Kelvin-Helmholtz instability in strongly coupled dusty plasmas: 2-D studies

In the previous Chapter 5, we have provided a detailed description about the linear and nonlinear aspects of KH instability in weakly coupled dusty plasma medium. But, as we have mentioned earlier that due to high charge of the dust species, it can be easily found in the strongly coupled regime. Here, in this Chapter, we have employed the generalized hydrodynamic description of dusty plasmas to include the the effect of strong coupling on the sheared flows in such systems. In the presence of strong coupling, we report the existence of a local instability along with KH instability in strongly coupled dusty plasma medium. We have also observed the phenomenon of recurrence of instability due to competition of strong coupling and compressibility of medium. Apart from this, we have provided a criteria for growth of KH instability in such system.

6.1 Introduction

Plasma can often be found in strongly coupled regime, which is defined by the condition of having the inter particle potential energy being comparable or exceeding the thermal kinetic energy of the particles. This condition can be expressed in terms of coupling parameter $\Gamma$ for strongly coupled plasmas [28]. The strongly coupled plasma medium have invoked considerable research interest due to the novel features associated with this state [9, 11] and its occurrence in a variety of realistic situations. The normal electron - ion plasma can be in a strong coupling regime
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when cooled by laser radiation in electrostatic traps and cyclotrons [136, 137]. They tend to crystallize and form "Coulomb crystals as well as liquids" [138]. Furthermore, the crystallization of electrons at 2-D Helium and Hydrogen surfaces are other examples of such a state [139]. While these experiments require fairly complicated apparatus, it is rather easy to produce a dusty plasma medium in strongly coupled state in laboratory. This is so as the inter particle separation is small and each of the dust particles can acquire a large number of electrons to be in a highly charged state, e.g. \((10^4 - 10^6)\) electronic charges. A number of microgravity and gravity experiments show dust crystallizations in the strongly coupled limit [26, 27, 81].

Normally, the coupling parameter \(\Gamma\) remains well below unity for most of high temperature plasmas. For instance, for \(Z \sim 1\), a plasma at a temperature of \(10^6K\) can even at a high density of the order of \(10^{26} cm^{-3}\) would be in a weakly coupled regime with the coupling parameter below unity. This is why only either high density cold plasmas or dusty plasmas with very high charge on dust particles can fulfill the condition of \(\Gamma \geq 1\) [28]. As dusty plasmas can be found in the strong coupling regime even at low densities, it provides one with a unique opportunity to investigate the behavior of a strongly coupled plasma medium over the phase domains right from gaseous to liquid to that of solid.

The weakly coupled dusty plasmas \((\Gamma < 1)\) can be easily treated like a fluid. However, in the very strong coupling limit where crystallization occurs, the fluid model is clearly not an adequate description. There is, however, an intermediate regime of the coulomb coupling parameter in which the the dusty plasma exhibits properties which are intermediate to fluid and solid like behavior. This is a rather interesting phase, as in this case the medium behaves like a visco-elastic system [99]. The particles are not rigidly fixed at any locations like in a solid medium. They can wander around, but retain a certain memory of their dynamics. The visco-elastic medium is often depicted in terms of a Generalized Hydrodynamic (GHD) model [59]. In the context of dusty plasma it was adopted by Kaw et al. and was found to provide a good description for the transverse shear waves supported by the strongly coupled dusty plasma medium [23, 77].

The dusty plasma medium is often observed with significant amount of sheared flows, for instance in cometary tails, protoplanetary disks, etc. In some of these cases the medium can be in a strongly coupled state. It is well known that shear
flows are susceptible to the well known fluid Kelvin - Helmholtz (KH) instability [126, 127]. We had in our recent studies shown the effect of compressibility and dispersion on the KH mode for the dusty plasma medium in the weak coupling regime [97]. In this Chapter our aim is to study KH instability in context to strongly coupled dusty plasma fluid and in particular seek the influence of the transverse shear waves (which are the normal modes of the medium) on the KH instability.

The Chapter has been organized as follows. Section 6.2 provides the details of the governing equations for the visco-elastic dust fluid. In section 6.3 we study the linear regime of the KH instability for such a visco-elastic fluid. We choose a specific tangent hyperbolic form of sheared flow profile for this purpose. The effect and role of strong coupling on the growth rate are discussed and comparison with the weak coupling limit is provided in this section. We also show the existence of local instability in the strong coupling limit, this is not possible for the normal hydrodynamic fluids. In section 6.4 we describe the numerical results obtained from the numerical simulation of the Generalized Hydrodynamic (GHD) model. We show that the growth of perturbed energy agrees with the prediction of the linear studies presented in section 6.3. In the nonlinear regime the simulations show fascinating characteristics. A phenomena of the recurrence of the KH instability is seen due to the repeated sharpening of the shear flow. In addition these cyclic events are associated with a bursts of activity in terms of the emission of transverse and compressional waves.

The emission and propagation of transverse shear wave from the sheared flow equilibrium is a natural outcome. The sheared equilibrium configuration chosen initially for these studies can in general be viewed as the superposition of the eigen state of the transverse shear waves. At low amplitudes (when each of the constitutive transverse shear normal mode is in linear domain) such an initial configuration would lead to independent propagation of all the modes. However, a sheared flow configuration being unstable to the KH destabilization, the sheared configuration of each of the mode by itself would be susceptible to the excitation of KH instability. We seek the possible excitation of KH instability in the sheared configuration of one single mode of the propagating transverse shear wave in section 6.5. In section 6.6 we summarize our observations.
6.2 **Governing equations**

The dynamics of strongly coupled dusty plasma medium has been described using GHD set of Eqs. (2.1,2.2,2.3). The description of model and its parameters have been provided in Chapter 2. We have chosen a simplified system geometry for our studies, where variations are confined in 2-D \(x - y\) plane and the third dimension of \(z\) is the axis of symmetry. The equilibrium flow is assumed to be directed along \(y\) and assumed to be sheared along \(x\). Thus the flow direction (i.e. \(y\)) is assumed periodic at boundary while the shear direction (i.e. \(x\)) is considered to be non periodic at boundary.

For the purpose of our investigation we take the basic equilibrium flow to have the following form:

\[
v_0 = v_y(x) \hat{y}
\]  

(6.1)

In the next section we study the stability of this flow in the 2-D \(x - y\) plane, wherein the role of strong coupling effects would be investigated.

6.3 **Linear studies**

We linearize the Eqs. (2.1,2.2,2.3) in the presence of the equilibrium flow defined by Eq. (6.1). The field variables are perturbed such that,

\[
\vec{v} = v_{1x} \hat{x} + [v_y(x) + v_{1y}] \hat{y}; \quad n = n_0 + n_1; \quad \phi = \phi_1
\]  

(6.2)

Here, the fields with subscript ‘1’ denote the perturbed fields and those with subscript ‘0’ represent the equilibrium. The linearized equations upon Fourier analyzing in time and the \(y\) coordinate yields the following:

\[
-i \Omega n_1 + n_0 v'_{1x} + i k_y n_0 v_{1y} = 0
\]

\[
(1 - i \tau_m \Omega) \left(-i \Omega v_{1x} + \alpha n_1 - \phi' \right) = \eta \left( v''_{1x} - k_y^2 v_{1x} \right)
\]

\[
(1 - i \tau_m \Omega) \left(-i \Omega v_{1y} + v_{1x} v'_0 + i k_y \alpha n_1 - i k_y \phi_1 \right) = \eta \left( v''_{1y} - k_y^2 v_{1y} \right)
\]

\[
\phi''_1 - k_y^2 \phi_1 = n_1 + (\mu_e \sigma_i + \mu_i) \phi_1
\]  

(6.3)
Here, $\Omega = \omega - k_y v_{y0}$, the superscript $t$ shows the derivative with respect to $x$ and $\alpha = \mu_d \gamma_d T_d / T_i Z_d$.

The parameter $\sqrt{\alpha}/V_0$ in such a case represents the ratio of sound speed with dust flow velocity and hence is effectively the typical inverse Mach number of the dusty plasma flow under consideration. Here $V_0$ is the amplitude of initial sheared flow velocity. Thus by choosing the value of $\alpha$ ranging from 0 $\rightarrow$ $\infty$ one can carry out investigation for an incompressible dusty plasma medium to a compressible case.

The assumption of incompressibility simplifies the system of equations, as one can then explicitly choose $\nabla \cdot \vec{v} = 0$. Also the continuity equations can then be dropped. A further simplification is possible by concentrating only on long wavelength quasi neutral responses. For this case the left hand side of the Poisson’s equation can also be ignored. We discuss this particular simplified incompressible limit in the next sub-section 6.3.1. The general case is then discussed in the subsequent sub-section.

### 6.3.1 Incompressible dust fluid

The incompressibility assumption simplifies the set of linearized equations (6.3) wherein they can be represented in terms of $v_{1x}$ alone

\[
k_y \Omega (1 - i \tau_m \Omega)^2 v_{1x} - \left( v_{1x} \frac{\Omega v_{1x}''}{k_y} \right) (1 - i \tau_m \Omega)^2 + \frac{i \eta}{k_y} \left( v_{1x}'' - k_y^2 v_{1x}'' \right) (1 - i \tau_m \Omega) \\
+ \eta \tau_m v_{y0} \left( v_{1x}'' - k_y^2 v_{1x}' \right) = i \eta k_y \left( v_{1x}'' - k_y^2 v_{1x} \right) (1 - i \tau_m \Omega)
\]

Rearranging the above equation we can write Eq. (6.4) alternatively as

\[
v_{1x}''' - \frac{i k_y \tau_m v_{y0}'}{(1 - i \tau_m \Omega)} v_{1x}'' - 2 k_y^2 v_{1x}'' + \frac{i k_y^3 \tau_m v_{y0}'}{(1 - i \tau_m \Omega)} v_{1x}' + k_y^4 v_{1x} \\
- \frac{i k_y^3 \Omega (1 - i \tau_m \Omega)}{\eta} v_{1x} + \frac{i k_y (1 - i \tau_m \Omega)}{\eta} \left( v_{1x} v_{y0}' + \frac{\Omega v_{1x}'}{k_y} \right) = 0
\]

In the incompressible case, the fluid velocity can be expressed in terms of a stream function so as to have $v_{1x} = \partial \Psi / \partial y$ and $v_{1y} = -\partial \Psi / \partial x$, the Eq. (6.5) then can be
written in terms of stream function $\Psi$ as

$$
\left[ \frac{d^2}{dx^2} - k_y^2 \right]^{\frac{3}{2}} \Psi = \frac{ik_y}{\eta} \left[ \frac{-\Omega}{k_y} \left( \frac{d^2}{dx^2} - k_y^2 \right) - \frac{d^2 v_{y0}}{dx^2} \right] \Psi
$$

$$
+ \frac{ik_y \tau_m d v_{y0}}{(1 - i \tau_m \Omega)} \left[ \frac{d^3}{dx^3} - k_y^2 \frac{d}{dx} \right] \Psi
$$

(6.6)

It can be easily seen that Eq. (6.6), in the limit of $\tau_m = 0$, reduces to the linearized equations discussed by Drazin for the KH mode in viscous fluids [126]. In the absence of equilibrium flow, one obtains dispersion relation for transverse shear wave from Eq. (6.5) as

$$
\omega = \frac{-i}{2 \tau_m} \pm 0.5 \sqrt{-\frac{1}{\tau_m^2} + \frac{4 \eta k^2}{\tau_m}}
$$

(6.7)

In the limit of strong coupling (i.e. $\omega \tau_m >> 1$), the dispersion relation for the pure transverse shear could be written as

$$
\frac{\omega^2}{k^2} = \frac{\eta}{\tau_m}
$$

(6.8)

We next consider the local limit, wherein the equilibrium velocity flow is assumed to vary rather slowly in comparison to the perturbation scales. In this limit $v_{y0}$ and its derivative can be treated as parameters in a local sense. The system can then be Fourier analyzed in the $x$ coordinate as well. This yields

$$
|k|^4 - \frac{k_x k_y \tau_m v'_0}{(1 - i \tau_m \Omega)} |k|^2 - \frac{i \Omega}{\eta} (1 - i \tau_m \Omega) |k|^2 + \frac{ik'_y}{\eta} (1 - i \tau_m \Omega) v''_0 = 0
$$

(6.9)

Fig. 6.1 shows a two dimensional growth rate plot for existing local instability. It could be seen clearly that the local instability growth is symmetric for both $k_x$ and $k_y$ directions. Equation (6.9) is a cubic equation in $\Omega$ and reduces to a quadratic form for the case when $\omega \tau_m >> 1$ and $v'_0 = 0$, but the second derivative term $v''_0$ is taken as finite. In can be shown that in this case we get a stable system as

$$
\Omega = \left( \frac{1}{2} \right) \left[ \frac{\frac{k_y v''_0}{|k|^2} \pm \sqrt{\left( \frac{k^2 v''_0}{|k|^4} + \frac{4 \eta |k|^2}{\tau_m} \right)}}{|k|^2} \right]
$$
However, we observe that the presence of finite $v'_{y0}$ can result in producing a local instability for the system. This has been illustrated by the plots of the Fig. (6.2), which shows a finite imaginary positive value of $\omega$ for various set of parameters. Such a local instability is altogether absent in the case of sheared flows in neutral hydrodynamic fluids. Thus, this is one of the new features associated with strong coupling properties of the system. Furthermore, it can also be seen that the transverse shear waves acquires a weak dispersion in the presence of $v'_{y0}$.

6.3.2 The general case

The eigen values of the complete general set of linearized Eq. (6.3) can be obtained numerically for specific choices of the flow profile. The set of equations involves four fields and takes considerable time to seek the eigen spectrum and the parameter scan for the study of the influence of strong coupling effects. As an alternative one
Figure 6.2: Dispersion relation for Eq. (6.9) with finite \( \nu_{y0} \) parameter while other parameters are \( \eta = 0.1, \tau_m = 20 \) for subplot (a) and (b) while \( \eta = 10, \tau_m = 20 \) for subplot (c) and (d). \( \nu_{y0} \) is taken to be zero. For all subplots, \( \nu_{y0} \) is 0, 0.4 and 0.8 represented by circle, star and square respectively.

can also employ a simplified case of quasi neutral response for which the dispersive effects appearing in Poisson equation can be ignored and one has instead a simple algebraic relationship between the potential and the density perturbations.

\[
\phi_1 = -n_1 / (\mu_e \sigma_i + \mu_i) \tag{6.10}
\]

This assumption has been invoked for simplicity and leads to a more simplified set. We have shown in our earlier work on weakly coupled dusty plasma system studies that dispersive effects reduce the growth rate, however, the reduction is insignificant at high values of the \( \alpha \) parameter. We have in our linear studies,
therefore, often chosen to consider the quasi neutral case, defined by the following simplified equation

\[-i\Omega n_1 + n_0 v'_1 + i k_y n_0 v_1 = 0\]
\[(1 - i \tau_m \Omega) \left( -i \Omega v_1 + \alpha_1 n_1 \right) = \eta \left( v''_1 - k_y^2 v_1 x \right)\]
\[(1 - i \tau_m \Omega) \left( -i \Omega v_{1y} + v_{1x} v'_0 + i k_y \alpha_1 n_1 \right) = \eta \left( v''_{1y} - k_y^2 v_{1y} \right),\]

(6.11)

to study the effects of strong coupling. Here \(\alpha_1 = \alpha / n_0 + 1 / (\mu_e \sigma_t + \mu_t)\). Choosing Tangent Hyperbolic equilibrium sheared flow profile with \(\epsilon\) as the shear width as shown in the following equation

\[\vec{v}_0 = V_0 \tanh(\frac{x}{\epsilon}) \hat{y}\]

(6.12)

We discretized the set of Eqs. (6.11) on a spatial grid of \(x\) coordinate and obtain the eigenvalues \(\omega\) numerically by standard procedures. The positive imaginary part of which provides for the growth rate \(\gamma\).

We show in Fig. (6.3) and Fig. (6.4) the role of varying the two parameters associated with strong coupling \(\eta\) and \(\tau_m\) respectively, on the growth rate of the KH mode. We have the plot of \(\gamma / V_0\) vs. \(k_y \epsilon\) for various cases in the two figures. In Fig. (6.3), the incompressible weakly coupled dusty plasma system (the hydrodynamics case) for reference, has been shown by a black thick line. For the rest of the plots the value of Mach number has been chosen to be 0.707, and hence they all have effects due to compressibility. The plot with circles (red in color online) is again for a weakly coupled dust fluid obeying hydrodynamical equations. For such a Mach number when the value of \(\eta\) parameter is increased keeping \(\tau_m\) fixed, the growth rate is found to decrease and the threshold wavenumber for instability also reduces. The variation in growth rate with respect to \(\tau_m\) has another interesting aspect. We observe that for flows with any given specific Mach number and a specific value of \(\eta = \eta_s\) say, the two curves for \(\eta = 0\), \(\tau_m = 0\) (weakly coupled inviscid dust shown by red circles in Fig. (6.4)) and \(\eta = \eta_s\), \(\tau_m = 0\) (weakly coupled viscous dust system, shown by diamonds in Fig. (6.4)) define the upper and lower limit of the growth rate respectively for any value of \(\tau_m\). As \(\tau_m\) is increased for all cases of \(\eta = \eta_s\) the growth rate increases from the lower curve and merges with the upper curve at very high values of \(\tau_m\). It has been observed by us earlier in the
context of 1-D simulations as well [96] that the strongly coupled dust behaviour described by the GHD set of equations, at very high values of \( \tau_m \), behaves similar to an inviscid weakly coupled hydrodynamic dust fluid.

It appears that in the limit of \( \tau_m \to \infty \) the unity from the operator \( 1 + \tau_m d/dt \) (Eq. (2.1)) can be ignored. Dividing the equation by \( \tau_m \), one can then ignore \( \eta/\tau_m \nabla^2 \vec{v} \) from right hand side. Thus one is left with an equation which has the form of the weakly coupled fluid system with an additional time derivative acting on all the terms.

### 6.4 Nonlinear studies

We have also investigated the nonlinear regime of the instability numerically. For this purpose the complete set of equations defined by Eq. (2.1,2.2,2.3) have been utilized. The quasineutral assumption is considered for these numerical studies. The assumption of incompressibility have not been invoked a priori for any of
the cases studied by simulations. A flux corrected scheme proposed by Boris et al. [107], have been adopted for the purpose of evolving the set of equations (2.1,2.2) in the 2-D $x - y$ plane. As the scheme numerically solves the continuity form of equations with possibility of inclusion of various source terms, we split Eq. (2.1) as two separate equations of the following form

\begin{equation}
\left[ 1 + \tau_m \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \right] \vec{\psi} = \eta \nabla^2 \vec{u} \\
\left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \vec{v} + \alpha \frac{\nabla n}{n} - \nabla \phi = \vec{\psi}
\end{equation}

(6.13)

The initial condition is chosen as

\begin{align}
v_x &= v_{1x} = V_1 k_{yp} \cos(k_{yp}y) \exp(-x^2/\sigma^2) \\
v_y &= v_{1y} = V_0 \tanh\left(\frac{x}{\epsilon}\right) + V_1 \left(\frac{2x}{\sigma^2}\right) \sin(k_{yp}y) \exp(-x^2/\sigma^2)
\end{align}

(6.14)
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The first term in the expression for \( v_y \) defines the equilibrium flow of Tangent hyperbolic profile defined in Eq. (6.12). Here \( k_{yp} \) is a mode over which the perturbation has been excited to facilitate the growth of instability. The perturbation is taken such that its effects die out in nonperiodic direction before it reaches to boundary. The value of parameters chosen in typical run are \( V_0 = 5.0, V_1 = 10^{-2}, \sigma = 0.8 \) and \( \epsilon = 0.5 \). The evolution is tracked by studying the evolution of the total perturbed kinetic energy, \( E_{PKE} = \iint \left( \overline{v} - \overline{v}_0 \right) dx dy \). The spatial profiles of velocity, density and potential obtained from simulation is also preserved at regular intervals. The evolution of the \( E_{PKE} \) for one typical simulation case has been shown in the plot of Fig. (6.5). It clearly shows that during the initial phase the linear instability mechanism is operative. The growth rate obtained from simulations are observed to match well with the predictions of the linear analysis. In the nonlinear regime the behaviour of the \( E_{PKE} \) evolution is somewhat distinct in the strongly coupled case than what is seen in the other weakly coupled fluid cases. For in-

Figure 6.5: The Perturbed kinetic energy(log scale) Vs. time for the strong coupling dust fluid. \( \eta = 5 \) and \( \tau_m = 20 \) has been chosen for this case. Other parameters in simulation are same as Fig. (6.3).

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stance the simulations of incompressible fluid shows saturation and a constancy of $E_{PKE}$ and the compressible and the dispersive cases show periodic oscillations in $E_{PKE}$ in the nonlinear regime. These periodic oscillations were attributed to the rotation of the vortex structure that ultimately forms as an aftermath of the KH instability in these cases. While for the incompressible fluid case the final saturated structure typically has a circular form, its rotation does not cause any periodic change in $E_{PKE}$. For the compressible and dispersive cases the vortices that finally form have an elliptical shape. Their rotation then shows up as periodic oscillations in the $E_{PKE}$. The rotation frequency of the vortex was shown to match with the observed oscillation in $E_{PKE}$ in these cases. It was also observed that when the vortices merge an irreversible sudden increase in the value of $E_{PKE}$ is observed. In the strong coupling case, though the $E_{PKE}$ in the nonlinear regime exhibits non-stationarity it does not show any periodic characteristics. We would see subsequently that this is associated with the elasticity of the medium causing the sharpening of the shear layer and recurrence of the KH excitation phenomena for a couple of times.

We show in Fig. (6.6) and Fig. (6.7) the snapshots of the $2-D$ spatial profile of the curl and the divergence of the velocity field respectively for the simulation run corresponding to plot of $E_{PKE}$ evolution shown in Fig. (6.5). The initial state as chosen is clearly divergenceless ($\nabla \cdot \vec{v} = 0$) and constitutes uniform strip of vorticity. During the linear phase $t = 10.01$ (comparison of timing can be seen from Fig. (6.5)) the usual bending of the vorticity strip due to the KH instability can be seen. At later stage it breaks up in anisotropic vorticity patches. Apart from the vorticity patches at the main central region, the emission of transverse waves separating from the central region and moving towards the boundaries can also be discerned clearly from the snapshots. These emissions are caused by the local instability which is possible in the case of the strongly coupled medium and about which we discussed in section 6.3 earlier. The spatial profile of divergence shown in Fig. (6.7) illustrates the compressional nature of these emissions.

These vorticity patches, however, are observed to change their shape as they rotate. This is quite unlike the other cases (e.g. incompressible and compressible weakly coupled fluids). Here the vorticity patches get stretched against the background flow, as they rotate. The elastic nature of the medium in this case lets the vorticity patch get extended. The extended structures then coalesce again to form
a very thin vorticity strip (see blue colored patches) as shown in the snapshots of Fig. (6.6) at \( t = 18.69 \) and \( t = 19.62 \). During this extension and coalescence phase there is an intense activity in terms of the emission of shear waves. This is reminiscent of the process when an elastic medium as it snaps back produces intense oscillations.

It is interesting to observe that the thin central vorticity strip developed after the coalescence is now again sharp enough to suffer KH destabilization. This again results in the formation of rotating vortices \(( t = 22.30 \)). The same process then repeats itself. At a later stage one also observed that smaller lumps of vorticity gets separated from the central region. This is when the medium yields and breaks apart as it is no longer possible for it to sustain the strain. Some of these smaller
structures upon reaching the region of uniform background flow form circular patterns and are seen to be well preserved for quite a long duration. This entire repetitive nature of the process can be summarized as follows:

* **Initial Configuration:** Initially, the shear scale $\epsilon = \epsilon_{init}$ is sharp enough to cause destabilization of shear flow (Fig. 6.6, subplot at $t = 0$ and $t = 10.01$).

* **Nonlinear regime:** In this regime, the effective shear width $(\epsilon = \epsilon_{eff})$ is broader and the growth of KH mode is no longer sustained. The saturated KH mode forms elliptical vortices (Fig. 6.6, subplots at $t = 14.00$, $t = 22.30$, and $37.70$).

* **Elliptical vortices:** The elliptical vortices formed in nonlinear regime rotate
and get stretched by the background flow. Basically, the elastic nature of
the medium stretches the elliptical vortices further to form sharp shear flow
structures (Fig. 6.6, subplots at $t = 15.89$, $t = 24.34$, and $t = 40.12$).

* Coalescence of sharper elliptical vortices: Elongation of elliptical vor-
tices leads to formation of a sharper shear width and hence this configura-
tion is once again susceptible to KH destabilization (Fig. 6.6, subplots at $t = 18.69$
and $t = 26.37$).

* Recurrence: Thus the phenomena of KH destabilization recurs in this case
of strong coupling. (Fig. 6.6, subplots at $t = 19.62$ and $t = 32.06$).

6.5 KH destabilization of transverse shear wave propagation

The sheared velocity flow is susceptible to KH destabilization process. Clearly, the
sheared configuration of flow in the transverse shear wave mode also ought to be
susceptible to this instability. However, as shown in Fig. 6.8 when we choose an
initial configuration with a sinusoidal perturbation of the form

$$v_0 = V_0 \sin K_y y \hat{x}$$

(6.15)

with $V_0 = 1e - 3$ and $K_y = 0.6382$ the wave propagates smoothly without any
distortion. For this particular case we had chosen $\eta = 5$ and $\tau_m = 100$. The
analytical phase velocity in this case being $v_{ph} \sim \sqrt{\eta/\tau_m} = 0.05$ also gets verified
by the numerical evolution of the profile. Fig. 6.9 shows the comparison of the
dispersion relation between the analytical and the simulation results. We also
observe from our simulations (Fig. 6.8) that the amplitude diminishes due to the
dissipative effect of $\eta$. However, even though the flow had a sheared configuration
in this mode there is no trace of the KH instability.

We have carried out simulations for the magnitude of $V_0$ to 0.1 and 2.0 shown in
Figs. 6.10 and 6.11 respectively. For $V_0 = 1e-3$ again there is no trace of instability.
However, for $V_0 = 2.0$ the distortions confirming the presence of KH instability can
be easily seen. From these simulations it appears that the KH destabilization is
preferred when the amplitude of the shear flow is high. A simple interpretation for these observations can be provided by considering the comparison of the growth period of the KH instability and the time period of the transverse mode. When the growth rate of the KH mode is slower than the transverse shear wave frequency, the wave propagation does not get hindered by the instability process. On the other hand when the reverse is true the instability manifests itself. A comparison of the growth rate further testifies to this. The typical growth rate of the KH mode can be approximated as $\gamma_{kh} \sim K_y V_0$ (the step velocity case). For the two cases of Fig. 6.10 and Fig. 6.11 the transverse shear wave frequency is from Eq. 6.7 is $\omega_T \sim 0.2236$. For the case (Fig. 6.10) in which the KH is suppressed we have $\omega_T > \gamma_{kh}$ and for the other case (Fig. 6.11) we have $\omega_T < \gamma_{kh}$.

### 6.6 Summary

The fluid Kelvin-Helmholtz instability in the context of a strongly coupled dusty plasma medium has been investigated in detail. In particular it is of interest to understand the role of visco-elasticity and the existence of transverse shear waves in a strongly coupled medium on the fluid KH instability. A generalized fluid
hydrodynamic model (GHD), which captures these aspects of the strong coupling state, has been used to represent the behaviour of dusty plasma medium in this regime.

A complete parametrization of the KH growth rate, in terms of the memory relaxation parameter employed in the GHD model to depict strong coupling effect, have been carried out. It is observed that the growth rate of KH mode reduces in a strongly coupled medium. Furthermore, in addition to the KH mode a local instability driven by the shear flow is also found to exist in the strongly coupled medium. The existence of this local mode causes emission of transverse shear modes. These linear results were verified in the nonlinear simulations conducted by us.

The simulations showed interesting phenomena of recurrence of the KH mode in the nonlinear regime. The KH mode typically saturates by generating vorticity structures which typically have much broader width for sustaining the KH mode. The 2-D constraints of the normal fluid on enstrophy can only make the scale lengths associated with the shear scale get further broadened due to coalescence in the case of normal fluids. However, the GHD fluid has no such constraint in
Figure 6.10: Vorticity of a initial flow as a transverse perturbation in velocity as given by Eq. (6.15). The parameters are \( V_1 = 1e^-3, \eta = 5 \) and \( \tau_m = 100 \) and the perturbation wave number correspond to two mode numbers (\( K_y = 0.6382 \)) in the system.

2-D. In fact the elastic nature of the strongly coupled fluid causes the individual vortices as they rotate to get stretched and form sharper flow structures. Thus unlike a 2-D normal fluid in a 2-D visco-elastic fluid one observes formation of short scale structures. These sharp structures are then again susceptible to the KH instability. This cycle was observed to get repeated several times in some of our simulations. It ultimately stops as a result of system exhausting up its free energy associated with the shear flows. An additional channel of free energy exhaustion is associated with shear flow is the local instability supported by the medium due to which strong emission of transverse as well as compressional wave are observed. We have demonstrated a rich variety of response in a 2-D strongly coupled visco-elastic dusty plasma medium by our simulations conducted here for an equilibrium shear flow configuration. Further studies to understand the competition between the local instability and the KH mode in getting rid of the free energy associated with the shear flow in a visco-elastic medium is necessary. It is also clear from
Figure 6.11: Vorticity of a initial flow as a transverse perturbation in velocity as given by Eq. (6.15). The parameters are $V_1 = 2.0$, $\eta = 5$ and $\tau_m = 100$ and the perturbation wave number correspond to two mode numbers ($K_y = 0.6382$) in the system.

our simulations that in a 2-D visco-elastic medium the possibility of formation of short scales does exist. Thus unlike the hydrodynamic fluid the cascade behaviour of the energy spectrum is quite distinct and should be investigated in detail.