been used. Lens' index formula depends only on the distance between the two successive diffraction orders displayed in F.T. plane. The comparator or X-Y recorder can be used to calculate the distance between two consecutive orders. The reported method may have a wide scope in this field.
USE OF DIFFRACTION EFFECTS FOR EVALUATING THE REFRACTIVE INDICES OF LIQUIDS AND LENSES

4.0 INTRODUCTION:

The chapter presents a method for evaluating lens' index which is based on diffraction phenomenon of light. It accounts the various factors which affect the different parameters of a test lens. Most significant parameters of a lens are its focal length and the refractive index of its material. The optical configuration is the same as used for coherent optical processing. The test lens immersed in liquid inside a glass cell acts as an optical processor. The Fourier-transform of an object placed in front of the test lens is displayed in its back focal plane. In other words, it is the diffraction pattern of the input object placed in collimated light before the front face of the test lens. Sometimes, it is also termed as the spatial frequency spectrum. The back focal plane is called as Fourier-transform plane or spatial frequency plane. The remarkable characteristics of this Fourier-transform is to provide the scope for filtering out desired frequencies at this plane. Thus, sometimes it is also
known as the spatial filtering plane.

The proposed technique presents good results similar to the existing methods. The effects of thickness parameters of the walls of the glass cell and liquid column inside the cell have been omitted, as the lens' index formula is not dependent on such factors.

The separation between consecutive diffraction dots are considered. Thus, lens index formula depends only on the distance between the two successive diffraction dots.

4.1 EXPERIMENTAL SET-UP:

The details of the experimental set-up are given in the figure (4.1).

The optical arrangement consists of a glass cell filled with a liquid. The test lens is immersed in the liquid inside the glass cell. The combination of liquid and lens acts like a lens whose focal length depends upon the refractive index of the liquid.

Parallel wave-front of light incides on an array of double slit. The double slit diffracts the incident
$n = \text{Refractive index of lens' material}$

$n_L = \text{Refractive index of liquid}$

$2d = \text{Separation between slits}$

$f = \text{Focal length of the test lens}$

**FIG. 4.1 - EXPERIMENTAL SET-UP FOR EVALUATING THE REFRACTIVE INDICES OF LIQUIDS AND LENS' MATERIALS.**
light. The diffracted wavefronts are collected by the test lens and converges to its back focal plane. This plane being the Fourier-transform consists of diffraction pattern or the Fourier-transform of double slits. It has dark and bright bands.

The separation between two consecutive bands or fringes can easily be calculated which is termed as fringe width. The fringes may be recorded and their separation is then calculated by a comparator or by a travelling telescope.

4.2 THEORY:

According to the Fourier-transformation properties of lenses, a converging lens always produces the Fourier-transform of input transparencies, in its back focal plane. For instance, when an input object or transparency function $g(x,y)$ is illuminated by a parallel wavefront of monochromatic light at the front side of the lens, the diffraction pattern of this input function $g(x,y)$ is Fourier-transformed in the back focal plane of test lens. This relation between input object and its Fourier-transform can be represented as, Goodman (1968).
\[ G(p,q) = c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \exp \left[ 2\pi i \left( x \cdot x_f + y \cdot y_f \right) \right] dx dy \quad \ldots \quad (4.1) \]

where,

\( (x,y) \) - co-ordinates in object plane.

\( (x_f,y_f) \) - co-ordinates in Fourier-transform plane.

Using the spatial frequency co-ordinates \((p,q)\) which are

\[
\begin{align*}
  p &= x_f / \lambda f \\
  q &= y_f / \lambda f
\end{align*}
\] .... (4.2)

These relations also denote the separation between diffraction orders.

The equation (4.1) can be rewritten as

\[ G(p,q) = c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \exp \left[ 2\pi i (px + qy) \right] dx dy \quad \ldots \quad (4.3) \]

It is concluded from the equation (4.3) that \( G(p,q) \) and \( g(x,y) \) are the Fourier-transform (F.T.) of each other. It can be written as

\[ G(p,q) \xrightarrow{\text{F.T.}} g(x,y) \quad \ldots \quad (4.4) \]
Therefore, the function 'G' indicates the amplitude distribution in the back focal plane of the test lens.

4.2.1 Separation Between Successive Diffraction Dots:

The diffraction equation for a double slit can easily be derived. If the slits 'S_1' and 'S_2' are at a distance '2d' apart, the optical path difference between two rays coming out from 'S_1' and 'S_2' can be calculated at any arbitrary point 'P' in the observation plane (i.e. Fourier-transform). Thus, these two rays suffer an optical path difference ($\Delta$) equal to

$$\Delta = 2d \sin \theta \quad \ldots \ldots (4.5)$$

Let us consider that m\textsuperscript{th} fringe or m\textsuperscript{th} order of diffraction lies on this arbitrary point 'P' which is situated at a distance $y_m$ from the optical axis.

When $\theta$ is very small $\sin \theta \approx \theta \approx \tan \theta$

Hence,

$$\Delta = 2d \cdot (y_m/f) \quad \ldots \ldots (4.6)$$

The equation can be used to explain the conditions for dark and bright bands.
4.2.2 CONDITION FOR DARK BAND:

When the path difference is an even multiple of \((\lambda/2)\), the point will be dark, So,

\[ \Delta = 2d \left( \frac{y_m}{f} \right) = 2m(\lambda/2) = m\lambda \quad \ldots \quad (4.7) \]

where,

\[ m = 1, 2, 3, \ldots \ldots \]

4.2.3 CONDITION FOR BRIGHT BAND:

When the path difference is an odd multiple of \((\lambda/2)\), the point will be bright. So,

\[ \Delta = 2nd \left( \frac{y_m}{f} \right) = (2m+1) \left( \frac{\lambda}{2} \right) \]

\[ = \left( m + \frac{1}{2} \right) \quad \ldots \quad (4.8) \]

4.2.4 FRINGE SPACE:

The separation between two successive fringes \([m^{th} \text{ and } (m+1)^{th}\) - say] is termed as the fringe width or fringe space.

Therefore, by using either equation (4.7) or (4.8)

\[ \lambda = \frac{(2d/f) \cdot (y_{m+1} - y_m)}{2d} \]

or,

\[ y_{m+1} - y_m = \left( \frac{\lambda}{2d} \right) \cdot f \]
or,
\[(\Delta y) = (\lambda/2d).f\] .... (4.9)

If \(\Delta y = w\)

or,
\[\frac{1}{f} = \frac{(\lambda/2d)}{w}\] .... (4.10)

It is obvious from equation (4.9) and (4.10) that the fringe space depends upon \(f\), \(2d\), and \(\lambda\).

If the monochromatic He-Ne laser light (\(\lambda = 633\)nm) is used with a particular double slit where '2d' is fixed, the factor \((\lambda/2d)\) is a constant (say-C).

Now equation (4.10) is rewritten as

\[\frac{1}{f} = \frac{C}{w}\] .... (4.11)

4.2.5 RELATION BETWEEN FOCAL LENGTH AND FRINGE SPACE:

The value of \((1/f)\) can be written from the lens' formula (2.4) when the lens is immersed in the liquid inside the glass cell. The value of \((1/f)\) along with equation (4.11) can be shown as

\[\frac{1}{f} = (n-n_L)(c_1-c_2) + (n-n_L)t_c_1c_2/n \]
\[= \frac{C}{w}\] .... (4.12)
Now reducing the lens' formula for a plano-convex lens or a thin lens (where the factor \((n-n_L)^2 t c_1 c_2/n\) is negligible), the equation (4.12) is modified and rewritten as denoted by equation (4.13).

\[
\frac{1}{f} = \frac{C}{w} = (n-n_L) K
\]

...(4.13)

where,

\[K = c_1 \text{ or } (c_1-c_2)\]

### 4.2.6 LENSA'S INDEX FORMULA:

As reported earlier only one liquid has been used in glass cell rather than a mixture of two or more liquids. The different liquids \(i^{th}\) and \(j^{th}\) say) are used independently. The focal length will change as it depends on \(n_L\). Consequently the fringe width \((w)\) will also change. So, equation (4.13) may be written for \(i^{th}\) and \(j^{th}\) liquids as follows

\[
\frac{1}{f_i} = \frac{C}{(w)_i} = (n-n_i) K
\]

...(4.14)

and

\[
\frac{1}{f_j} = \frac{C}{(w)_j} = (n-n_j) K
\]

...(4.15)

Dividing equation (4.14) by (4.15), we get,
\[ \frac{f_j}{f_i} = \frac{(w)_j}{(w)_i} = \frac{(n-n_i)}{(n-n_j)} \] .... (4.16)

On solving the equation (4.16) in terms of fringe width

\[ n = \frac{(n_jw_j - n_iw_i)}{(w_j - w_i)} \] .... (4.17)

Lenses' index relationship in terms of focal length parameter will be

\[ n = \frac{n_jf_j - n_if_i}{(f_j-f_i)} \] .... (4.18)

Thus, in the proposed technique, lenses' index relation denoted by equation (4.17) is used.

### 4.2.7 POSSIBLE UNCERTAINTIES:

The errors in refractive index parameter (\(\Delta n\)) can be calculated by differentiating equation (4.17) as follows;

\[ \Delta n = \frac{(w_j-w_i)(n_j\Delta w_j-n_i\Delta w_i)-(n_jw_j-n_iw_i)(\Delta w_j-\Delta w_i)}{(w_j-w_i)^2} \] .... (4.19)

By solving equation (4.19), we get,
$$\Delta n = \frac{(n_j - n_i) [w_j \Delta w_i - w_i \Delta w_j]}{(w_j - w_i)^2} \quad \ldots \quad (4.20)$$

But, $\Delta w_i = \Delta w_j = \xi$ (Say)

$$\Delta n = \frac{(n_j - n_i)(w_j - w_i) \xi}{(w_j - w_i)^2} \quad \ldots \quad (4.21)$$

Therefore, maximum uncertainty is

$$\Delta n = \frac{(n_j - n_i) \xi}{(w_j - w_i)} \quad \ldots \quad (4.22)$$

4.2.8 PERCENTAGE ERROR:

The percentage error can also be calculated as detailed below,

By taking logarithm of equation (4.17)

$$\log n = \log (n_j w_j - n_i w_i) - \log (w_j - w_i) \quad \ldots \quad (4.22)$$

on differentiating equation (4.22), we get

$$\frac{s_n n}{n} = \frac{(n_j s_w - n_i s_w)}{(n_j w_j - n_i w_i)} - \frac{(s_w) (w_j - w_i)}{(w_j - w_i)} \quad \ldots \quad (4.23)$$

on solving equation (4.23),

$$\frac{s_n}{n} = \frac{(n_j - n_i) \xi}{(n_j w_i - n_i w_i)} \quad \ldots \quad (4.24)$$
Hence, the percentage error can be calculated from the above expression as shown in Table : (4.2). The percentage error for all the possible set of liquids has been calculated and mentioned in Table (4.2).

4.3 EXPERIMENTAL PROCEDURE :

The optical components and necessary auxiliary gadgets are arranged as shown in figure (4.1). The diffraction dots, displayed in the back focal plane of the test lens can be viewed easily. The separation between various orders can be calculated or measured by different techniques such as :

(1) X-Y recorder
(2) Photodetector as scanner
(3) Photographic recording.

The photographs have been taken in the back focal plane of the test lens immersed in different liquids. The focal length will be different for different liquids. Therefore, according to equation (4.12) the fringe separation will also be different as it is the function of focal length.

Photographs of PLATE No. (4.1) correspond to the
PLATE No. 4.1: DOUBLE SLIT DIFFRACTION PATTERNS WITH DIFFERENT LIQUIDS
double slit pattern with different liquids. The distance between successive spots is measured by a comparator. The shrinkage effect of the photographic plates or films have been neglected.

4.4 OBSERVATIONS:

Wavelength of the light in use ($\lambda$) = 633 nm

Room Temperature (T) = 25°C

Details of Test lens:

Focal length of lens (f) = 198.934 mm

Curvatures of the lens ($c_1$) = 102.915 mm

($c_2$) = 0

Calculated value of lens' index (n) = 1.51703
<table>
<thead>
<tr>
<th>S. No.</th>
<th>Liquids</th>
<th>Liquid Index</th>
<th>Value of W (in mm)</th>
<th>Value of f (in mm)</th>
<th>Value of (l/w) (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Air</td>
<td>1.0000</td>
<td>0.169</td>
<td>198.934</td>
<td>5.917</td>
</tr>
<tr>
<td>2.</td>
<td>Water</td>
<td>1.3330</td>
<td>0.474</td>
<td>558.316</td>
<td>2.109</td>
</tr>
<tr>
<td>3.</td>
<td>Freon</td>
<td>1.3600</td>
<td>0.556</td>
<td>644.300</td>
<td>1.798</td>
</tr>
<tr>
<td>4.</td>
<td>Xylene</td>
<td>1.4949</td>
<td>3.836</td>
<td>4588.070</td>
<td>0.260</td>
</tr>
</tbody>
</table>
### Table 4.2
**Possible Percentage Error in Estimating Lens' Index**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Liquids (ith &amp; jth) pair</th>
<th>((n_j - n_i))</th>
<th>(n_jw_i - n_iw_i)</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Air-Water</td>
<td>0.330</td>
<td>0.4628</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Air-Freon</td>
<td>0.360</td>
<td>0.7561</td>
<td>4.761 \times 10^{-2}</td>
</tr>
<tr>
<td>3.</td>
<td>Air-Xylene</td>
<td>0.4949</td>
<td>5.5643</td>
<td>8.894 \times 10^{-3}</td>
</tr>
<tr>
<td>4.</td>
<td>Water-Freon</td>
<td>0.0270</td>
<td>0.1243</td>
<td>2.172 \times 10^{-2}</td>
</tr>
<tr>
<td>5.</td>
<td>Water-Xylene</td>
<td>0.1619</td>
<td>5.1026</td>
<td>3.172 \times 10^{-3}</td>
</tr>
<tr>
<td>6.</td>
<td>Feron-Xylene</td>
<td>0.1349</td>
<td>4.9782</td>
<td>2.709 \times 10^{-3}</td>
</tr>
</tbody>
</table>

\(\xi\) is the least count of the comparator, which is \(0.001\) mm
Values of lens' index by liquid immersion method

1. \( n_{(A-N)} \) = 1.517514
2. \( n_{(A-F)} \) = 1.517709
3. \( n_{(A-X)} \) = 1.517708
4. \( n_{(W-F)} \) = 1.516073
5. \( n_{(W-X)} \) = 1.517725
6. \( n_{(F-X)} \) = 1.517760

Average value

\( \text{1.517331} \)

where, \( A \) - denotes Air
\( W \) - denotes Water
\( F \) - denotes Freon
\( X \) - denotes Xylene

4.5 RESULTS AND DISCUSSIONS:

This is a simple and easy method for evaluating the refractive indices of liquids and lenses. It is very economic also because it does not need any sophisticated instrumentation.

The salient features of this proposed method are that the wave-length of light (\( \lambda \)) or the separation can be evaluated. For this purpose a graph between (w Vs f) has been plotted as shown in fig. (4.2).
GRAPH BETWEEN $\omega$ Vs $f$

$$\omega = \left( \frac{\lambda}{2d} \right) f$$

**FIG. 4.2**
For instance equation (4.11) is rewritten as

\[ w = Cf \]

\[ \text{..... (4.25)} \]

The gradient of the straight line represented by above equation denotes the value of \((\lambda/2d)\). Hence, either the value of wavelength of light \((\lambda)\) or the separation between two slits \((2d)\) can easily be found at a time.

4.6 REFRACTIVE INDEX OF UNKNOWN A LIQUID:

Equation (4.14) is rewritten here which is valid for plano-convex or a thin lens

\[ \frac{(\lambda/2d)}{w_L} = (n-n_L)K \]

\[ \text{..... (4.26)} \]

Where, \(L\) denotes the liquid

or,

\[ \frac{(\lambda/2d.K)}{w_L} = n-n_L \]

\[ \text{..... (4.27)} \]

The above equation can be expressed as

\[ n_L = n - \frac{Z}{w_L} \]

\[ \text{..... (4.28)} \]

Where,

\[ Z = (\lambda/2d.K) = \text{constant} \]
It we plot a graph between $n_L$ vs $1/w_L$ a straighgt line is drawn as shown in Fig. (4.3) where intercept gives the value of refractive index of the lens. The refractive index of an unknown liquid can easily be found from the same straight line and hence a quick identification of the liquid can be made.

The shrinkage effect of films or photoplate may also affect the accuracy of the distance between successive diffraction dots. Thus, to eliminate such a possibility an in-line arrangement of measuring the distance by using X-Y recorder can be made. The separation between two consecutive peaks of intensity determines the value of $w$. Some graphs plotted by X-Y recorder have been shown in Fig. (4.4 to 4.7). The exact value of separation of two peaks of intensity can easily be calculated by adjusting the velocity of the pen and conversion factor of the recorder.

Hence, it is concluded that any easily available means can be used to measure the distance between successive diffraction dots.
GRAPH BETWEEN $n_L$ Vs ($1/\omega_L$)

Where $n_L = n - \frac{x}{\omega}$

$L$ = Indicates liquids

$\omega$ = Fringe width

$n_L$ = Refractive index of liquid

$n$ = Refractive index of lens

FIG. 4.3
FIG. 4.4 : FOR WATER
FIG. 4.5: FOR-FREON