6.1 INTRODUCTION

In 5\textsuperscript{th} chapter a stochastic model for a system of non-identical units is developed by allowing operation in abnormal weather. The concept of priority for operation to main unit over duplicate unit has been introduced in chapter 3\textsuperscript{rd} with no operation of the system in abnormal weather. It is revealed that the idea of priority to operation is not helpful in making the system more profitable if system is not allowed to operate in abnormal weather. Thus, it becomes necessary to ascertain whether a system of non-identical units with priority for operation to main unit will have profit if it is allowed to operate in abnormal weather. Hence, the main object of this chapter is to fill this gap. For this, a stochastic model is developed for a system of non-identical units-one is original unit (called main unit) and the other is substandard unit (called duplicate unit). Initially, the original unit is operative and the substandard unit is kept as cold standby. Each unit has a constant failure rate from normal mode. A single server is provided immediately to carry out repair activities in normal weather. The operation of the units is also allowed in abnormal weather. Priority is given to the operation of the main unit over the duplicate unit. The units works as new after repair.

The distributions of the failure time of the units and change of weather conditions follow negative exponential whereas repair time of the units is arbitrarily distributed with different probability density functions. All random variables are statistically independent. The switch over is instantaneous and perfect. The expressions for some important reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), and availability, busy period of the server and profit function are derived in steady state using semi-Markov process and regenerative point technique. The variations in MTSF, availability and profit functions have been observed with respect to normal weather rate for arbitrary values of various parameters and costs. The MTSF and profit function of the present system model is compared with that of the models discussed in chapters 3\textsuperscript{rd} and 5\textsuperscript{th}.

6.2 NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>The set of regenerative states</td>
</tr>
<tr>
<td>MO/DO</td>
<td>Main/Duplicate unit is good and operative</td>
</tr>
<tr>
<td>MO\slash DO</td>
<td>Main/Duplicate unit is good and operating in abnormal weather</td>
</tr>
<tr>
<td>DCs</td>
<td>Duplicate unit is in cold standby mode</td>
</tr>
<tr>
<td>DCS</td>
<td>Duplicate unit is in cold standby mode in abnormal weather</td>
</tr>
<tr>
<td>$\lambda/\lambda_1$</td>
<td>Constant failure rate of Main /Duplicate unit</td>
</tr>
<tr>
<td>$\beta/\beta_1$</td>
<td>Constant rate of change of weather from normal to</td>
</tr>
</tbody>
</table>
abnormal/abnormal to normal weather

MFur/DFur  Main/duplicate unit failed and under repair
MFUR/DFUR Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr Main/duplicate unit failed and waiting for repair
MFWR/DFWR Main/duplicate unit failed and waiting for repair continuously from previous state

MFWR / DFwr Main/Duplicate unit failed and waiting for repair due to abnormal weather

MFWR / DFWR Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather

g(t)/G(t) pdf/cdf of repair time of Main unit

g1(t)/ G1(t) pdf/cdf of repair time of Duplicate unit

q_{ij}(t)/Q_{ij}(t) pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]

q_{ij,kr}(t)/Q_{ij,kr}(t) pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]

q_{ij,k,r(s)}^{n}(t)/Q_{ij,k,r(s)}^{n}(t) pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.

M_{i}(t) Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state

W_{i}(t) Probability that the server is busy in state Siupto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states

m_{ij} The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as
\[ m_{ij} = E(T_{ij}) = \int_0^\infty t \, d \{ Q_{ij}(t) \} = -q_{ij}^{(i)}(0), \]

where \( T_{ij} \) is the transition time from state \( S_i \) to \( S_j \); \( S_i, S_j \in E \).

The mean Sojourn time in state \( S_i \) this is given by

\[ \mu_i = E(T_i) = \int_0^\infty P(T_i > t) \, dt = \sum_j m_{ij}, \]

where \( T_i \) is the sojourn time in state \( S_i \).

\( \mathbb{S}/\mathbb{C}/\mathbb{C}^n \)
Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution n times

\( ** / * \)
Symbol for Laplace Stieltjes Transform (L.S.T.)/Laplace transform (L.T.)

\( ^\prime \) (desh)
Used to represent alternative result

The following are the possible transition states of the system

\[ S_0 = (\text{MO, DCs}), S_1 = (\text{MFur, DO}), S_2 = (\text{MO, DCs}), S_3 = (\text{MFwr, DO}), \]

\[ S_4 = (\text{MFUR, DFwr}), S_5 = (\text{MFWR, DFWR}), S_6 = (\text{MFwr, DFWR}), S_7 = (\text{MO, DFur}), \]

\[ S_8 = (\text{MFur, DFWR}), S_9 = (\text{MO, DFwr}), S_{10} = (\text{MFwr, DFUR}) \]

The states \( S_0, S_1, S_2, S_3, S_7, \) and \( S_9 \) are regenerative while the states \( S_4, S_5, S_6, S_8, \) and \( S_{10} \) are non regenerative as shown in figure 6.1.
6.3 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The differential transition probabilities are:

\[
\begin{align*}
\text{d}Q_{01}(t) &= \lambda e^{-(\lambda + \beta)t} \text{dt}, \\
\text{d}Q_{02}(t) &= \beta e^{-(\lambda + \beta)t} \text{dt}, \\
\text{d}Q_{10}(t) &= g(t) e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{13}(t) &= \beta e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{14}(t) &= \lambda e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{20}(t) &= \beta e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{23}(t) &= \lambda e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{31}(t) &= \beta e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{36}(t) &= \alpha e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{46}(t) &= \beta e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{47}(t) &= g(t) e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{57}(t) &= \lambda e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{70}(t) &= g(t) e^{-(\beta + \lambda)t} \text{dt}, \\
\text{d}Q_{80}(t) &= \beta e^{-(\beta + \lambda)t} \text{G}(t) \text{dt}, \\
\text{d}Q_{87}(t) &= g(t) e^{-(\beta + \lambda)t} \text{dt},
\end{align*}
\]

Fig. 6.1

- Up-state
- Failed-State
- Regenerative point
\[
dQ_9(t) = \beta_1 e^{-(\beta_1 + \lambda_1) t} dt, \quad \text{d}Q_{96}(t) = \lambda e^{-(\beta_1 + \lambda_1) t} dt, \quad \text{d}Q_{10.1}(t) = g_1(t) e^{-\beta_1} dt, \quad \text{d}Q_{10.5}(t) = \beta e^{-\beta_1} G_1(t) dt \]

\[
\ldots(6.1)
\]

Simple probabilistic considerations yield the following expressions for the non-zero elements

\[
p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t) dt , \text{ we have}
\]

\[
p_{01} = \frac{\lambda}{\beta + \lambda}, \quad p_{02} = \frac{\beta}{\beta + \lambda}, \quad p_{10} = g^*(\beta + \lambda_1), \quad p_{13} = \frac{\beta}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)),
\]

\[
p_{14} = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)), \quad p_{20} = \frac{\beta_1}{\beta_1 + \lambda_1}, \quad p_{23} = \frac{\lambda_1}{\beta_1 + \lambda_1}, \quad p_{31} = \frac{\beta_1}{\beta_1 + \lambda_1},
\]

\[
p_{35} = \frac{\lambda_1}{\beta_1 + \lambda_1} \cdot p_{46} = 1 - g^*(\beta), p_{47} = g^*(\beta), p_{58} = 1, p_{68} = 1, \quad p_{70} = g_1^*(\beta + \lambda),
\]

\[
p_{79} = \frac{\beta}{\beta + \lambda} (1 - g^*(\beta + \lambda)), \quad p_{7.10} = \frac{\lambda}{\beta + \lambda} (1 - g_1^*(\beta + \lambda)), \quad p_{86} = 1 - g^*(\beta),
\]

\[
p_{87} = g^*(\beta), p_{96} = \frac{\lambda}{\beta_1 + \lambda_1}, \quad p_{97} = \frac{\beta_1}{\beta_1 + \lambda_1} \cdot p_{10.1} = g_1^*(\beta), p_{10.5} = 1 - g_1^*(\beta),
\]

\[
p_{17.4} = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)) g^*(\beta), \quad p_{17.4}(6.8)^n = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)) (1 - g^*(\beta)),
\]

\[
p_{35.58} = \frac{\lambda_1}{\beta_1 + \lambda_1} (1 - g^*(\beta)), p_{35.58}(6.8)^n = \frac{\lambda_1}{\beta_1 + \lambda_1} g^*(\beta), p_{71.10} = \frac{\lambda}{\beta + \lambda} (1 - g_1^*(\beta + \lambda)) g_1^*(\beta),
\]

\[
p_{77.10.5,8} = \frac{\lambda}{\beta + \lambda} (1 - g_1^*(\beta + \lambda))(1 - g_1^*(\beta)) g^*(\beta),
\]

\[
p_{77.10.5,8}(6.8)^n = \frac{\lambda}{\beta + \lambda} (1 - g_1^*(\beta + \lambda))(1 - g_1^*(\beta))(1 - g^*(\beta)) - p_{97.10.6,8}^n = \frac{\lambda}{\beta_1 + \lambda_1} ,
\]

\[
\ldots(6.2)
\]

It can be easily verified that

\[
p_{01} + p_{02} = p_{10} + p_{13} + p_{17.4} + p_{17.4}(6.8)^n = p_{20} + p_{23} = p_{31} + p_{31} + p_{37.58} + p_{35.58}(6.8)^n = p_{46} + p_{47} = p_{58} = p_{68} = 1
\]

\[
p_{70} + p_{79} + p_{71.10} + p_{77.10.5,8} + p_{77.10.5,8}(6.8)^n = p_{86} + p_{87} = p_{97} + p_{97.10.6,8}^n = p_{10.1} + p_{10.5} = 1
\]

\[
\ldots(6.3)
\]

The mean sojourn times ($\mu_i$) in the state $S_i$ are

\[
\mu_0 = m_{01} + m_{02} = \frac{1}{\beta + \lambda}, \quad \mu_1 = m_{10} + m_{13} + m_{13} = \frac{1}{\beta + \lambda} (1 - g^*(\beta + \lambda_1)), \quad \mu_2 = m_{20} + m_{23} = \frac{1}{\beta_1 + \lambda_1}.
\]
\[ \mu_3 = m_{31} + m_{35} = \frac{1}{\beta_1 + \lambda_1}, \mu_4 = m_{46} + m_{47} = \frac{1}{\beta} (1-g^*(\beta)), \mu_5 = m_{58} = \frac{1}{\beta_1}, \mu_6 = m_{68} = \frac{1}{\beta}, \]
\[ \mu_7 = m_{70} + m_{79} + m_{7,10} = \frac{1}{\beta + \lambda} (1-g_1^*(\beta + \lambda)), \mu_8 = m_{86} + m_{87} = \frac{1}{\beta} (1-g^*(\beta)), \]
\[ \mu_9 = m_{96} + m_{97} = \frac{1}{\beta_1 + \lambda}, \mu_{10} = m_{10,1} + m_{10,5} = \frac{1}{\beta} (1-g_1^*(\beta)), \]
\[ \mu'_1 = m_{10} + m_{13} + m_{17,4} + m_{17,4,(6,8)} = \frac{(1-g^*(\beta + \lambda))(\beta \beta_1 g^*(\beta) + \lambda_1 (\beta + \beta_1)(1-g^*(\beta))}{\beta \beta_1 (\beta + \lambda_1) g^*(\beta)}, \]
\[ \mu'_1 = m_{31} + m_{37,8} + m_{37,8,(6,8)} = \frac{\beta (\beta_1 + \lambda) g^*(\beta) + \lambda_1 (\beta + \beta_1)(1-g^*(\beta))}{\beta \beta_1 (\beta_1 + \lambda_1) g^*(\beta)}, \]
\[ \mu'_2 = m_{70} + m_{79} + m_{7,10} + m_{77,10,5,8} + m_{77,10,5,8,(6,8)} = \frac{\beta (1-g^*(\beta)) + \lambda (1-g^*(\beta))(\beta + \beta_1)}{\beta \beta_1 (\beta + \lambda) g^*(\beta)}, \]
\[ \mu'_9 = m_{97} + m_{97,(6,8)} = \frac{\beta (\beta_1 + \lambda) g^*(\beta) + \lambda (\beta + \beta_1)(1-g^*(\beta))}{\beta \beta_1 (\beta_1 + \lambda_1) g^*(\beta)} \]
\[ \ldots (6.4) \]

### 6.4 RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let \( \varnothing_i(t) \) be the cdf of first passage time from regenerative state \( S_i \) to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for \( \varnothing_i(t) \):

\[ \varnothing_0(t) = Q_{01}(t) \varnothing_1(t) + Q_{02}(t) \varnothing_2(t) \]
\[ \varnothing_1(t) = Q_{10}(t) \varnothing_0(t) + Q_{13}(t) \varnothing_3(t) + Q_{14}(t) \]
\[ \varnothing_2(t) = Q_{20}(t) \varnothing_0(t) + Q_{23}(t) \varnothing_3(t) \]
\[ \varnothing_3(t) = Q_{31}(t) \varnothing_1(t) + Q_{35}(t) \]
\[ \ldots (6.5) \]

Taking L.S.T. of above relations (6.5) and solving for \( \varnothing_0^{**}(s) \)

\[ \varnothing_0^{**}(s) = \frac{Q_{01}^{**}(s)(Q_{13}^{**}(s) + Q_{14}^{**}(s)Q_{35}^{**}(s)) + Q_{02}^{**}(s)Q_{23}^{**}(s)(Q_{33}^{**}(s)Q_{35}^{**}(s) + Q_{35}^{**}(s))}{(1 - Q_{13}^{**}(s)Q_{31}^{**}(s))(1 - Q_{02}^{**}(s)Q_{23}^{**}(s)) - Q_{10}^{**}(s)(Q_{01}^{**}(s) + Q_{02}^{**}(s)Q_{23}^{**}(s)Q_{35}^{**}(s))} \]
\[ \ldots (6.6) \]
We have

\[ R^*(s) = \frac{1 - \phi^\infty_0(s)}{s} \]  

\[ \cdots \text{(6.7)} \]

The reliability of the system model can be obtained by taking inverse Laplace transform of (6.7). The mean time to system failure (MTSF) is given by

\[ MTSF = \lim_{s \to 0} \frac{1 - \phi^\infty_0(s)}{s} = \frac{N_1}{D_1} \]

\[ \cdots \text{(6.8)} \]

\[ N_1 = (1-p_{13}p_{31})(p_{02}p_{23}+\mu_0)+p_{01}(\mu_1+p_{13}p_{41})+p_{02}p_{23}(\mu_1p_{31}+\mu_3) \]

\[ D_1=(1-p_{13}p_{31})(1-p_{02}p_{23})-p_{10}(p_{01}+p_{02}p_{23}p_{31}) \]

\[ \cdots \text{(6.9)} \]

### 6.5 STEADY STATE AVAILABILITY

Let \( A_i(t) \) be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state \( S_i \) at \( t = 0 \). The recursive relations for \( A_i(t) \) are given as:

\[ A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \]

\[ A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{13}(t) \odot A_3(t) + (q_{17.4}(t) + q_{17.4,6.8}(t)) \odot A_7(t) \]

\[ A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{23}(t) \odot A_3(t) \]

\[ A_3(t) = M_3(t) + q_{31}(t) \odot A_1(t) + (q_{37.58}(t) + q_{37.5,8.6}(t)) \odot A_7(t) \]

\[ A_7(t) = M_7(t) + q_{70}(t) \odot A_0(t) + q_{71.10}(t) \odot A_1(t) + (q_{77.10,5,8}(t) + q_{77.10,5,8.6}(t)) \odot A_7(t) \]

\[ + q_7(\text{G}) \odot A_9(t) \]

\[ A_9(t) = M_9(t) + (q_{97}(t) + q_{97.6,8}(t)) \odot A_7(t) \]

\[ \cdots \text{(6.10)} \]

Where \( M_i(t) \) is the probability that the system is up initially in state \( S_i \in E \) is up at time \( t \) without visiting to any other regenerative state, we have

\[ M_0(t) = e^{-(\beta + \lambda)t}, M_1(t) = e^{-((\beta + \lambda)t) \cdot G_1(t)}, M_2(t) = e^{-(\beta + \lambda)t}, M_3(t) = e^{-(\beta + \lambda)t}, \]

\[ M_7(t) = e^{-(\beta + \lambda)t} \cdot G_7(t), M_9(t) = e^{-((\beta + \lambda)t)} \]

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Taking L.T. of above relations (6.10) and solving for $A_0(s)$, we get

$$M_0(s) + q_{02}^*(s)M_2(s) = \left(1 - q_{77.10,5}(8,6)^*\right)(s) - \left(q_{q_{97}(s)}^*(s) + q_{q_{97,6,8}^*}(s)q_{q_{79}(s)}^*(s)(1 - q_{q_{13}(s)}q_{q_{31}(s)})
\right)$$

The steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{N_2}{D_2}$$

where

$$N_2 = (\mu_0 + \mu_2p_2)\left(1 - p_{13,31}\right)p_{70,71.10}(p_{14,13,35}) + (\mu_7 + p_{79,49})(p_{35, p_{01,13} + p_{02,23}} + p_{14, p_{01,23} + p_{02,31}}) + p_{14, p_{01,23} + p_{02,31}} + p_{13,13,13,13} + p_{23,23,23,23} + m_1(p_{01, p_{02,23} + p_{31}}) - p_{71.10, p_{02,23} + p_{14,31,31} - p_{35,41}}$$

$$D_2 = (\mu_0 + \mu_2p_2)\left(1 - p_{13,31}\right)p_{70,71.10}(p_{14,13,35}) + (\mu_7 + p_{79,49})(p_{35, p_{01,13} + p_{02,23}} + p_{14, p_{01,23} + p_{02,31}}) + p_{14, p_{01,23} + p_{02,31}} + p_{13,13,13,13} + p_{23,23,23,23} + m_1(p_{01, p_{02,23} + p_{31}}) - p_{71.10, p_{02,23} + p_{14,31,31} - p_{35,41}}$$
6.6 BUSY PERIOD ANALYSIS OF THE SERVER

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant ‘$t$’ given that the system entered regenerative state $S_i$ at $t=0$. The recursive relations for $B_i(t)$ are as follows:

$$B_0(t) = q_0(t) \oplus B_1(t) + q_2(t) \oplus B_2(t)$$

$$B_1(t) = W_1(t) + q_{10}(t) \oplus B_0(t) + q_{13}(t) \oplus B_3(t) + (q_{17.4}(t) + q_{17.4,6.8}(t)) \oplus B_7(t)$$

$$B_2(t) = q_{20}(t) \oplus B_0(t) + q_{23}(t) \oplus B_3(t)$$

$$B_3(t) = q_{31}(t) \oplus B_1(t) + (q_{37.58}(t) + q_{37.5,6.8}(t)) \oplus B_7(t)$$

$$B_7(t) = W_7(t) + q_{70}(t) \oplus B_0(t) + q_{71.10}(t) \oplus B_1(t) + (q_{77.10.5.8}(t) + q_{77.10.5.6.8}(t)) \oplus B_7(t) + q_{79}(t) \oplus B_9(t)$$

$$B_9(t) = (q_{97}(t) + q_{97,6.8}(t)) \oplus B_7(t)$$

...(6.15)

where $W_i(t)$ be the probability that the server is busy in state $S_i$ due to failure up to time $t$ without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,

$$W_1(t) = e^{-\beta + \lambda t} G(t) + (\lambda e^{-(\beta + \lambda t)} \oplus 1) G(t)$$

$$W_7(t) = e^{-\beta + \lambda t} G(t) + (\lambda e^{-(\beta + \lambda t)} \oplus 1) G(t)$$

...(6.16)

Taking L.T. of above relations (6.14) and solving for $B_6(s)$, we have

$$B_6(s) = \frac{W_1(s)(1 - q_{77.10.5.8}(s) - (q_{77.10.5.8}(s) + q_{77.10.5.6.8}(s))q_{97}(s)(q_{01}(s) + q_{02}(s)q_{23}(s)q_{31}(s)) + q_{71.10}(s)q_{02}(s)q_{23}(s)q_{57.518.67}(s) + W_7(s)(q_{77.10.5.6.8}(s)q_{01}(s)q_{13}(s) + q_{02}(s)q_{23}(s))}{(1 - q_{02}(s)q_{23}(s))(1 - q_{77.10.5.6.8}(s) - (q_{77.10.5.8}(s) + q_{77.10.5.6.8}(s))q_{97}(s)(1 - q_{13}(s)q_{31}(s)) - (q_{37.518.67}(s)q_{13}(s)q_{31.10}(s) - (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{71.10}(s) - q_{70}(s)(1 - q_{77.10.5.6.8}(s) - (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{71.10}(s) + q_{03}(s)q_{23}(s)q_{37.518.67}(s) - q_{70}(s)q_{37.518.67}(s)q_{03}(s)q_{13}(s) + q_{02}(s)q_{23}(s) + (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{03}(s)q_{23}(s)q_{37.518.67}(s))}{(1 - q_{02}(s)q_{23}(s))(1 - q_{77.10.5.6.8}(s) - (q_{77.10.5.8}(s) + q_{77.10.5.6.8}(s))q_{97}(s)(1 - q_{13}(s)q_{31}(s)) - (q_{37.518.67}(s)q_{13}(s)q_{31.10}(s) - (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{71.10}(s) - q_{70}(s)(1 - q_{77.10.5.6.8}(s) - (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{71.10}(s) + q_{03}(s)q_{23}(s)q_{37.518.67}(s) - q_{70}(s)q_{37.518.67}(s)q_{03}(s)q_{13}(s) + q_{02}(s)q_{23}(s) + (q_{77.10.5.6.8}(s) + q_{77.10.5.6.8}(s))q_{03}(s)q_{23}(s)q_{37.518.67}(s))}$$

...(6.17)
The time for which server is busy due to repair is given by
\[
B_0^*(\infty) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3}{D_2}
\] …(6.18)
\[
N_3 = W_1^*(0)(p_70(p_{01}+p_{02}p_{23}p_{31})+p_{02}p_{23}p_{71.10}p_{33})+W_7^*(0)(p_{01}(p_{14}+p_{13}p_{35})+p_{02}p_{23}(p_{14}p_{31}+p_{35}))and D_2 \text{ is already mentioned.}
\] …(6.19)

**6.7 EXPECTED NUMBER OF VISITS BY THE SERVER**

Let \( N_i(t) \) be the expected number of visits by the server in \((0,t]\) given that the system entered the regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( N_i(t) \) are given as:

\[
N_0(t) = Q_{01}(t)S(1+N_1(t)) + Q_{02}(t)S_N(t)
\]

\[
N_1(t) = Q_{10}(t)SN_0(t) + Q_{13}(t)SN_3(t) + Q_{17.4}(t)S_N(t) + Q_{17.4.(6,8)}(t)S(1+N_7(t))
\]

\[
N_2(t) = Q_{20}(t)SN_0(t) + Q_{23}(t)SN_3(t)
\]

\[
N_3(t) = Q_{31}(t)S(1+N_1(t)) + (Q_{37.58}(t) + Q_{37.5.(8,6)}(t))S(1+N_7(t))
\]

\[
N_7(t) = Q_{70}(t)SN_0(t) + Q_{71.10}(t)SN_1(t) + (Q_{77.10.5.8}(t) + Q_{77.10.5.(8,6)}(t))S(1+N_7(t))
\]

\[
N_9(t) = (Q_{79}(t) + Q_{79.(6,8)}(t))S(1+N_7(t))
\]

\[
N_9(t) = (Q_{97}(t) + Q_{97.(6,8)}(t))S(1+N_7(t))
\]

Taking L.S.T. of relations (6.20) and solving for \( N_0^*(s) \). We obtain
The expected numbers of visits per unit time by the server are given by

\[ N_0^{**}(s) = \left( \frac{(Q_{01}^{**}(s) + Q_{02}^{**}(s) + Q_{23}^{**}(s) + Q_{31}^{**}(s))}{(1 - Q_{20}^{**}(s))(1 - Q_{77.10}^{**}(s))} \right) \]

…(6.21)

where

\[ N_a = \frac{p_{01}(1 - p_{13}p_{31} + p_{13} + p_{14}p_{46}) + p_{02}p_{23}(1 + p_{14}p_{46}p_{31}) - p_{71.10}(p_{01}(1 + p_{35}p_{13}) - p_{02}p_{23}(p_{35}p_{46}p_{14}) + p_{75.10}(p_{14}(p_{01} + p_{02}p_{23}) - p_{31}(p_{02}p_{23} + p_{01}p_{13}) - p_{14}p_{46}(p_{02}p_{23} + p_{14}p_{31}))}{D_2} \]

and \( D_2 \) is already specified. …(6.23)
6.8 PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as
\[ P = K_0 A_0 - K_1 B_0 + K_2 N_0 \]
where
\[ K_0 = \text{Revenue per unit up-time of the system} \]
\[ K_1 = \text{Cost per unit for which server is busy} \]
\[ K_2 = \text{Cost per unit visit by the server and } A_0, B_0, N_0 \text{ are already defined.} \]

6.9 PARTICULAR CASE

Suppose \( g(t) = e^{-at} \), \( g_1(t) = e^{-\eta t} \)

By using the non zero elements \( p_{ij} \), we can obtain the following results:

\[ p_{01} = \frac{\lambda}{\beta + \lambda}, \quad p_{02} = \frac{\beta}{\beta + \lambda}, \quad p_{10} = \frac{\alpha}{\alpha + \beta + \lambda}, \quad p_{13} = \frac{\beta}{\alpha + \beta + \lambda}, \quad p_{14} = \frac{\lambda}{\alpha + \beta + \lambda}, \quad p_{20} = \frac{\beta}{\beta + \lambda} \]

\[ p_{23} = \frac{\lambda}{\beta + \lambda}, \quad p_{31} = \frac{\beta}{\beta + \lambda}, \quad p_{35} = \frac{\lambda}{\beta + \lambda}, \quad p_{46} = \frac{\beta}{\alpha + \beta}, \quad p_{47} = \frac{\alpha}{\alpha + \beta}, \quad p_{88} = 1, p_{68} = 1 \]

\[ p_{70} = \frac{\lambda}{\alpha + \beta + \lambda}, \quad p_{79} = \frac{\beta}{\alpha + \beta + \lambda}, \quad p_{7.10} = \frac{\beta}{\alpha + \beta + \lambda}, \quad p_{7.11} = \frac{\beta}{\alpha + \beta}, \quad p_{86} = \frac{\beta}{\alpha + \beta}, \quad p_{87} = \frac{\alpha}{\alpha + \beta} \]

\[ p_{96} = \frac{\lambda}{\beta + \lambda}, \quad p_{97} = \frac{\beta}{\beta + \lambda}, \quad p_{10.1} = \frac{\alpha}{\alpha + \beta}, \quad p_{10.5} = \frac{\beta}{\alpha + \beta}, \quad p_{17.4} = \frac{\lambda}{\alpha + \beta + \lambda}, \quad p_{17.5} = \frac{\beta}{\alpha + \beta + \lambda}, \quad p_{17.6} = \frac{\alpha}{\alpha + \beta} \]

\[ p_{71.10} = \frac{\lambda}{\alpha + \beta + \lambda}, \quad p_{7.10.5} = \frac{\lambda}{\alpha + \beta + \lambda}, \quad p_{7.10.6} = \frac{\beta}{\alpha + \beta + \lambda}, \quad p_{97.6} = \frac{\beta}{\alpha + \beta + \lambda} \]

\[ \mu_0 = \frac{1}{\beta + \lambda}, \quad \mu_1 = \frac{1}{\alpha + \beta + \lambda}, \quad \mu_2 = \frac{1}{\beta + \lambda}, \quad \mu_3 = \frac{1}{\beta + \lambda}, \quad \mu_4 = \frac{1}{\alpha + \beta}, \quad \mu_5 = \frac{1}{\beta}, \quad \mu_6 = \frac{1}{\beta} \]

\[ \mu_7 = \frac{1}{\alpha + \beta + \lambda}, \quad \mu_8 = \frac{1}{\alpha + \beta + \lambda}, \quad \mu_9 = \frac{1}{\beta + \lambda}, \quad \mu_{10} = \frac{1}{\alpha + \beta + \lambda} \]

\[ \mu_1' = \frac{\alpha \beta \lambda + \beta \lambda}{\alpha \beta \lambda}, \quad \mu_3' = \frac{\alpha \beta \lambda}{\alpha \beta \lambda} \]

\[ \mu_7' = \frac{\alpha \beta (\alpha + \beta + \lambda) + \lambda \beta (\alpha + \beta + \lambda)}{\alpha \beta (\alpha + \beta) (\alpha + \beta + \lambda)}, \quad \mu_9' = \frac{\beta \alpha + \lambda (\alpha + \beta + \lambda)}{\alpha \beta (\beta + \lambda)} \]
\[ W_1^*(0) = \frac{(\alpha + \lambda)}{\alpha (\alpha + \beta + \lambda)} \quad \text{and} \quad W_7^*(0) = \frac{(\alpha_i + \lambda)}{\alpha_i (\alpha_i + \beta + \lambda)} \]

MTSF \( (T_0) = \frac{N_1}{D_1} \), Steady state availability \( (A_0) = \frac{N_2}{D_2} \),

Busy period of the server \( (B_0) = \frac{N_1}{D_2} \),

Expected number of visits by the server \( (N_0) = \frac{N_1}{D_2} \)

Where

\[ N_1 = (\lambda_i(\beta + \lambda + \beta_1) + \lambda \beta)(\alpha + \beta + \lambda_1) + (\beta + \beta_1 + \lambda)\alpha \beta_1 + \lambda(\beta_1 + \lambda_1 + \beta) \]

\[ D_1 = \lambda \left( (\lambda_i(\beta_1 + \lambda_1 + \beta + \alpha) + \alpha \beta_1) (\beta + \lambda + \beta_1) - \alpha(\beta_1(\beta_1 + \lambda_1 + \beta + \lambda) + \lambda_1) \right) \]

\[ V = \alpha + \beta + \lambda + \beta_1 \]

\[ V_1 = (\beta_1 + \lambda_1)(\beta_1 + \lambda) + \beta \beta_1 \]

\[ N_2 = \alpha \beta_1(\alpha + \beta)(\beta_1 + \lambda)(\beta + \lambda + \beta_1)(\alpha_1(\alpha \beta_1 + \lambda_1(V - \lambda))(\alpha_1 + \beta + \lambda) - \lambda \lambda_1(\lambda + 2 \beta_1)) + \alpha_1(\alpha + \beta + \lambda)((\beta \lambda + (\alpha \beta_1)(\alpha_1 + \beta_1)(\beta V + (\beta_1(V - \lambda)) - \alpha + \lambda \lambda_1)(\beta + \lambda + \beta_1)(\alpha_1 + \beta)(\beta V + V_1)))) \]

\[ D_2 = \alpha \beta_1(\beta_1 + \lambda)(\alpha + \beta)(\beta + \lambda + \beta_1)(\alpha_1(\alpha \beta_1 + \lambda_1(V - \lambda))(\alpha_1 + \beta + \lambda) - \lambda \lambda_1(\lambda + 2 \beta_1)) + (\alpha + \beta)(\alpha \beta_1(\alpha_1 + \beta_1 + \lambda) + \lambda \lambda_1(\beta \lambda + (\alpha \beta_1)(\alpha_1 + \beta_1)(\beta V + (\beta_1(V - \lambda))))(\beta V + ((\alpha \beta_1)(\alpha + \beta + \lambda_1)(\beta V + V_1)) - \beta \alpha_1 \lambda_2 \lambda^2 \alpha(\beta_1 + \lambda)(\alpha + \beta)) \]

\[ N_3 = \frac{(\beta \alpha_1^2 (\alpha + \beta)(\beta_1 + \lambda)(\alpha + \lambda_1)(V_1(\alpha_1 + \beta + \lambda) + \beta \lambda \lambda_1) + \alpha \lambda_1(\alpha_1 + \beta)(\alpha_1 + \lambda)(V_1 + \beta V)) \lambda}{\alpha_1} \]

\[ N_4 = \alpha \beta_1(\beta_1 + \lambda)(\lambda(\alpha_1 + \beta)(\alpha_1 + \beta_1)(\beta_2((\beta_1 + \lambda)(\alpha + \beta + \lambda_1) + \beta_1(\beta + \lambda + \beta_1)) + (\alpha + \beta)(\beta_1 + \lambda_1)(\alpha + \beta + \lambda_1)(\beta + \lambda + \beta_1 - \lambda^2(\alpha_1 + \beta_1 + \lambda_1(\alpha + \beta)(\beta_1(\beta + \lambda_1)(\alpha + \beta + \lambda_1)))) \]
### 6.10 TABLES AND GRAPHS

Table 6.1: MTSF vs. Normal Weather Rate ($\beta_1$)

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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<td>9.859135</td>
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<td>9.933243</td>
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<td>9.963189</td>
<td>15.97259</td>
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<td>9.989462</td>
<td>15.98348</td>
<td>14.38572</td>
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</table>

Fig. 6.2

MTSF vs. Normal Weather Rate ($\beta_1$)

- $\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$
- $\alpha=1.5$
- $\beta=0.05$
- $\lambda=0.3$
- $\lambda_1=0.4$
Table 6.2: Availability vs. Normal Weather Rate ($\beta_1$)

<table>
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<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
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<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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</thead>
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<td>0.941612</td>
<td>0.936057</td>
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<td>0.959919</td>
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<td>1.2</td>
<td>0.945063354</td>
<td>0.918024</td>
<td>0.94186</td>
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Fig. 6.3
Table 6.3: Profit vs. Normal Weather Rate ($\beta_1$)

<table>
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<th>Normal Weather Rate ($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300$</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
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<th>$\alpha_1=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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Fig. 6.4
Table 6.4: MTSF Difference between Model 3.1 and Model 6.1

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<th>Normal Weather Rate($\beta_1$)</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
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<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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MTSF Difference vs. Normal Weather Rate($\beta_1$)

Fig. 6.5
Table 6.5: MTSF Difference between Model 5.1 and Model 6.1

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<tr>
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<th>$\alpha$=2,$\alpha_1$=2.5,$\beta$=0.01, $\lambda$=0.5,$\lambda_1$=0.6</th>
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<th>$\beta$=0.05</th>
<th>$\lambda$=0.3</th>
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<td>2</td>
<td>0.588109652 0.969869 0.561228 0.721992 1.122119</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MTSF Difference vs. Normal Weather Rate($\beta_1$)

<table>
<thead>
<tr>
<th>Normal Weather Rate($\beta_1$)</th>
<th>MTSF Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.580527137</td>
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<tr>
<td>1.2</td>
<td>0.581923908</td>
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<tr>
<td>1.3</td>
<td>0.583115332</td>
</tr>
<tr>
<td>1.4</td>
<td>0.584140523</td>
</tr>
<tr>
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<td>0.58502965</td>
</tr>
<tr>
<td>1.6</td>
<td>0.585806286</td>
</tr>
<tr>
<td>1.7</td>
<td>0.586489075</td>
</tr>
<tr>
<td>1.8</td>
<td>0.587092917</td>
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<td>1.9</td>
<td>0.587629842</td>
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<tr>
<td>2</td>
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</tr>
</tbody>
</table>

Fig. 6.6
### Table 6.6: Profit Difference between Model 6.1 and Model 3.1

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300$</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha_1=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>26.39275</td>
<td>24.16135</td>
<td>112.0964</td>
<td>31.01574</td>
<td>24.72451</td>
<td>22.55397</td>
<td>17.48349</td>
</tr>
<tr>
<td>1.2</td>
<td>24.25483</td>
<td>21.97397</td>
<td>103.2459</td>
<td>28.49997</td>
<td>20.65276</td>
<td>22.55397</td>
<td>17.48349</td>
</tr>
<tr>
<td>1.3</td>
<td>22.36426</td>
<td>20.03706</td>
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<td>20.65276</td>
<td>22.55397</td>
<td>17.48349</td>
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<tr>
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<td>20.68142</td>
<td>18.31067</td>
<td>88.36735</td>
<td>24.37256</td>
<td>18.97472</td>
<td>16.15002</td>
<td>12.88306</td>
</tr>
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<td>16.76276</td>
<td>82.065</td>
<td>22.65977</td>
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<td>12.88306</td>
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<td>11.98563</td>
</tr>
<tr>
<td>1.8</td>
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<td>12.95311</td>
<td>66.53122</td>
<td>18.52018</td>
<td>13.86717</td>
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<td>11.98563</td>
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<tr>
<td>1.9</td>
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<td>11.90212</td>
<td>62.24305</td>
<td>17.39665</td>
<td>12.88306</td>
<td>11.98563</td>
<td>11.98563</td>
</tr>
</tbody>
</table>

### Fig. 6.7

#### Profit Difference vs. Normal Weather Rate ($\beta_1$)

- $K_0=5000$
- $K_1=350$
- $K_2=300$
- $\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$
- $\alpha=1.5$ (green line)
- $\alpha_1=2$ (blue line)
- $\beta=0.05$ (red line)
- $\lambda=0.3$ (orange line)
- $\lambda_1=0.4$ (purple line)
Table 6.7: Profit Difference between Model 5.1 and Model 6.1

<table>
<thead>
<tr>
<th>Normal Weather Rate($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300, \alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha=2$</th>
<th>$\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>43.45133</td>
<td>86.33412</td>
<td>11.99672</td>
<td>56.69034</td>
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<td>8.095439</td>
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<tr>
<td>1.2</td>
<td>46.81184</td>
<td>89.71414</td>
<td>15.42196</td>
<td>61.57716</td>
<td>-36.1827</td>
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<tr>
<td>1.3</td>
<td>49.47605</td>
<td>92.36334</td>
<td>18.14604</td>
<td>64.41432</td>
<td>-33.7027</td>
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<td>1.4</td>
<td>51.87909</td>
<td>94.75215</td>
<td>20.60417</td>
<td>66.95811</td>
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<td>103.8654</td>
<td>29.99538</td>
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<tr>
<td>2</td>
<td>62.46805</td>
<td>105.2721</td>
<td>31.44661</td>
<td>78.00996</td>
<td>-21.5834</td>
<td>42.19454</td>
</tr>
</tbody>
</table>

**Fig. 6.8**

**Profit Difference vs. Normal Weather Rate($\beta_1$)**

- $K_0=5000$
- $K_1=350$
- $K_2=300$
- $\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$
- $\alpha=1.5$
- $\alpha_1=2$
- $\beta=0.05$
- $\lambda=0.3$
- $\lambda_1=0.4$
6.11 CONCLUSION

For a particular case \( g(t) = ae^{\alpha t} \) and \( g_1(t) = e^{-\alpha t} \), the numerical results for MTSF, availability, profit of the system model are obtained for arbitrary values of various parameters and costs to show their trends with respect to normal weather rate \((\beta_1)\) as shown respectively in figures 6.2, 6.3 and 6.4. It is observed that MTSF keeps on increasing with the increase of normal weather rate \((\beta_1)\) and repair rate \((\alpha)\) of the main unit. However, it decreases with the increase of failure rates \((\lambda, \lambda_1)\) and abnormal weather rate \((\beta)\). Figures 6.3 and 6.4 indicate that availability and profit of the system go on increasing with increase of normal weather rate \((\beta_1)\) and repair rates \((\alpha, \alpha_1)\) of the units. But their values decline with the increase of abnormal weather \((\beta)\) and failure rates \((\lambda, \lambda_1)\). Thus study reveals that a system of non-identical units operating in different weather conditions can be made more reliable and profitable to use by increasing repair rate of the main unit.

6.12 COMPARATIVE STUDY

i. Comparison of MTSF

a. MTSF Comparison between Model 3.1 and Model 6.1

Figure 6.5 indicates that MTSF difference (model 3.1-model 6.1) decreases with increase of normal weather rate \((\beta_1)\) and failure rates \((\lambda, \lambda_1)\) of the units. But, it increases with the increase of abnormal weather rate \((\beta)\) and repair rate \((\alpha)\) of the main unit. Thus, the study reveals that the idea to allow operation of the system in abnormal weather is not useful in improving reliability of the system and system model 3.1 has more MTSF as compared to system model 6.1.

b. MTSF Comparison between Model 5.1 and Model 6.1

Figure 6.6 indicates that MTSF difference of the system models (model 5.1-model 6.1) keeps on increasing with the increase of normal weather rate \((\beta_1)\) while it declines with the increase of repair rate \((\alpha)\), abnormal weather rate \((\beta)\) and failure rates \((\lambda, \lambda_1)\) of the units. Thus, we can say that concept of priority to operation to main unit over duplicate unit is not helpful in improving the reliability of the system if system is allowed to work in abnormal weather.
ii. Comparison of Profit

a. Profit Comparison between Model 6.1 and Model 3.1

Figure 6.7 highlights that profit difference of the models (model 5.1-model 6.1) goes on decreasing with the increase of normal weather rate ($\beta_1$) and failure rates of the units. However, it increases with increase of repair rates of the units and abnormal weather rate ($\beta$). The effect of abnormal weather rate ($\beta$) on profit difference is much more as compared to other parameters. The results indicate that model 6.1 is profitable over model 3.1. As a whole, it is analyzed that the idea to operate system in abnormal weather with priority for operation to main unit over duplicate unit is helpful in making the system more profitable.

b. Profit Comparison between Model 5.1 and Model 6.1

The profit difference of the models (model 6.1-model 3.1) follows a upward trend with the increase of weather rates ($\beta$ and $\beta_1$), failure rates ($\lambda$ and $\lambda_1$) and repair rate ($\alpha_1$) of the duplicate unit while there is a decline in profit difference with the increase of repair rate($\alpha$) of the main unit. Thus, we conclude that priority for operation to main unit over duplicate unit would be helpful in making the system more profitable in either of the following ways

- When failure rate of the main unit is considerably small as compared to the duplicate unit
- When repair rate ($\alpha_1$) of the duplicate unit is less than that of main unit.