Conclusion

The present thesis covers several novel phenomena in one and two dimensions associated with the dusty plasma medium in both weak and strong coupling regimes. A Generalized Hydrodynamic (GHD) fluid model coupled with Poisson’s equation has been adopted for the purpose to describe the dusty plasma medium. The GHD model takes care of the visco-elastic properties of the medium in the strong coupling regime. Both analytical and numerical simulation studies have been done on the collective behavior of the dusty plasma medium in the two regimes of weak and strong coupling.

The summary of the interesting results obtained in this thesis has been presented in section 8.1. The future scope of the research studies carried out in this thesis have been listed point wise in section 8.2.

8.1 Main results of the Thesis

8.1.1 1-D studies

The 1-D coupled set of GHD and Poisson’s equation was solved by using the flux corrected scheme and the Poisson solver respectively. The numerical code was benchmarked by reproducing the linear dispersion relation of the Dust Acoustic Wave (DAW) numerically.

We have made certain interesting observations pertaining to the characteristic nonlinear solutions for the coupled GHD and Poisson set of equations in one dimensions for the dusty plasma medium. These observations are summarized as
Observation of stable singular cusp structure in weakly coupled dusty plasma medium

The equations for the dusty plasma medium in the weak coupling limit reduce to the usual fluid equation for the charged dust fluid which is influenced by the scalar potential field through the Poisson’s equation. It can be shown by using the reductive perturbative scheme, that in the limit of weak nonlinearity, the equations can be cast into a well known KdV form. Our simulations reproduce the characteristics properties of the KdV soliton solutions in the weak amplitude limit, thereby benchmarking the code in the nonlinear regime too.

At higher amplitudes the exact analytical form of solutions can be obtained by constructing Sagdeev potential. At smaller amplitude the solutions of the Sagdeev potential are identical to KdV solutions. However, with increasing amplitude the form starts deviating from the KdV solutions. The Sagdeev potential provides an upper limit beyond which localized solutions propagating with constant velocity do not exist. This is due to the wave breaking at the limiting amplitude. At the limiting amplitude the structure has a singular cusp form for the density and velocity fields.

We observe that by choosing an initial condition with amplitude higher than that provided by the Sagdeev limit, the evolution invariable deforms into two or more localized structures. The higher amplitude structures are found to be invariably the singular cusp solutions of the Sagdeev potential at the wave breaking limit. These solutions are observed to be fairly robust and evolve stably during the entire course of simulation (several hundreds of dust acoustic period).

Our numerical observations have experimental relevance as such singular cusp structures have been observed in dusty plasma experiments by Teng et al. [1].

The new paradigm of Hunter Saxton (HS) equation for the Strongly coupled dusty plasma

The application of reductive perturbative approach to study the weakly nonlinear response of the strongly coupled dust fluid leads to an entirely new
paradigm. The equation in this case do not reduce to the well known KdV form. They have an altogether different form from which it is apparent that periodic and all localized smooth solutions are not permissible in this particular case. In fact we have shown that in the limit when the elastic effects due to correlation dominate over the Boltzmann screening and dispersive effects, the equation takes the form of the Hunter Saxton (HS) equation. The HS equation is known to permit shock solutions which are either conservative or dissipative. This, therefore, offers an entirely new paradigm for the strongly coupled dusty plasma medium. The HS equations have been explored previously in context of the study of directors field in liquid crystals and we have first time predicted the applicability of such an equation in the context of dusty plasma medium.

The numerical simulations in 1-D with the complete set of coupled GHD and Poisson system with localized initial conditions are found to evolve and exhibit shock formation.

8.1.2 2-D studies

After carrying out a comprehensive investigation in 1-D, we have chosen to investigate the characteristics features exhibited by the dusty plasma medium in 2-D. We have chosen for this purpose to study the well known shear flow driven fluid Kelvin Helmholtz (KH) instability as well as turbulent characteristics for this medium. The important findings from our studies are as follows.

- **Compressible and dispersive effects on the nature of Kelvin-Helmholtz instability in weakly coupled dusty plasma medium**

  The growth rate as a function of wavenumber have been obtained by numerically finding the eigen value of the system for specific shear flow profiles. We have chosen to consider the case of a tangent hyperbolic profile of the shear flow in our studies. We observe that both compressibility and dispersive effects present in the case of dusty plasma medium reduce the growth rate of the KH mode.
We have also carried out perturbative analytical calculation to obtain the correction in growth rate due to the presence of compressibility. It is clear from such a calculation also that the presence of compressibility reduces the KH growth rate. The reason for this being that the free energy in the shear flow now finds an additional channel of release by the excitation of compressible modes in the system.

The nonlinear 2-D simulations with an initial shear flow profile shows the KH excitations. The linear growth rate obtained numerically from simulations are found to match with the eigen value estimates. In the nonlinear regime of the simulations the vortices get formed at the shear layer. The usual process of vortex coalescence is also observed in this case. The only difference in this case is that the coalesced vortices have an elliptical instead of the circular forms. The rotating elliptical structures in this case produce reversible oscillations in the perturbed kinetic energy evolution in the nonlinear regime. At each coalescence, however, an irreversible jump in perturbed kinetic energy is observed.

- **Existence of transverse shear waves in strongly coupled dusty plasma medium**

The strongly coupled dusty plasma medium supports transverse shear wave in addition to compressible DAW's. However, the transverse nature of the mode requires two or higher dimension for its existence. We have simulated this mode numerically and have been able to verify the linear dispersion characteristics of this particular mode. The phase velocity of the propagating waves are observed to match with the analytical expression, viz.,

\[ v_{ph} \sim \sqrt{\eta/\tau_m}. \]

- **Shear flow instability in strongly coupled dusty plasma medium**

The linear stability analysis of nonlocal KH mode for the 2-D sheared flow (the specific case of Tangent Hyperbolic sheared flow profile was studied) in the strongly coupled medium shows that the growth rate lies between the KH growth rates for the viscous and the inviscid hydrodynamic fluid. As the values of \( \tau_m \), the relaxation parameter is increased (i.e. with increase in coupling parameter \( \Gamma \)), we observe that the growth rate increases but
continues to reside between the purely viscous and the inviscid hydrodynamic limits.

In addition to nonlocal KH mode instability, the shear flow in strongly coupled medium also supports a local instability. This is quite unlike the case of weakly coupled dusty plasma which does not support any growing local mode. It is also found that while $v'_0$ is responsible for the existence of KH instability, the $v''_0$ is prime cause for local sheared flow instability. Here, $'$ indicates spatial derivative.

• Phenomena of Recurrence in strongly coupled shear flow instability

The nonlinear regime of the sheared flow instability for the strongly coupled medium was studied by simulations. It is observed that in the nonlinear state the system exhibits an interesting interplay between vortex coalescence and its stretching followed by the KH instability repeatedly. This recurrence phenomena occurs for a couple of cycles and eventually it is followed by certain small scale vortex formation which are well separated from the original shear scale. The formation of small scale structures in this particular case is in distinct contrast to the inverse cascade behavior exhibited by ordinary fluids in 2-D.

• 2-D turbulence in Visco-Elastic fluids: A scale separation in the power spectra

The interplay between vortex coalescence and small scale formation observed in the context of KH instability suggests that the behavior of spectral cascade for the 2-D strongly coupled medium is distinct from the ordinary fluids. To study the phenomena of spectral cascade behavior we have studied the evolution of a given spectrum of random fluctuation. The evolved spectrum exhibits a power law, with a break in spectrum. The location of spectral break, however, is observed to evolve with time. Furthermore, the break is prominent when the initial spectral excitations are at short scales. These features can be understood by realizing the existence of two distinct time scale regime in the model. For time scales longer than the relaxation time $\tau_m$, the GHD dynamics is similar to that of the ordinary fluid, exhibiting phenomena of inverse cascade. However, for shorter time scales the elastic...
nature of the fluid plays a predominant role. In this short time scale elastic limit the system does not respect the conservation of two invariants and direct cascade occurs. This is also apparent from the existence of local instabilities for the sheared system in the case of strong coupling. The interplay of these two dynamical features of the spectral cascade results in a spectral break.

8.2 Future scope of the work

The thesis results provide a direction towards lot of possibilities which could be explored in weakly as well strongly coupled plasmas. The formation of cusp structures, recurrence of vortex formation in sheared flow, the nature of turbulence are few of the interesting results predicted in strongly coupled dusty plasmas which needs to be verified by different possible experimental and numerical means. Here, we will present a point wise discussion over further possibilities of explorations in this particular area.

- The Generalized Hydrodynamic model is a phenomenological model which successfully explains many collective features of strongly coupled dusty plasmas. The model is a variant of Maxwell’s model extensively used to explain visco-elastic fluids. In the present model, we have considered only one constant relaxation time scale. In actual dusty plasma medium or in other visco-elastic materials, multiple relaxation time scales are possible. Similarly, the viscosity which we have considered as a constant parameter could have a functional form depending on space and time. Inclusion of these properties will help us to look at more realistic view of dusty plasma as visco-elastic fluids.

- Present thesis assumes electrons and ions as inertialess species while studying the collective dynamics of dusty plasma medium, as the time scales and length scales for collective dynamics in these different species are well separated. But certain phenomena like dust void formation, dust rotation (observed in dusty plasma experiments) require the dynamics of ions to be included along with dust dynamics. For example, ion streaming is important for the formation of dust voids and the phenomenon of dust rotation occurs due to magnetized ions. These magnetized ions rotate fast and because
of momentum transfer, dust also starts rotation [147,148]. To understand such experimental observations in the strong coupling limit, one would need to incorporate ion dynamics along with the dust dynamic in the GHD and Poisson coupling.

- Throughout the thesis, the studies of dusty plasma medium assume the dust particles carrying constant charge. But in a realistic situation, the charge on dust species varies in time depending on a charging mechanism [57]. Inclusion of the mechanism of charge fluctuation may be interesting.

- In present thesis, we have employed GHD fluid model to study dusty plasma medium assuming the medium as visco-elastic fluid. Another way to study such medium is the particle approach (i.e. Molecular Dynamics). With the present computational facilities, realistic particle simulations are possible for dusty plasmas having typical density of the order $10^6 - 10^8/m^2$ in laboratory experiments. In such studies, the inter particle potential between dust particles has been taken of Yukawa potential form. The large scale simulations could be carried out to make a comparative study of the two approaches. Also, the particle studies will provide an opportunity to include the finite size effects of dust particles on the collective dust dynamics.

- Recently, experiments were performed to study turbulent flows in visco-elastic fluids [101]. Similar experimental studies could help us to understand the similarities and differences of dusty plasmas with visco-elastic fluids.

- The dusty plasmas were found to show exceptional similarities with polymer liquids in various experiments [149,150]. The explorations to study such similarities opens up an entirely new domain of research.