Chapter 2

Theory and Modeling Techniques

2.1 OPTICAL PROPERTIES OF MICROSTRUCTURED FIBERS

In this section, we intend to survey the linear and nonlinear properties of the holey cladding structures. The two types of microstructured fibers discussed in this thesis are photonic crystal fibers (PCF) and photonic quasi-crystal fibers (PQF). Here, the PCFs are reviewed before studying the PQF properties, since the former type turns out to be the base for the PQF design. The properties of both the solid and hollow core PCFs and PQFs are discussed in this section. The properties such as dispersion, confinement loss, mode area, bi-refringence and bending losses are surveyed in detail for both PCFs and PQFs. Further, the nonlinear propagation equation is derived for the microstructured fibers.

2.1.1 PHOTONIC CRYSTAL FIBER (PCF)

In 1991, Phillip Russel coined a name for his proposed fiber called holey fiber. This fiber has microscopic hollow channels running through its length, which form the cladding/core of the fiber. It has been proved that a photonic band gap exists whenever there is a significant index difference in the dielectric-air structure. Later on, successful efforts were made to trap the light within the bandgap, Russell (2007). In 1995, T. A. Birks demonstrated the existence of photonic band gap in two dimensional air-silica structure, Birks et al. (1995). In the subsequent year, J.C. Knight developed the first solid core PCF, Knight et al. (1996). This is the major breakthrough in the optical fiber era. In 1997, the first endlessly single mode PCF was developed to guide the light for a wide range of wavelengths, Birks et al. (1997). Till now, various types of fibers have been developed such as ultra large mode area, dispersion shifted, dispersion flattened, hollow core photonic bandgap, multi core, high birefringence, etc, Russell (2006). A large number of review papers and books have also been written on photonic crystal fibers, e.g. A. Bjarklev et al. (2003); Federica Poli et al. (2007); Zolla (2005). The applications of PCF encompass different domains like optical fiber communication, fiber lasers and amplifiers, fiber sensors, biomedical instrumentation, optical measurement etc. So there has been
an interest to explore its novel characteristics for future applications, Arismar Cerqueira (2010). Types of PCFs and their properties are discussed in the following sections.

**a. Solid core PCF**: Solid core PCFs generally have higher refractive index for the core compared to the cladding. It has more degrees of freedom in the fiber design, namely, the hole diameter, distance between the center of two adjacent holes, hole location and refractive index of solid medium which ultimately result in various novel properties of the fiber. Because of the presence of holes in the cladding, the index contrast between core and cladding increases which is unimaginable in conventional fiber. By choosing very high contrast i.e., by introducing larger diameter holes in the cladding, very high nonlinear fiber can be designed, Begum et al. (2009). On the other hand, by introducing an asymmetry in the cladding, high bi-refringence property can be obtained, Hansen et al. (2001). With dual core fiber, it is possible to enhance the dispersion to a larger extent, Yang, S. et al. (2006). By choosing appropriate diameter of the hole and its location, we can shift zero dispersion to any desired wavelength, Bhattacharya and Konar (2012). Fig. 2.1 shows the SEM image of a solid core PCF which exhibits endlessly single mode behavior over large spectral width. The key aspects which are normally studied in the case of solid core PCFs are chromatic dispersion, bending loss, effective area and confinement loss. In what follows, we discuss the large mode area (LMA) PCF and rare earth doped fiber which form the cornerstone in the high energy fiber amplifier applications.
b. Large mode area PCF: Large mode area fibers are extensively used to handle high power laser pulses. The first LMA PCF with a core diameter roughly fifty times the operating wavelength was reported, Knight et al. (1998). LMA fiber with single mode propagation is the requirement for the propagation of high power pulses in the fiber. The main limitation for the enhancement of the power in a standard single mode fiber is due to the occurrence of various nonlinear effects such as self phase modulation, Stimulated Raman scattering and stimulated Brillouin scattering. These can be overcome by having larger mode area for the fiber. The LMA is achieved in the conventional fiber by reducing NA. To increase the LMA to larger values, NA should be further reduced in conventional fiber. This may increase the bending loss of the fiber. The maximum diameter of LMA in conventional fiber is 30 µm and increasing beyond that value results in drastic increase in bending loss. This limitation is overcome by having periodic holey cladding in the fiber.

LMA PCF has a capability of scaling up of the mode area of the fiber with minimum bending loss. There are various designs in practice for achieving LMA in PCF. Normally PCFs have hexagonal holey structures in the cladding. This structure experiences high bending loss and the degradation of the beam quality in the event of increasing the diameter of the fiber. Thus the research in LMA PCF focuses on increasing the mode area by preserving other properties within a nominal limit.

c. Ytterbium doped PCF: The first doped PCF reported by Wadsworth et al. (2000) finds wide applications in fiber laser development. Then comes the cladding pumping in doped PCF which has increased the efficiency by two fold, Furusawa et al. (2001). The large mode area doped fiber proposed in 2003, had an output power of 80 W with 78% slope efficiency, Limpert et al. (2003b). This has provided necessary impetus towards developing high power lasers using Yb doped LMA PCFs. Latest reported work has a mode area of 5000 µm² with a high output power of 94 W, Boullet et al. (2008). Hence, the Yb-doped holey fibers are more likely to be the candidates for amplification of high power laser pulses and there are more properties to be explored.

d. Hollow core PCF: Cregan et al. (1999) report the very first hollow core PCF that finds wide range of applications. The phenomena by which the propagation occurs in the case of hollow core PCF is called bandgap guidance which is highly wavelength de-
Figure 2.2: Cross sectional SEM image of a hollow core photonic crystal fiber (manufactured by Blazephotonics Ltd.)

This would mean that only certain frequencies that lie within the bandgap are guided with very low loss. The frequencies outside the bandgap are rather leaky. In this type of fiber, the loss due to scattering and absorption is very low since the propagation is through air medium. Fig.2.2 shows the SEM image of the fabricated fiber from Blazephotonics Ltd. The applications of this fiber essentially include particle guidance, higher harmonic generation, high energy pulse compression, etc. Schmidt et al. (2012); Heckl et al. (2009, 2011). Photonic band gap fibers are classified as solid and hollow type and the choice between then depends upon the nature of the application.

2.1.2 PHOTONIC QUASI-CRYSTAL FIBER (PQF)

Photonic Quasi-Crystal: Understanding of the basics of photonic crystal is a prerequisite for studying about the photonic quasi-crystal. Photonic crystal is an artificial periodic structure which results in photonic band gap (PBG) which ensures that a particular frequency range of electromagnetic wave is prohibited inside the structure. The complete PBG occurs due to the interplay between Bragg scattering resonances of periodic dielectric array and the Mie resonances of individual dielectric scattering centers, John D. Joannopoulos et al. (2008). The presence of PBG helps control the flow of light i.e., inhibition of spontaneous emission, Yablonovitch (1987). This finds wide applications in radiation sources, photovoltaic devices, telecommunication devices, sensors, etc. Photonic Quasi-Crystal (PQC) is an aperiodic structure with long range quasi-periodic translational order. But it has rotational as well as mirror symmetry. The existence of
PBG in quasi crystal has been reported, Chan et al. (1998). The photonic band gap of PQC is highly isotropic compared to conventional photonic crystal, Zoorob et al. (2000); Gauthier and Mnaymneh (2005). It also exhibits bandgap for the materials which have lower index constrast, Zito et al. (2009). The various quasi periodic orders give rise to interesting waveguiding properties and confinement which, in turn, result in novel photonic applications, Cheng et al. (1999); Jin et al. (1999). Example of a 12 fold quasi-crystal is shown in Fig. 2.3. Applications like particle acceleration and focusing properties of PQC have attracted applications in integrated photonics technologies, Di Gennaro et al. (2008b,a). Over the years, photonic quasi-crystals have been playing an indispensable role for enhancing the light extraction in group III-Nitrides based light emitting diodes (LEDs), Shields, P. A. et al. (2009); Li, X. H. et al. (2011) and Organic LEDs, Koo, W. H. et al. (2012). In addition, they also help increase the light trapping capability in thin film silicon solar cell, Koo, W. H. et al. (2012).

**Solid core photonic quasi-crystal fiber** : The PQF is an extension of third dimension of the photonic quasi-crystal and it has been first reported by Kim, S. et al. (2007). The ultimate light propagation mechanism inside the solid core PQF is the total internal reflection. The motivation for developing this kind of fiber stemmed from the unusual properties of the quasi periodic structure, not observable in periodic structure. Kim, S. et al. (2007) believed that by using quasi periodic cladding in the fiber, the basic properties of the fiber like dispersion, confinement loss, mode area, bending loss etc. could be enhanced. It is reported that the six fold symmetry PQF, shown in Fig. 2.4, exhibits large cut off ratio of 0.525 for endlessly single mode operation against 0.45 for PCF. This hap-
pens to be a desired condition for broadband supercontinuum generation. Also, this fiber exhibits ultra-flat zero dispersion for a bandwidth of 200 nm in optical communication wavelength range. These properties have led to a major breakthrough in microstructure fiber technology. The second reported work in PQF is the large negative dispersion with dual core technology, Kim, K. and Kee, C.S. (2009). Due to the circular structure of the outer core, the dispersion of the fiber is enhanced. Only a few other studies have been reported in PQFs relating to V-parameters, impact of ring core in chromatic dispersion and broadband negative dispersion, Zhao et al. (2010); Kim et al. (2010); Li et al. (2010). Finally a very large dispersion in addition to the large mode area characteristics is reported, Sivabalan, S. and Raina, J.P. (2011).

b. Hollow core Photonic quasi-crystal fiber: The only difference between hollow core PQF and conventional bandgap fiber lies in the arrangement of holes in the cladding. 2D photonic quasi-crystal works by the principle of bandgap guidance. As we discussed above, with various quasi-crystal designs, we end up with different novel guiding properties of the structure. Similarly, many unusual properties may emerge in the hollow core PQF for various geometries of the cladding. The first hollow core PQF was proposed by Sun, X. and Hu, D.J.J. (2010). The Fig. 2.5 shows the 7 missing holes of hollow core six-fold symmetry fiber. The novel properties in this fiber are two-photonic bandgap with low loss guidance and absence of surface modes in the fiber. The hollow core PQF can be further explored with various types of quasi structure in the cladding.
2.1.3 NONLINEAR PROPAGATION EQUATIONS OF MICROSTRUCTURED FIBERS

Electromagnetic wave propagation in any medium is governed by Maxwell’s equations, Agrawal (2007). From Maxwell’s equation, considering the linear and nonlinear effects, the pulse envelope equation can be derived as the NLS equation.

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (2.1)
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.2)
\]

\[
\nabla \cdot \mathbf{B} = 0, \quad (2.3)
\]

\[
\nabla \cdot \mathbf{D} = \rho, \quad (2.4)
\]

where \( \mathbf{H} \) is the magnetic field vector, \( \mathbf{E} \) is the electric field vector, \( \mathbf{B} \) and \( \mathbf{D} \) are the magnetic and electric flux densities, respectively. The current density vector is \( \mathbf{J} \) and the charge density is \( \rho \). In the optical fiber, \( \mathbf{J} \), \( \rho \) and \( \mathbf{M} \) are zero. Therefore the electric and the magnetic flux density vectors are defined as

\[
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad (2.5)
\]

\[
\mathbf{B} = \mu_0 \mathbf{H}, \quad (2.6)
\]

where \( \mu_0 \) is the permeability of the free space, \( \varepsilon_0 \) is the permittivity of the free space and \( \mathbf{P} \) and \( \mathbf{M} \) are the induced electric and magnetic polarizations. By taking the curl of Eq. (2.2) and using Eqs. (2.1), (2.5) and (2.6), one can obtain

\[
\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, t) \quad \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} \quad \mu_0 \frac{\partial^2 \mathbf{P}(\mathbf{r}, t)}{\partial t^2} = 0, \quad (2.7)
\]
where \( c = 1/\sqrt{\mu_0\varepsilon_0} \) is the light velocity in the vacuum. This equation reduces to

\[
\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2},
\]

(2.8)

The induced polarization can be written as

\[
\mathbf{P}(\mathbf{r}, t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-t_1) \mathbf{E}(\mathbf{r}, t_1) dt_1 +
+ \varepsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^{(2)}(t-t_1, t-t_2) \mathbf{E}(\mathbf{r}, t_1) \mathbf{E}(\mathbf{r}, t_2) dt_1 dt_2
+ \varepsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) \mathbf{E}(\mathbf{r}, t_1) \mathbf{E}(\mathbf{r}, t_2) \mathbf{E}(\mathbf{r}, t_3) dt_1 dt_2 dt_3
\]

(2.9)

where \( \chi^{(j)} \) is the \( j \)th order susceptibility. If the medium response is instantaneous compared to the pulse duration \( (t_1/T_0 \ll 1 \) where \( T_0 \) is the pulse width and \( t_1 \) is the nonlinear response time of the medium) then Eq. (2.9) may be approximated by

\[
\mathbf{P}(\mathbf{r}, t) = \varepsilon_0 \left[ \chi^{(1)} \cdot \mathbf{E}(\mathbf{r}, t) + \chi^{(2)} : \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) + \chi^{(3)} : \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) + \ldots \right]
\]

(2.10)

As discussed earlier, the linear effects are due to \( \chi^{(1)} \). \( \chi^{(2)} \) is the second order susceptibility. In centro-symmetric molecules all the even order susceptibilities vanishes thus the contribution of second order susceptibility is negligible. In silica fiber the lowest dominant nonlinearity is due to \( \chi^{(3)} \) as they are made up of SiO\(_2\) molecules. Hence the linear and a nonlinear response can be written as

\[
\mathbf{P}_L(\mathbf{r}, t) = \varepsilon_0 \chi^{(1)} \mathbf{E}(\mathbf{r}, t)
\]

(2.11)

\[
\mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \chi^{(3)} \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t)
\]

(2.12)

and \( \mathbf{P}_L \) denotes the linear part while \( \mathbf{P}_{NL} \) denotes the nonlinear part of the induced polarization vector.

In order to derive basic propagation equation, it is necessary to make several simplifying assumptions before solving Eq. (2.8).

- \( \mathbf{P}_{NL} \) is treated as a small perturbation to \( \mathbf{P}_L \). This is justified because nonlinear effects are weak in silica fibers.
- The optical field is assumed to maintain its polarization along the fiber length so that a scalar approach is valid.

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• The optical field is assumed to be quasi-monochromatic, i.e., the pulse spectrum, centered at \( \omega_0 \), is assumed to have a spectral width \( \Delta \omega \) such that \( \Delta \omega / \omega_0 \ll 1 \).

According to the slowly-varying-envelope approximation it is useful to separate the rapidly varying part of the electric field by writing it in the form of

\[
E(r, t) = \frac{1}{2} \hat{x} \left[ U(r, t)e^{-i\omega_0 t} + U^*(r, t)e^{i\omega_0 t} \right],
\]

where \( \hat{x} \) is the polarization unit vector of the light assumed to be linearly polarized along the x axis, \( U(r, t) \) is a slowly-varying function of time (relative to the optical period) and \( U^* \) means the complex conjugate of \( U \). Eq. (2.14) is substituted into Eq. (2.11) and (2.12) and a similar form is used in the polarization vector as in Eq. (2.14):

\[
P_L(r, t) = \frac{1}{2} \hat{x} \left[ P_L(r, t)e^{-i\omega_0 t} + P_L^*(r, t)e^{i\omega_0 t} \right],
\]

\[
P_{NL}(r, t) = \frac{1}{2} \hat{x} \left[ P_{NL}(r, t)e^{-i\omega_0 t} + P_{NL}^*(r, t)e^{i\omega_0 t} \right],
\]

A Fourier-transformation is applied on Eq. (2.8) and Eq. (2.15) are substituted and (2.16) in that where we express \( P_L(r, t) \) and \( P_{NL}(r, t) \) with their relation to \( U(r, t) \). The obtained wave equation will have a form of

\[
\nabla^2 \tilde{U} + \epsilon(\omega)k_0^2 \tilde{U} = 0,
\]

where \( \tilde{U} \) denotes the Fourier-transform of \( U(r, t) \), \( k_0 = \omega_0/c \) and

\[
\epsilon(\omega) = 1 + \frac{\chi^{(1)}_{xx}(\omega)}{2} + \epsilon_{NL}
\]

whose nonlinear part \( \epsilon_{NL} \) can be written as

\[
\epsilon_{NL} = \frac{3}{4} \chi^{(3)}_{xxx} |U(r, t)|
\]

Eq. (2.17) is known as Helmholtz equation and can be solved by using the method of separation of variables

\[
\tilde{U}(r, \omega - \omega_0) = F(x, y)\tilde{U}(z, \omega - \omega_0)e^{ib_0z}
\]

where \( \tilde{U}(r, \omega - \omega_0) \) is a slowly varying function of \( z \), \( \beta_0 \) is the wave number and \( F(x, y) \) is a function which corresponds to the transverse electric modes in the \( (x, y) \) plane if the \( z \)-axis is identical to the propagation direction, Eq. (2.17) leads to the following two equations for \( F(x, y) \) and \( \tilde{U} \)

\[
\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + (\epsilon_0(\omega)k_0^2 - \bar{\beta}^2) F = 0,
\]

\[
\frac{\partial^2 \tilde{U}}{\partial z^2} + 2i\beta_0 \frac{\partial \tilde{U}}{\partial z} + (\bar{\beta}^2 - \beta_0^2) \tilde{U} = 0.
\]
In obtaining Eq. (2.22), the second derivative can be neglected since $\tilde{U}(z, \omega)$ is assumed to be a slowly varying function of $z$. The wave number $\tilde{\beta}$ is determined by solving the eigenvalue equation (2.21). The eigenvalue $\tilde{\beta}$ can be written in the form of

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta,$$  \hspace{1cm} (2.23)

where $\Delta\beta$ is a perturbation term and $\beta(\omega)$ is the frequency dependent mode propagation constant. Thus, from Eq. (2.22) we obtain

$$\frac{\partial \tilde{U}}{\partial z} - \frac{i}{2} \left\{ [\beta(\omega)^2 + 2\beta(\omega)\Delta\beta] \frac{1}{\tilde{\beta}_0} - \beta_0 \right\} \tilde{U} = 0 \hspace{1cm} (2.24)$$

It is useful to expand $\beta(\omega)$ in a Taylor-series around the carrier frequency $\omega_0$ as

$$\beta - \beta_0 = \frac{\partial \beta}{\partial \omega} \bigg|_{\omega_0} (\omega - \omega_0) + \frac{1}{2} \frac{\partial^2 \beta}{\partial \omega^2} \bigg|_{\omega_0} (\omega - \omega_0)^2 + \frac{1}{6} \frac{\partial^3 \beta}{\partial \omega^3} \bigg|_{\omega_0} (\omega - \omega_0)^3 + \cdots \hspace{1cm} (2.25)$$

Neglecting the terms that are higher than third order, take the inverse Fourier transform using

$$U(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}(z, \omega - \omega_0) \exp[-i(\omega - \omega_0)t]d\omega \hspace{1cm} (2.26)$$

The resulting equation for $U(z,t)$ becomes

$$\frac{\partial U}{\partial z} + \beta_1 \frac{\partial U}{\partial t} + \frac{i}{2} \beta_2 \frac{\partial^2 U}{\partial t^2} + \frac{i}{6} \beta_3 \frac{\partial^3 U}{\partial t^3} - i \Delta\beta U = 0. \hspace{1cm} (2.27)$$

The term with $\Delta\beta$ includes the effect of fiber loss and nonlinearity. It can be evaluated from Eq. (2.21) using a first-order perturbation theory:

$$\Delta\beta = -\frac{\alpha}{2} + i\gamma |U|^2 \hspace{1cm} (2.28)$$

where $\gamma$ is the nonlinear coefficient defined by

$$\gamma = \frac{n_2 \alpha_0}{c A_{eff}} \hspace{1cm} (2.29)$$

where $n_2$ is nonlinear refractive index and $A_{eff}$ is effective core area which is given in the form of

$$A_{eff} = \frac{\int \int_{-\infty}^{\infty} |F(x,y)|^2 dxdy}{\int \int_{-\infty}^{\infty} |F(x,y)|^4 dxdy} \hspace{1cm} (2.30)$$

where $F(x,y)$ is the transverse mode field distribution. Substituting Eq. (2.28) into Eq. (2.27) and making a variable transformation with $T = t - \beta_1 z$, one can obtain NLS equation as, Agrawal (2007)

$$\frac{\partial U(z,T)}{\partial z} = -\frac{\alpha}{2} U - i \frac{\beta_2}{2} \frac{\partial^2 U}{\partial T^2} - i \frac{\beta_3}{6} \frac{\partial^3 U}{\partial T^3} + i\gamma |U|^2 U. \hspace{1cm} (2.31)$$
Attenuation is described by the first term at the right-hand side in Eq. (3.2), GVD corresponds to the second term, TOD is the third term and nonlinearity, SPM is the fourth term with the intensity dependence. The Eq. 2.31 is used in the analysis of pulse stretching and pulse compression of high energy laser pulses in third and fifth chapters.

2.2 DESIGN ASPECTS OF PQF

The basic principle of operation of a solid core PQF is different from that of a hollow core PQF since both differ in their structural design. Here, we concentrate on the design of LMA solid as well as hollow core PQF. The important parameters which decide their properties are pitch (\( \Lambda \)) and the air hole diameter ‘d’. In the proposed design, we use only six fold and twelve fold PQ structure in the cladding. To some extent, the designing procedures that are followed for PCFs may be extended in order to achieve the desired properties in PQFs. Owing to the fact that the fiber parameters are highly interdependent, necessary optimizations are made while designing the fiber with two or more required properties.

2.2.1 INDEX GUIDING PQF

The index guiding PQF works on the principle of modified total internal reflection. The core is solid and has higher refractive index compared to that of the cladding. For designing the index guiding PQF, the desired parameters have to be fixed for a particular operating wavelength. Here, we consider large mode area, low confinement loss, single mode operation for larger bandwidth and high dispersion. The wavelength of operation is centered around 1.06\( \mu m \).

**Single mode operation** : First, we discuss the design procedure for obtaining single mode property in PQF. V-parameter is generally used to confirm the single mode propagation in the fiber. Mortensen et al. (2003) formulated the V parameter for PCF and same has been defined in Eq.2.32. Zhao et al. (2010) investigated and found that the V parameter which is used for PCF may be applicable to PQF too, with a condition that the fiber experiences a smooth change in refractive index in the cross-section .

\[
V_{\text{eff}}(\lambda) = \frac{2\pi \Lambda}{\lambda} [n_{\text{co}}^2(\lambda) - n_{\text{cl}}^2(\lambda)]^{1/2},
\]

where \( \Lambda \) is the pitch of the fiber. The primary condition for single mode operating
regime is found to be $V_{\text{eff}} < \pi$. The other design parameter which is to be considered for single mode operation is the $\frac{d}{\Lambda}$. The Fig. 2.6 shows the V parameter value for various $\frac{d}{\Lambda}$ for a range of wavelengths. From the figure, it is obvious that $\frac{d}{\Lambda}$ should be less than 0.525 for PQF to have single mode propagation. It should be noted here that the V-parameter for PCF (0.406 for the triangular lattice) is lesser than that of PQF. On account of the increase of $\frac{d}{\Lambda}$ ratio, it is possible to design higher index difference PQF structures which, in turn, could enhance the overall desired properties of the fiber.

![Figure 2.6: V parameter for six fold symmetric PQF for various $\frac{d}{\Lambda}$, Kim, S. et al. (2007)](image)

**Dispersion:** Another important property of the fiber is the dispersion. Eq. 2.33 shows the total dispersion of the PQF which is the sum of geometrical and material dispersions, Kim, S. et al. (2007).

$$D(\lambda) \approx D_g(\lambda) + D_m(\lambda) = \frac{\lambda}{c} \frac{d^2 n_g(\lambda)}{d\lambda^2} - \frac{\lambda}{c} \frac{d^2 n_m(\lambda)}{d\lambda^2},$$

(2.33)

where $n_g(\lambda)$ and $n_m(\lambda)$ are geometrical modal index and refractive index of the silica/doped material. Here, $D_m(\lambda)$ is approximated by the Sellmeier equation, whereas $D_g(\lambda)$ depends on the fiber structural parameters, namely, 'd' and $\Lambda$. By varying 'd' and $\frac{d}{\Lambda}$, it is possible to catch up with the required dispersion. To design higher dispersion fibers, two concentric cores are needed in the fiber. Further, tuning of $\frac{d}{\Lambda}$ is carried out to let that mode cross from inner core to outer core within the desired wavelength range so
that the dispersion in the fiber can be appreciably increased. The dual core PQF design is elaborately discussed in the next chapter.

**Low confinement loss** : To get the minimum confinement loss, the number of rings of holes should be more. However, this might result in increased complexity in the engineering of fabrication. Therefore, an optimum number of rings is to be determined with the permissible compromise on confinement loss. Another important parameter that decides the confinement loss is the \( \frac{d}{\Lambda} \). Here, higher the value of \( \frac{d}{\Lambda} \), lower is the confinement loss. But higher \( \frac{d}{\Lambda} \) ratio results in reduced effective area and hence increased nonlinearity. Therefore, higher values of \( \frac{d}{\Lambda} \) is generally applied only for highly nonlinear fibers. It is necessary that during the optimization of \( \frac{d}{\Lambda} \) value for a nominal confinement loss, sufficient care should be taken to ensure the single mode propagation requirement over larger spectral width.

**Effective Area** : The design steps to be followed to have required effective area \( (A_{eff}) \) for the six fold symmetric PQF are the same as that of the triangular PCF since the first ring in both the fibers is identical. Although, the design procedure for PCF can be adopted for the for six fold symmetry PQF, it may vary slightly for twelve fold symmetry structure. The effective area of the fiber is given as, Agrawal (2007)

\[
A_{\text{eff}} = \frac{\int dr_{\perp} I_n(r_{\perp})^2}{\int dr_{\perp} I_n^2(r_{\perp})},
\]

(2.34)

where \( I_n(r_{\perp}) \) is the intensity distribution in the cross section of the fiber. The intensity distribution in the fiber depends on the total magnitude of air holes present in the fiber structure. Normally for large mode area fiber, the pitch is kept at a higher value so that \( \frac{d}{\Lambda} \) is lesser than 0.525, ensuring single mode operation. From the \( \frac{d}{\Lambda} \) ratio, ’d’ value can be calculated and the same is to be optimized to meet the desired characteristics. Higher the value of air hole size, more will be the light confinement to the core and hence lesser will be the effective area. Further, the effective area of the fiber does decide the nonlinearity, spotsize, bending loss, splice loss and numerical aperture of the fiber, Mortensen (2002). Very large mode area (VLMA) fiber does not follow any of the above mentioned design criteria. Elaborate design procedure for VLMA is discussed in the fourth chapter.
In a bandgap PQF, the refractive index of the core is lesser than that of the cladding. The core can be formed by enlarging the centre hole or filling the centre hole with reverse doped solid material. Here, we discuss only the hollow or air filled core PQF design procedure. The core of the fiber can be formed by removing 7 cells or 19 cells according to the desired fiber diameter. However, the 19 cell structure does not turn out to be the correct choice for high dispersion requirement. The cladding can be of six, eight, ten or twelve fold symmetry. Since the fiber follows the bandgap principle, the first step in the design is to find the bandgap for the proposed fiber structure. Normally, finite element method or plane wave expansion method is used for determining bandgap in PQF.

While designing the hollow core PQF, the desired parameters have to be fixed for a particular operating wavelength. Here, we look for single mode operation, large mode area and high dispersion at 1.06 \( \mu \)m. The basic design parameters for a hollow core PQF are \( \frac{d}{\Lambda} \) ratio, pitch and diameter of the core. The range of \( \frac{d}{\Lambda} \) for PQF varies from 0.9 to 0.98. One of the advantages of PQF is that for lower \( \frac{d}{\Lambda} \) ratio, larger gap can occur for the hollow core fiber. The single mode propagation requirements which were discussed for the solid core PQF are not applicable in the case of hollow core PQF. Here, the \( \Lambda \) may be fixed depending upon the diameter of the fiber and \( \frac{d}{\Lambda} \) should be within the above mentioned range. The exact \( \frac{d}{\Lambda} \) ratio could be determined by trial and error so that the desired characteristics are met at the particular wavelength. Larger the hole size in the cladding, more will be the field confinement in the core, including the higher order modes (HOM). This can be avoided by having HOM distribution in the cladding by choosing an optimum diameter of the hole. This results in higher losses to HOMs and hence, the fiber essentially supports only the single mode propagation.

The diameter of the core is highly sensitive to the bandwidth of the bandgap. In PQF, the number of bandgaps are high compared to the PCFs. The dispersion in this fiber is naturally large in comparison with the conventional bandgap fibers. The reason behind this is that the slope of the effective index for various wavelengths is high in PQF compared to PCF, wherein the effective index variation is linear for the range of wavelengths. The reason for this behavior may be attributed to the aperiodic cladding structure. The detailed design for the hollow core large mode area PQF is discussed in chapter 5.
2.3 NUMERICAL TECHNIQUES

There are various numerical methods in vogue to find the solutions of Maxwell’s equations derived for the microstructure optical fiber (MOF). Since MOF exhibits higher index difference between core and cladding, the electromagnetic analysis eventually turns complex. The methods normally used for conventional fibers are less likely used in MOF as they provide only the approximate solutions.

Maxwell’s vector equation which is used in the analysis of MOF is written as, Russell (2006)

\[ (\nabla^2 + k^2 \varepsilon(r_T) + [\nabla \ln \varepsilon(r_T)] \wedge \nabla \wedge) H_T = \beta^2 H_T, \]  

(2.35)

where \( \varepsilon(r_T) \) is the dielectric constant \( r_T \) is position in the transverse plane, \( k = \omega / c \) is the wave vector

The scalar approximate of Eq. (2.35) is given as

\[ \nabla^2 H_T + [k^2 \varepsilon(r_T) - \beta^2] H_T = 0. \]  

(2.36)

For solving the eigen value Eq.(2.35) or (2.36), the various methods have been reported, Saitoh, K. and Koshiba, M. (2005). The choice of the numerical techniques would depend on the fiber geometry. Some techniques rely on their symmetry to increase the accuracy of the solution and a few other methods provide better solution for highly periodic structure but with less complexity. If the MOF has perfect circular holes, multipole method is the best and fastest method, White et al. (2002); Kuhlmyey et al. (2002). Hermite-Gaussian functions are used in the expanding fields that result in solution for Eq.(2.35), Dangui et al. (2006). The next accurate method is the finite difference time domain (FDTD) technique. But, it requires a large memory space for solving higher dimension fiber structure, Qiu (2001). A relatively less frequently used approach is the source-model technique which uses two sets of elementary sources to determine the field both inside and outside the silica regions, Hochman and Leviatan (2004). Finally, the fastest and versatile method for complex structure is the finite element method (FEM) which is discussed in detail in the next section, Saitoh and Koshiba (2002). The above discussed methods are used to analyze the field distributions in the PQF. In order to study the linear and nonlinear propagation of laser pulses in the fiber, split step Fourier method (SSFM) is used, Agrawal (2007). Here, we employ SSFM to analyze the stretching and compression phenomena in PQF. To determine the bandgap of hollow core holey fiber,
the plane wave expansion method or FEM is widely adopted. Finally to solve the fiber amplifier model equations, Runge-Kutta method is used.

2.3.1 FINITE ELEMENT METHOD

Finite element method is a general and proven technique for the engineering applications like structural analysis. It is an appropriate tool for solving the partial differential equations. When compared to other numerical methods, FEM provides relatively more accurate solutions for complex shapes and structures. It basically involves the following four steps: i) discretizing the solution region into finite number of subregions ii) deriving the governing equations for a typical equation iii) assembling all the elements in the solution region and iv) solving the system of equations.

![Figure 2.7: PCF is divided into substructure using triangular elements (meshing)](image)

**Figure 2.7:** PCF is divided into substructure using triangular elements (meshing)

**a. Finite element formulation for the MOFs** The FEM is a full-vector analysis suitable for modeling MOFs with large air-holes and high index variations, and also for accurately predicting their properties, Obayya et al. (2005). Perfectly matched layer is the boundary condition that is used for the analysis of MOFs. As a first step in FEM, the fiber crosssection is divided into small segments using any one of the FEM meshing elements. Fig 2.7 shows the subdivided cross section of PCF using triangular elements. In FEM, there are different types of meshing elements such as linear, quadratic and cubic ones. The elements can be chosen according to the shape of the structure and the required accuracy of the solution.
The Eq. 2.36 can also be written as, Obayya et al. (2005)

\[ \nabla \times (n^{-2} \nabla \times H) - k_0^2 = 0, \quad (2.37) \]

where \( n \) is the refractive index and \( k_0 \) is the free space wave number. By applying the Garlerkin’s or FEM procedure the Eq. 2.37 becomes, Kenji Kawano and Tsutomu Kitoh (2001)

\[
[M] \frac{d^2 h_t}{dz^2} - 2 j n_0 k_0 [M] \frac{dh_t}{dz} + ([K] - n_0^2 k_0^2 [M]) h_t = 0. \quad (2.38)
\]

By assuming the \( z \) directive terms in Eq. (2.38) to zero, we obtain the following eigenvalue equation:

\[
[K] h_t = n_0^2 k_0^2 [M] h_t, \quad (2.39)
\]

where \([K]\) and \([M]\) are the finite element matrices, \( \{ h \} \) is the discretized magnetic field vector consisting of the edge and nodal variables. The matrices \([K]\) and \([M]\) are sparse allowing an efficient resolution of the equation by means of high performance algebraic solvers for both real and complex problems. To save computational efforts, structural symmetries can be exploited in the numerical simulations. Also to increase the speed of simulation and to improve the accuracy of the field values, one-quarter of the fiber cross section can be used for the meshing, which is shown in Fig. 2.8.

The next important step is to choose the solver which can solve the set of linear equations. There are variety of solvers used in FEM, the choice of which would depend on the required accuracy and the available memory space. Recently a number of commercial packages have appeared that could solve Maxwell’s equations using a finite element combined with fast matrix eigenvalue solvers. For e.g. COMSOL multiphysics is a powerful FEM package which combines different domains in the simulation, COMSOL4.0 (2012).

In order to enclose the computational domain without affecting the numerical solution, anisotropic perfectly matched layers (PML) are placed before the outer boundary. This is discussed in detail in the next subsection.

b. Perfectly Matched Layer (PML) From the implementation point of view, it is more practical to describe the PML as an anisotropic material with losses. The PML can have arbitrary thickness and it is assumed to be made of an artificial absorbing material. The material has anisotropic permittivity and permeability that match the permittivity and permeability of the physical medium outside the PML in such a way that there are no
reflections, Saitoh and Koshiba (2002). The Fig. 2.9 represents schematic diagram of transverse cross section of a PCF surrounded by PML regions at the edges of the computational window, where $x$ and $y$ are the transverse directions and $z$ is the propagation direction. PML regions I and II are defined by the $x$ and $y$ directions, respectively. Regions III correspond to the four corners and ‘$W$’ is width of the PML.

Figure 2.9: Schematic diagram of PCF with PML boundary

The del operator $\nabla$ in Eq. (2.36) is defined as

$$\nabla = \hat{x}\alpha_x \frac{\partial}{\partial x} + \hat{y}\alpha_y \frac{\partial}{\partial y} + \hat{z}\alpha_z \frac{\partial}{\partial z} \quad (2.40)$$
Here, the $\alpha_x, \alpha_y, \alpha_z$ are related to the PML boundary conditions applied at the edge of the computational window. The $\alpha_s$ are determined using the following equation, Saitoh and Koshiba (2002)

$$\alpha = 1 - j \frac{3\lambda \rho_d^2}{4\pi nW^3} \ln \left( \frac{1}{R_t} \right), \quad (2.41)$$

where $\rho_d$ is the distance inside the PML measured from the interface between the PML and the edge of the computational window and $R_t$ is a theoretical reflection coefficient at the interface between the PML and the edge of the computational window.

The permittivity and permeability tensors in the PML region are expressed as

$$[\varepsilon]_{PML} = \varepsilon_0 n^2 [L], \quad [\mu]_{PML} = \mu_0 [L], \quad (2.42)$$

where $\varepsilon_0$ and $\mu_0$ are permittivity and permeability of free space and 'n' is the refractive index with

$$[L] = \begin{bmatrix} \frac{\alpha_x \alpha_y}{\alpha_z} & 0 & 0 \\ 0 & \frac{\alpha_x \alpha_z}{\alpha_y} & 0 \\ 0 & 0 & \frac{\alpha_x \alpha_y}{\alpha_z} \end{bmatrix} \quad (2.43)$$

The PML parameters $\alpha_x$ and $\alpha_y$ are given in Table I, and $\alpha_z$ will be unity where the wave propagation is assumed to be along the $z$ direction.

### 2.3.1.1 CALCULATION OF FIBER PARAMETERS USING FEM

In this thesis, we calculate dispersions, confinement loss, bending loss, mode area and overlap factor. These parameters can be calculated by the following equations.

**a. Dispersion :** The total dispersion $D$ of the PQF which includes waveguide and material dispersion ($D_m$) can be calculated from the following equation, Saitoh and Koshiba (2005)

$$D = \frac{\lambda}{c} \frac{d^2 n_{eff}}{d\lambda^2} + D_m. \quad (2.44)$$

<table>
<thead>
<tr>
<th>PML parameter</th>
<th>Region I</th>
<th>Region II</th>
<th>Region III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_x$</td>
<td>$\alpha$</td>
<td>1</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>1</td>
<td>$\alpha$</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters $\alpha_x$ and $\alpha_y$
Here, the $n_{\text{eff}}$ is determined for the specific fiber using FEM software. From the calculation of dispersion, $D$, the dispersion parameter $\beta_2$ is calculated using the following equation.

$$D = -\frac{2\pi c}{\lambda^2} \beta_2.$$  \hspace{1cm} (2.45)

Here, the material dispersion ($D_m$) of silica can be included in the calculation by varying the material’s refractive index to the corresponding wavelength. This can be calculated using the Sellmeier approximation as given in Appendix B.

**b. Confinement loss**: Another important characteristic of PQF is the confinement loss, which decides the amount of field confinement in the core. It is calculated using the following equation,

$$L_c = 8.686 \times k_0 n_{\text{eff}}.$$  \hspace{1cm} (2.46)

**c. Bending loss**: The bending loss of PQFs are calculated with the bending refractive index which is the effective refractive index due to bending of the fiber.

$$n_{\text{bend}} = n(x, y) \exp\left(\frac{x}{R_{\text{bend}}}\right),$$  \hspace{1cm} (2.47)

$R_{\text{bend}}$ denotes the effective bend radius.

**d. Nonlinear coefficient**: Nonlinear coefficient is defined by the equation,

$$\gamma = \frac{n_2 \alpha_0}{c A_{\text{eff}}},$$  \hspace{1cm} (2.48)

where $n_2$ is nonlinear refractive index and $A_{\text{eff}}$ is effective core area which is presented in what follows.

**e. Effective mode area**: \hspace{1cm}  

$$A_{\text{eff}} = \left(\int \int_S |F(x, y)|^2 dxdy\right)^2,$$  \hspace{1cm} (2.49)

where $F(x, y)$ is the transverse mode field distribution and the term $S$ represent the whole fiber cross section.

**f. Gain overlap factor**: \hspace{1cm}  

$$\Gamma_x = \frac{\int_0^{r_d} \int_0^{2\pi} |E_x|^2 dxdy}{\int_0^{r_d} \int_0^{2\pi} |E_x|^2 dxdy},$$  \hspace{1cm} (2.50)

where $E(x)$ is the transverse electric field distribution and $r_d$ is the radius of the doped region.
The bandgap represents the range of wavelengths that are forbidden for propagation through the PQF structure. It is normally plotted between the wavenumber and the frequency of the light. The Bandgap can be determined in two ways by using the finite element method. In the first one, the unit cell of the PQF is used to analyze the bandgap for the desired fiber. The unit cell is the one which repeats to form the complete fiber. Here, the periodic boundary conditions are used since the unit cell is placed periodically in the structure. Then the eigenmodes that support in the unit cell takes the form as
\[
\psi = e^{ikx}u(x),
\]
where \(\psi\) is the scalar field intensity, \(k\) is the Bloch-quasimomentum vector and \(u(x)\) is the value of the eigenfunctions at spatial position \(x\). The resultant change in the gradient vector operator:
\[
\nabla \rightarrow \nabla + ik,
\]
This results in a eigensystem of the form:
\[
(S + kP + k^2 T)\psi = \lambda T\psi,
\]
where \(S\) and \(T\) are the Dirichlet and metric matrices. \(P\) is a vector of matrices assembled from elemental matrices that are defined in terms of the basis function \(\alpha_i\).
\[
S = \int_\Omega \nabla \alpha_m . \nabla \alpha_n d\Omega
\]
(2.54)
\[
T = \int_\Omega \alpha_m . \alpha_n d\Omega
\]
(2.55)
\[
P = \int_\Omega (\alpha_i \nabla \alpha_j - \alpha_j \nabla \alpha_i) d\Omega
\]
(2.56)
The elemental matrices are assembled to form generalized eigenvalue problem which is solved as a function of \(k\). For each value of \(k\), the solver computes a set of \(i\) eigenvalues. These eigenvalues are plotted as a function of \(k\) to give a band diagram. The typical plot which the FEM package provides is shown in Fig. 2.10. From this plot, the number of frequencies allowed in a particular direction can be determined. The second method of calculating the band diagram is by determining the effective index of the fundamental modes supported by the fiber. Then the plot is drawn between effective refractive index and the normalized wavelength. This could provide the information about the bandgaps that exists for a particular PQF. This method is followed in this thesis to determine the bandgaps of the six/twelve fold symmetry hollow core PQF.
2.3.2 SPLIT STEP FOURIER METHOD

The NLS equation as given in Eq. (2.57), cannot be solved analytically for the general case of arbitrarily shaped pulses launched into the fiber. However, powerful numerical procedures have been developed over the years to solve it. There are two main types of numerical schemes generally applied to the NLS equation, namely, the finite difference method, such as the Crank-Nicolson and the function approximation methods such as SSFM. Among them, the SSFM has proved to be the most robust technique. The SSFM is the preferred choice of technique for solving the NLS equation due to its easy implementation and speed compared to other methods, notably, time-domain finite-difference methods, Agrawal (2007).

\[
\frac{\partial A}{\partial z} + \alpha A + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{i\beta_3}{6} \frac{\partial^3 A}{\partial t^3} = i\gamma \left( |A|^2 A + \frac{i}{\alpha_0} \frac{\partial(|A|^2 A)}{\partial T} - T_R A \frac{\partial |A|^2}{\partial T} \right), \quad (2.57)
\]

In SSFM, it is assumed that the dispersion and nonlinearity act independently over a short distance of the fiber. So, the mathematical terms due to dispersion and nonlinearity are separated and decoupled in the NLS equation. The simplified version of Eq. 2.57 is presented below, Agrawal (2007)

\[
\frac{\partial A}{\partial z} = (\hat{D} + \hat{N}) A, \quad (2.58)
\]

where \( \hat{D} \) is the differential operator that represents the dispersion and attenuation in the medium and \( \hat{N} \) includes all the nonlinear effects that arise in the medium. The
operators are written as

\[ \hat{D} = -\frac{\alpha}{2} A - \frac{i \beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{i \beta_3}{6} \frac{\partial^3 A}{\partial t^3} \]  

(2.59)

and

\[ \hat{N} = i \gamma \left( |A|^2 A + \frac{i}{\omega_0} \frac{\partial (|A|^2 A)}{\partial T} - T_R A \frac{\partial |A|^2}{\partial T} \right), \]  

(2.60)

where \( A(z, T) \) is the complex field envelope at step \( z \) and time \( T \).

The numerical integration is based on splitting each step \( \Delta z \), so that only the nonlinear terms in the equations are taken as the first substep, and the GVD and loss terms are dealt in the second substep. At the latter stage, the corresponding linear equation(s) are solved by means of the Fourier transform. More specifically, propagation from \( z \) to \( z + h \) is carried out in two steps as shown in Fig. 2.11. In the first step, the nonlinearity acts alone, and \( \hat{D} = 0 \) in Eq. (2.57). In the second step, dispersion acts alone, and \( \hat{N} = 0 \) in Eq. (2.57). Mathematically, the field envelope at \( z + h \) is written as, Agrawal (2007),

\[ A(z+h, T) \approx \exp \left( h\hat{D} \right) \exp \left( h\hat{N} \right) A(z, T) \]  

(2.61)

The exponential operator \( \exp(h\hat{D}) \) can be evaluated in the Fourier domain using the prescription

\[ \exp \left( h\hat{D} \right) B(z, T) = F_T^{-1} \exp \left( h\hat{D} (-i\omega) \right) F_T B(z, T), \]

where \( F_T \) denotes the Fourier-transform operation, \( \hat{D}(-i\omega) \) is obtained from Eq. (2.59) by replacing the operator \( \partial / \partial T \) by \(-i\omega\), and \( \omega \) is the frequency in the Fourier domain. As \( \hat{D}(i\omega) \) is just a number in the Fourier space, the evaluation of Eq. (2.62) is straightforward. The linear operator \( \hat{D} \) is most efficiently solved in the spectral domain, while the nonlinear operator \( \hat{N} \) is more favorably solved in the time domain. Assuming a discrete signal description in the time and frequency domain, the Fast Fourier Transform (FFT) is used for the conversion between the two. The use of the FFT algorithm makes numerical evaluation of Eq. (2.62) relatively fast. It is for this reason that the SSFM is faster by two orders of magnitude compared with most finite-difference schemes. The accuracy of the SSFM can be improved by adopting a different procedure to propagate the optical pulse over one segment from \( z \) to \( z + h \). In this procedure Eq. (2.61) is replaced by

\[ A(z+h, T) \approx \exp \left( \frac{h}{2} \hat{D} \right) \exp \left( \int_z^{z+h} \hat{N}(z')dz' \right) \exp \left( \frac{h}{2} \hat{D} \right) A(z, T), \]  

(2.62)
The efficiency of the split-step method depends on both the time (or frequency) domain resolution and on the distribution of step sizes along the fiber. If $\Delta z$, the so-called split-step size, becomes too large the condition for separable calculation of $\hat{D}$ and $\hat{N}$ breaks, and the algorithm delivers wrong results. So, careful determination of the optimum split-step size is important in order to use minimal computational effort for a given accuracy. Typically, $\Delta z$ is adaptively adjusted to be very small. The step size $\Delta z$ should be a small fraction, thereby requiring $> 1000$ steps/(shortest linear or nonlinear length) along the fiber length. As the speed of the FFT is proportional to $N_t \log_2 N_t$, where $N_t$ is the number of signal samples in the time or frequency domain, careful determination of the simulation band width and the time window is important for minimizing computational effort for a given specific accuracy constraints. For temporal and spectral computation windows spanning $T_{span}$ and $F_{span}$, respectively, the sampling theorem imposes the condition $T_{span} F_{span} = N_p$, where $N_p$ is the number of discretization points. For a simulation of femtosecond range pulse parameters considered here, the broadest stretching or compression typically demands that $F_{span} \sim 1000$ THz and $T_{span} \sim 20$ ps so that a very large number of points $Np \geq 2^{15}$ are required.

2.3.3 RUNGE-KUTTA METHOD

It is a method for numerically integrating ordinary differential equations by using a trial step at the midpoint of an interval to cancel out lower-order error terms. Here, we use RK
Figure 2.12: Flow chart of RK Method

fourth order method to solve the governing Eqs. 2.63 and 2.64, relating to the pump and signal power in the Yb-doped fiber amplifier, Paschotta, R. et al. (1997)-Hilaire, S. et al. (2006). Figure 2.12 shows the general flow chart of RK method which is used to solve the amplifier equations. The MATLAB code for the algorithm is given in Annexure-A.

\[
\frac{dP_p}{dz} = \Gamma_p [\sigma_e(\lambda_p)N_2(z) - \sigma_a(\lambda_p)N_1(z)]N_{tot}P_p(z), \quad (2.63)
\]

\[
\frac{dP_s}{dz} = \Gamma_s [\sigma_e(\lambda_s)N_2(z) - \sigma_a(\lambda_s)N_1(z)]N_{tot}P_s(z). \quad (2.64)
\]

2.4 SUMMARY

In this chapter, the basic linear and nonlinear characteristics of PCF and PQFs have been discussed. PQF properties are explained from the basics of two dimensional photonic quasi-crystal and well established photonic crystal fibers. Here, both solid and hollow core PQF design procedures have been discussed elaborately for the desired parameters such as single mode operation, large mode area, high dispersion, low confinement loss and minimum bending loss. As a next step, the various numerical techniques used in
the modeling of microstructure fibers are surveyed and the merits and de-merits of each technique have been outlined. Finally, the methods used in this thesis such as FEM, RKM and SSFM are discussed in detail. Under FEM, the procedure to determine the various parameters such as confinement loss, effective mode area, overlap factor, bending loss and dispersion of the microstructure fibers are delineated. Further, the algorithm for solving the NLS equation using SSFM has also been elaborated. Finally the RK method steps are discussed for solving the amplifier equations.