Abstract

In this thesis a study of Lie algebra bundles is carried out in four directions.

Pull back, clutching and collapsing constructions are defined for Lie algebra bundles. As in the case of vector bundles it has been proved that these constructions are unique up to homotopy equivalence of the maps under consideration in the case of locally trivial Lie algebra bundles over compact Hausdorff spaces.

The ideas of Lie algebras of finite type and Lie rings are introduced and an equivalence between the category of Lie algebra bundles and finitely generated projective Lie rings is established.

In the point of view of the algebraic structure on the Lie algebra bundles the decomposition theorem for Lie algebra bundles over arbitrary characteristic is carried out. This is done by defining the concepts such as characteristic semisimple, characteristic simple and completely semisimple Lie algebra bundles. Further, a decomposition theorem for Lie algebra bundles having a symmetric invariant non degenerate bilinear form is also proved.

In analogy with the theory of cohomology of Lie algebras, the concept of module bundle extensions and Lie algebra bundle extensions have been developed and proofs
for the First and the Second Whitehead Lemmas for semisimple Lie algebra bundles are given. Further, by using the First Whitehead Lemma, the equivalence between reflexive Lie algebra bundles and semisimple Lie algebra bundles is established.

The thesis is concluded with some observations and suggestion for further research.