4.1 INTRODUCTION

Keeping in view of the high failure rates and repair costs, there are many practical situations where the idea of giving priority for operation and repair to one unit over the other becomes necessary. Researchers including Kadyan et al. (2004), Chander (2005 and 2007) and Sharma et al. (2010) have carried out stochastic analysis of system Models by using the concept of priority either for operation to any unit or to any repair activity. Also, in the previous chapter we have investigated stochastic Model of repairable system of non-identical units with priority for operation to main unit over the duplicate unit subject to different weather conditions.

In the present chapter, to make the study more real and useful, we have developed a stochastic Model of a repairable system of two non-identical units – one is original (called main unit) and other is substandard (called a duplicate unit) with the perception of priority for operation and repair activities to the main unit over the duplicate unit subject to two types of weather conditions - normal and abnormal. Initially, the main unit is operative and duplicate unit is kept as spare in cold standby. Each unit has direct complete failure from normal mode. Both units are capable of performing the system functions well with different degree of reliability and desirability. There is a single server who visits the system immediately whenever needed in normal weather conditions. The operation and repair activity of the system are not allowed in abnormal weather. After repair, each unit works as new. All random variables are statistically independent and uncorrelated. The switch devices are perfect.

The failure times of the units and time of change of weather conditions follow exponential distributions. And, repair times of the units are arbitrarily distributed. Various reliability and performance measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server, expected no of visits by the server and profit function are obtained using semi-Markov process and regenerative point technique. The graphical behavior of MTSF, availability and profit functions with respect to normal weather rate has also been examined for a particular case and fixed cost parameters. The profit function of the present chapter is compared with the profit function of chapter 2nd and chapter 3rd.
4.2 NOTATIONS

E \quad \text{The set of regenerative states}

MO/DO \quad \text{Main/Duplicate unit is good and operative}

\underline{MWO / DWO} \quad \text{Main/Duplicate unit is good and waiting for operation in abnormal weather}

DCs \quad \text{Duplicate unit is in cold standby mode}

\underline{DCs} \quad \text{Duplicate unit is in cold standby mode in abnormal weather}

\lambda / \lambda_1 \quad \text{Constant failure rate of Main /Duplicate unit}

\beta / \beta_1 \quad \text{Constant rate of change of weather from normal to abnormal/abnormal to normal weather}

MFur/DFur \quad \text{Main/duplicate unit failed and under repair}

MFUR/DFUR \quad \text{Main/duplicate unit failed and under repair continuously from previous state}

MFwr/DFwr \quad \text{Main/duplicate unit failed and waiting for repair}

MFWR/DFWR \quad \text{Main/duplicate unit failed and waiting for repair continuously from previous state}

\underline{MFwr / DFwr} \quad \text{Main/Duplicate unit failed and waiting for repair due to abnormal weather}

\underline{DFWR} \quad \text{Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather}

\text{pdf/cdf of repair time of Main unit}

\text{pdf/cdf of repair time of Duplicate unit}

\text{pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in } (0,t]

\text{pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in } (0,t]

\text{pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.}
\( M_i(t) \) Probability that the system is up initially in regenerative state \( S_i \) at time \( t \) without visiting to any other regenerative state

\( W_i(t) \) Probability that the server is busy in state \( S_i \) up to time \( t \) without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states

\( m_{ij} \) The conditional mean sojourn time in regenerative state \( S_i \) when system is to make transition in to regenerative state \( S_j \). Mathematically, it can be written as

\[
m_{ij} = E(T_{ij}) = \int_0^\infty [Q_{ij}(t)] dt = -q_{ij}^{\prime\prime}(0)
\]

where \( T_{ij} \) is the transition time from state \( S_i \) to \( S_j \); \( S_i, S_j \in E \).

\( \mu_i \) The mean sojourn time in state \( S_i \) this is given by

\[
\mu_i = E(T_i) = \int_0^\infty P(T_i > t) dt = \sum_j m_{ij}
\]

where \( T_i \) is the sojourn time in state \( S_i \).

\( \mathbb{S}/\mathbb{C}/\mathbb{C}^n \) Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution \( n \) times

\( ** / * \) Symbol for Laplace Stieltjes Transform (L.S.T.)/ Laplace transform (L.T.)

\( \prime \) (desh) Used to represent alternative result

The following are the possible transition states of the system

\( S_0 = (MO, DCs), S_1 = (MFur, DO), S_2 = (\text{MWO}, \text{DCs}), S_3 = (\text{MFwr}, \text{DO}), S_4 = (\text{MFUR,DFwr}), S_5 = (MO,DFur), S_6 = (\text{MFwr}, \text{DWO}), S_7 = (\text{MFur,DFWR}), S_8 = (\text{MWO}, \text{DFwr}), S_9 = (\text{MFur,DFwr}) \)

The states \( S_0, S_1, S_2, S_3, S_5, S_8, S_9 \) are regenerative while the states \( S_4, S_6, S_7 \) are non-regenerative as shown in figure 4.1.
### 4.3 Transition Probabilities and Mean Sojourn Times

The differential transition probabilities are:

\[
dQ_{01}(t) = \lambda e^{-(\lambda + \beta)t} \, dt,
\]

\[
dQ_{02}(t) = \beta e^{-(\lambda + \beta)t} \, dt,
\]

\[
dQ_{10}(t) = g(t) \, dt,
\]

\[
dQ_{13}(t) = \beta e^{-(\beta + \lambda)t} \overline{G(t)} \, dt,
\]

\[
dQ_{14}(t) = \lambda_1 e^{-(\beta + \lambda)t} \overline{G(t)} \, dt,
\]

\[
dQ_{20}(t) = \beta_1 e^{-\beta t} \, dt,
\]

\[
dQ_{31}(t) = \beta_1 e^{-\beta t} \, dt,
\]

\[
dQ_{32}(t) = g(t) e^{-\beta t} \, dt,
\]

\[
dQ_{36}(t) = \beta e^{\beta t} \overline{G(t)} \, dt,
\]

\[
dQ_{50}(t) = g_1(t) e^{-\beta \lambda t} \, dt,
\]

\[
dQ_{58}(t) = \beta e^{-(\beta + \lambda)t} \overline{G_1(t)} \, dt,
\]

\[
dQ_{59}(t) = \lambda e^{-(\beta + \lambda)t} \overline{G_1(t)} \, dt,
\]

\[
dQ_{67}(t) = \beta_1 e^{-\beta t} \, dt,
\]

\[
dQ_{75}(t) = g(t) e^{-\beta t} \, dt,
\]

\[
dQ_{76}(t) = \beta e^{\beta t} \overline{G(t)} \, dt,
\]

\[
dQ_{85}(t) = \beta_1 e^{-\beta t} \, dt,
\]

\[
dQ_{86}(t) = \beta e^{\beta t} \overline{G(t)} \, dt.
\]
Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t) \, dt \], we have

\[ p_{01} = \frac{\lambda}{\beta + \lambda}, \quad p_{02} = \frac{\beta}{\beta + \lambda}, \quad p_{10} = g^*(\beta + \lambda), \quad p_{13} = \frac{\beta}{\beta + \lambda_i} (1 - g^*(\beta + \lambda_i)), \]

\[ p_{14} = \frac{\lambda_i}{\beta + \lambda_i} (1 - g^*(\beta + \lambda_i)), \quad p_{20} = 1, \quad p_{31} = 1, \quad p_{45} = g^*(\beta), \quad p_{46} = 1 - g^*(\beta), \quad p_{50} = g_t^*(\beta + \lambda), \]

\[ p_{58} = \frac{\beta}{\beta + \lambda} (1 - g_1^*(\beta + \lambda)), p_{59} = \frac{\lambda}{\beta + \lambda_i} (1 - g_1^*(\beta + \lambda)), p_{67} = 1, p_{75} = g^*(\beta), p_{76} = 1 - g^*(\beta), \]

\[ p_{85} = 1, p_{95} = g^*(\beta), p_{96} = 1 - g^*(\beta), p_{15.4}^{(6,7)} = \frac{\lambda_i}{\beta + \lambda_i} (1 - g^*(\beta + \lambda_i)) g^*(\beta), \]

\[ p_{15.4(6,7)}^n = \frac{\lambda_i}{\beta + \lambda_i} (1 - g^*(\beta + \lambda_i)) (1 - g^*(\beta)), p_{95.6(6,7)}^n = 1 - g^*(\beta) \]

\[ \ldots(4.2) \]

It can be easily verified that

\[ p_{01} + p_{02} = p_{10} + p_{13} + p_{16.4} + p_{16.4(6,7)}^n = p_{20} = p_{31} = p_{45} + p_{46} = p_{50} + p_{58} + p_{59} = p_{67} = p_{75} + p_{76} = p_{85} = 1 \]

\[ p_{95} + p_{95.6(6,7)}^n = 1 \]

\[ \ldots(4.3) \]

The mean sojourn times (\( \mu_i \)) in the state \( S_i \) are

\[ \mu_0 = m_{01} + m_{02} = \frac{1}{\beta + \lambda}, \quad \mu_1 = m_{10} + m_{13} + m_{14} = \frac{1}{\beta + \lambda_i} (1 - g^*(\beta + \lambda_i)), \mu_2 = m_{20} = \frac{1}{\beta_i}, \]

\[ \mu_3 = m_{31} = \frac{1}{\beta_i}, \quad \mu_4 = m_{45} + m_{46} = \frac{1}{\beta} (1 - g^*(\beta)), \mu_5 = m_{50} + m_{58} = m_{59} = \frac{1}{\beta + \lambda} (1 - g_1^*(\beta + \lambda)), \]

\[ \mu_6 = m_{67} = \frac{1}{\beta_i} \mu_7 = m_{75} + m_{76} = \frac{1}{\beta} (1 - g^*(\beta)), \mu_8 = m_{85} = \frac{1}{\beta_i}, \mu_9 = m_{95} + m_{96} = \frac{1}{\beta} (1 - g^*(\beta)), \]

\[ \mu_1' = m_{10} + m_{13} + m_{15.4} + m_{15.4(6,7)}^n = \frac{(1 - g^*(\beta + \lambda_i))(\beta \beta_i g^*(\beta) + \lambda_i (\beta + \beta_i)(1 - g^*(\beta)))}{\beta_i (\beta + \lambda_i) g^*(\beta)}, \]

\[ \mu_9' = m_{95} + m_{95.6(6,7)}^n = \frac{(\beta + \beta_i)(1 - g^*(\beta))}{\beta_i g^*(\beta)} \]

\[ \ldots(4.4) \]
4.4 RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state $S_i$ to a failed state. Regarding failed state as absorbing state, we have following recursive relations for $\phi_i(t)$:

$$
\phi_0(t) = Q_{01}(t)\phi_1(t) + Q_{02}(t)\phi_2(t) \\
\phi_1(t) = Q_{10}(t)\phi_0(t) + Q_{13}(t)\phi_3(t) + Q_{14}(t) \\
\phi_2(t) = Q_{20}(t)\phi_0(t) \\
\phi_3(t) = Q_{31}(t)\phi_1(t)
$$

...(4.5)

Taking L.S.T. of above relations (4.4) and solving for $\phi_0^*(s)$, we get

$$
\phi_0^*(s) = \frac{Q_{14}^*(s)Q_{01}^*(s)}{((1 - Q_{02}^*(s)Q_{20}^*(s))(1 - Q_{13}^*(s)Q_{31}^*(s)) - Q_{01}^*(s)Q_{10}^*(s))}
$$

...(4.6)

We have

$$
R^*(s) = \frac{1 - \phi_0^*(s)}{s}
$$

...(4.7)

The reliability of the system model can be obtained by taking inverse Laplace transform of (4.7).

The mean time to system failure (MTSF) is given by

$$
\text{MTSF} = \lim_{s \to 0} \frac{1 - \phi_0^*(s)}{s} = \frac{N_1}{D_1}
$$

...(4.8)

where

$$
N_1 = p_{01}(p_{13}\mu_3 + \mu_1) + (1 - p_{13})(\mu_0 + p_{02}\mu_2) \\
D_1 = p_{01}p_{14}
$$

...(4.9)
4.5 STEADY STATE AVAILABILITY

Let \( A_0(t) \) be the probability that the system is in up-state at instant \( t \) given that the system entered regenerative state \( S_i \) at \( t = 0 \). The recursive relations for \( A_i(t) \) are given as

\[
\begin{align*}
A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \\
A_1(t) &= M_1(t) + q_{10}(t) \odot A_0(t) + q_{13}(t) \odot A_3(t) + (q_{15.4}(t) + q_{15.4(6,7)\nu}(t)) \odot A_5(t) \\
A_2(t) &= q_{20}(t) \odot A_0(t) \\
A_3(t) &= q_{31}(t) \odot A_1(t) \\
A_5(t) &= M_5(t) + q_{50}(t) \odot A_0(t) + q_{58}(t) \odot A_8(t) + q_{59}(t) \odot A_9(t) \\
A_8(t) &= (q_{85}(t) + q_{85(6,7)\nu}(t)) \odot A_5(t)
\end{align*}
\]

…(4.10)

where \( M_i(t) \) is the probability that the system is up initially in state \( S_i \in E \) is up at time \( t \) without visiting to any other regenerative state, we have

\[
\begin{align*}
M_0(t) &= e^{-(\beta + \lambda)t}, \\
M_1(t) &= e^{-(\beta + \lambda)t} G(t), \\
M_5(t) &= e^{-(\beta + \lambda)t} G_1(t)
\end{align*}
\]

…(4.11)

Taking L.T. of above relations (4.9) and (4.10) and solving for \( A_0^*(s) \), we obtain

\[
A_0^*(s) = \frac{((1 - q_{13}(s)q_{13}^*(s))M_0^*(s) + q_{01}(s)M_1^*(s))(1 - q_{58}(s)q_{85}^*(s))}{((1 - q_{13}^*(s)q_{13}(s))(1 - q_{58}(s)q_{85}^*(s)) - q_{01}(s)q_{01}^*(s))1 - q_{58}(s)q_{85}^*(s)}
\]

…(4.12)

The steady state availability is given by

\[
A_0(\infty) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2}
\]

…(4.13)

where

\[
\begin{align*}
N_2 &= p_{50}(\mu_0(1-p_{13}) + \mu_1p_{01}) + \mu_5p_{01}p_{14} \\
D_2 &= p_{01}p_{14}(\mu_5 + \mu_8p_{58} + p_{59}\mu') + p_{50}(p_{01}(\mu_1 + p_{13}\mu_3) + (1-p_{13})(\mu_0 + p_{02}\mu_2))
\end{align*}
\]

…(4.14)
4.6 BUSY PERIOD ANALYSIS OF THE SERVER

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state $S_i$ at $t=0$. The recursive relations for $B_i(t)$ are as follows:

$B_0(t) = q_{01}(t)B_1(t) + q_{02}(t)B_2(t)$
$B_1(t) = W_1(t) + q_{10}(t)B_0(t) + q_{13}(t)B_3(t) + (q_{15.4}(t) + q_{15.4,6.7}(t))B_5(t)$
$B_2(t) = q_{20}(t)B_0(t)$
$B_3(t) = q_{31}(t)B_1(t)$
$B_5(t) = W_5(t) + q_{50}(t)B_0(t) + q_{58}(t)B_8(t) + q_{59}(t)B_9(t)$
$B_8(t) = q_{85}(t)B_0(t)$
$B_9(t) = (q_{95}(t) + q_{95,6.7}(t))B_5(t)$

... (4.15)

where $W_i(t)$ be the probability that the server is busy in state $S_i$ due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,

$W_1(t) = e^{-\beta t}G(t) + (\lambda t e^{-\beta t})G(t)$, $W_5(t) = e^{-\beta t}G(t) + (\lambda t e^{-\beta t})G(t)$, $W_9(t) = e^{-\beta t}G(t)$

... (4.16)

Taking L.T. of above relations (4.15) and solving for $B_0^*(s)$ we get

$B_0^*(s) = \frac{W_1^*(s)q_{01}(s)(1 - q_{55}(s)q_{45}(s) - q_{59}(s)(q_{95}(s) + q_{95,6.7}^*(s)))}{(1 - q_{13}(s)q_{11}(s))(1 - q_{15.4}(s)q_{15.4,6.7}^*(s)) - q_{01}(s)q_{10}(s)(1 - q_{55}(s)q_{85}(s) - q_{59}(s)(q_{95}(s) + q_{95,6.7}^*(s)))}$

... (4.17)

The time for which server is busy due to repair is given by

$B_0^*(\infty) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3}{D_2}$

... (4.18)

where

$N_3 = p_{01}p_{50}W_1^*(0) + p_{01}p_{14}(W_5^*(0) + W_9^*(0)p_{59})$ and $D_2$ is already mentioned.

... (4.19)
4.7 EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in $[0,t]$ given that the system entered the regenerative state $S_i$ at $t=0$. The recursive relations for $N_i(t)$ are given as:

$N_0(t) = Q_{01}(t) \otimes (1+N_1(t)) + Q_{02}(t) \otimes N_2(t)$

$N_1(t) = Q_{10}(t) \otimes N_0(t) + Q_{13}(t) \otimes N_3(t) + Q_{15.4}(t) \otimes N_5(t) + Q_{15.4}(67)^n(t) \otimes (1+N_5(t))$

$N_2(t) = Q_{20}(t) \otimes N_0(t)$

$N_3(t) = Q_{31}(t) \otimes (1+N_1(t))$

$N_5(t) = Q_{50}(t) \otimes N_0(t) + Q_{58}(t) \otimes N_8(t) + Q_{59}(t) \otimes N_9(t)$

$N_8(t) = Q_{85}(t) \otimes (1+N_5(t))$

$N_9(t) = Q_{95}(t) \otimes N_5(t) + Q_{95.67}(n(t) \otimes (1+N_5(t))$

Taking L.S.T. of relations (4.19) and solving for $N_0^+(s)$, we have

$$N_0^+(s) = \frac{((1 - Q_{13}^+(s)Q_{31}^+(s))(Q_{15.4}(67)^n(s) + Q_{15.4}(s)Q_{31}^+(s)))}{(1 - Q_{58}^+(s)Q_{59}^+(s)Q_{96}(s) + Q_{95.67}^+(s)s)Q_{58}^+(s))}$$

The expected numbers of visits per unit time by the server are given by

$$N_0^+(\infty) = \lim_{s \to 0} sN_0^+(s) = \frac{N_4}{D_2}$$

where

$N_4 = p_{01}(1+p_{14}p_{46})p_{50} + p_{14}(p_{58}+p_{59}p_{96})$ and $D_2$ is already specified.
4.8 PROFIT ANALYSIS

The profit incurred to the system Model in steady state can be obtained as

\[ P = K_0 A_0 - K_1 B_0 - K_2 N_0 \]

where

- \( K_0 \) = Revenue per unit up-time of the system
- \( K_1 \) = Cost per unit for which server is busy
- \( K_2 \) = Cost per unit visit by the server and \( A_0, B_0, N_0 \) are already defined.

4.9 PARTICULAR CASE

Suppose \( g(t) = \alpha e^{-\alpha t} \), \( g_1(t) = \alpha e^{-\alpha t} \)

By using the non-zero elements \( p_{ij} \), we can obtain the following results:

\[ p_{01} = \frac{\lambda}{\beta + \lambda}, p_{02} = \frac{\lambda}{\beta + \lambda}, p_{10} = \frac{\alpha}{\alpha + \beta + \lambda}, p_{13} = \frac{\beta}{\alpha + \beta + \lambda}, p_{14} = \frac{\lambda}{\alpha + \beta + \lambda}, p_{20} = 1, p_{31} = 1, \]

\[ p_{45} = \frac{\alpha}{\alpha + \beta}, p_{46} = \frac{\beta}{\alpha + \beta}, p_{50} = \frac{\alpha_1}{\alpha_1 + \beta + \lambda}, p_{58} = \frac{\beta}{\alpha_1 + \beta + \lambda}, p_{59} = \frac{\lambda}{\alpha_1 + \beta + \lambda}, p_{67} = 1, \]

\[ p_{75} = \frac{\alpha}{\alpha + \beta}, p_{76} = \frac{\beta}{\alpha + \beta}, p_{85} = 1, p_{95} = \frac{\alpha}{\alpha + \beta}, p_{96} = \frac{\beta}{\alpha + \beta}, p_{15.4} = \frac{\lambda}{\alpha + \beta + \lambda}, \frac{\alpha}{\alpha + \beta}, p_{15.4(6,7)} = \frac{\beta}{\alpha + \beta + \lambda}, \frac{\alpha}{\alpha + \beta}, \]

\[ p_{15.4(6,7)n} = \frac{\lambda}{\alpha + \beta + \lambda}, \frac{\beta}{\alpha + \beta}, p_{95.6(5,7)n} = \frac{\beta}{\alpha + \beta}, \]

\[ \mu_0 = \frac{1}{\beta + \lambda}, \mu_1 = \frac{1}{\alpha + \beta + \lambda}, \mu_2 = \frac{1}{\beta_1}, \mu_3 = \frac{1}{\alpha + \beta}, \mu_4 = \frac{1}{\alpha + \beta}, \mu_5 = \frac{1}{\alpha + \beta + \lambda}, \mu_6 = \frac{1}{\beta_1}, \mu_7 = \frac{1}{\alpha + \beta}, \mu_8 = \frac{1}{\alpha + \beta}, \mu_9 = \frac{1}{\alpha + \beta}. \]

\[ W_1^*(0) = \frac{(\alpha + \lambda_1)}{\alpha (\alpha + \beta + \lambda_1)}, W_5^*(0) = \frac{1}{\alpha_1 + \beta + \lambda}, W_9^*(0) = \frac{1}{\alpha + \beta}, \]

\[ MTSF (T_0) = \frac{N_1}{D_1}, \text{ Steady state availability (A_0) } = \frac{N_2}{D_2}, \]

Busy period of the server (B_0) = \( \frac{N_3}{D_2} \),

Expected number of visits by the server (N_0) = \( \frac{N_4}{D_2} \),

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Where

\[ N_1 = (\alpha + \lambda + \lambda_1)(\beta + \beta_1) \]

\[ D_1 = \lambda \lambda_1 \beta_1 \]

\[ N_2 = a \beta_1 (\alpha_1 (\alpha + \lambda) + \lambda_1 (\alpha_1 + \lambda)) \]

\[ D_2 = (\beta + \beta_1 ) (\alpha + \lambda ) (\lambda \lambda_1 + \alpha_1 (\alpha + \lambda_1)) \]

\[ N_3 = \frac{\beta \lambda (\alpha_1 (\alpha + \lambda_1)(\beta + \lambda) + \alpha \lambda_1 (\alpha + \beta + \lambda))}{(\alpha + \beta)} \]

\[ N_4 = \frac{\lambda (\alpha_1 (\alpha + \beta)(\alpha + \beta + \lambda_1) + \beta \lambda_1 (\alpha + \alpha_1 + \beta + \lambda))}{(\alpha + \beta)} \]
## 4.10 TABLES AND GRAPHS

### Table 4.1: MTSF vs. Normal Weather Rate ($\beta_1$)

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
</tr>
</thead>
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<td>8.745455</td>
<td>10.80303</td>
<td>16.25758</td>
<td>14.63182</td>
</tr>
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<td>8.738889</td>
<td>10.76389</td>
<td>16.24537</td>
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<td>10.73077</td>
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<td>14.59667</td>
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<td>10.65625</td>
<td>16.21181</td>
<td>14.59063</td>
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<td>10.63725</td>
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<td>16.19591</td>
<td>14.57632</td>
</tr>
</tbody>
</table>

### Fig. 4.2

MTSF vs. Normal Weather Rate ($\beta_1$)

- $\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.5$
- $\alpha=1.5$
- $\beta=0.05$
- $\lambda=0.3$
- $\lambda_1=0.4$
Table 4.2: Availability vs. Normal Weather Rate ($\beta_1$)

<table>
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<tr>
<th>Normal weather Rate($\beta_1$)</th>
<th>$\alpha=2, \beta=0.01, \lambda=0.5, \alpha_1=2.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha_1=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
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<td>1.1</td>
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<td>0.90588</td>
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</table>

Fig. 4.3
Table 4.3: Profit vs. Normal Weather Rate ($\beta_1$)

<table>
<thead>
<tr>
<th>Normal weather rate ($\beta_1$)</th>
<th>K_0=5000, K_1=350, K_2=300, $\alpha=2, \beta=0.01, \lambda=0.5$, $\alpha_1=2.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha_1=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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<td>4391.325</td>
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</table>

![Graph of Profit vs. Normal Weather Rate ($\beta_1$)](image-url)

**Fig. 4.4**
Table 4.4: Profit difference between Model 2.1 and 4.1

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300,$</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01,$</th>
<th>$\lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha_1=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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<tbody>
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<tr>
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<td>53.38572</td>
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</tr>
</tbody>
</table>

Profit Difference vs. Normal Weather Rate ($\beta_1$)

- $K_0=5000$
- $K_1=350$
- $K_2=300$

Fig. 4.5
Table 4.5: Profit difference between Model 3.1 and Model 4.1

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300, \alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
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<tbody>
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<td>6.2153</td>
<td>-37.3973</td>
</tr>
</tbody>
</table>

Fig. 4.6
4.11 CONCLUSION

The stochastic behavior of the mean time to system failure (MTSF), availability and profit function has been observed on the basis graphs drawn for a particular case as shown in figures 4.2, 4.3 and 4.4 respectively. From figure 4.2, it is analyzed the MTSF goes on decreasing with the increase of normal weather rate ($\beta_1$) and failure rates ($\lambda$ and $\lambda_1$). However, MTSF increases with increase of abnormal weather rate ($\beta$) and repair rate ($\alpha$) of the main unit. Figures 4.3 and 4.4, indicate that the values of availability and profit availability and profit keep on increasing with the increase of normal weather rate ($\beta_1$) and repair rates ($\alpha$ and $\alpha_1$). But their values decline as and when abnormal weather rate ($\beta$) and failure rates ($\lambda$ and $\lambda_1$) increase. Hence, the study reveals that a system of non-identical unit working under different weather conditions can be made more profitable by increasing repair rate of the main unit.

4.12 COMPARATIVE STUDY

i. Comparison of MTSF

MTSF Comparison between Model 2.1 and Model 4.1

It is observed that the MTSF of the Model 4.1 is same as that of Model 3.1. Therefore, the interpretation of the results on difference of MTSF of the Models 2.1 and 4.1 is same as that between Models 2.1 and 3.1 (see fig. 3.5).

ii. Comparison of Profit

a. Profit Comparison between Model 2.1 and Model 4.1

The profit difference between Model 2.1 and Model 4.1 is shown in the fig.4.5. It is observed that the profit difference (Model 2.1-Model 4.1) goes on decreasing with the increase of normal weather rate ($\beta_1$) and repair rate ($\alpha$) of main unit. The Model 2.1 is profitable over 4.1. Thus, the concept of priority for operation and repair to main unit over duplicate unit is not advisable when the system has no operation and repair activity in abnormal weather.
b. Profit Comparison between Model 3.1 and Model 4.1

The profit difference between Model 3.1 and Model 4.1 is shown in fig.4.6. The profit difference (Model 3.1-Model 4.1) goes on decreasing with the increase of weather rates (β and β₁) and repair rate (α) of the main unit. However, the difference increases with increase in failure rates (λ and λ₁) of the units and repair rate (α₁) of duplicate unit. Thus, we can say that the idea of giving priority for operation and repair to main unit instead of priority for operation only is much more beneficial if failure rate of duplicate unit is less than that of main unit.