Chapter 4

THERMOPHORESIS EFFECT ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OVER A VERTICAL WAVY SURFACE IN A POROUS MEDIUM WITH VARIABLE PROPERTIES
CHAPTER 4

Chapter-4. THERMOPHORESIS EFFECT ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OVER A VERTICAL WAVY SURFACE IN A POROUS MEDIUM WITH VARIABLE PROPERTIES

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4.1 Introduction

In this chapter, we investigate the effect of thermophoresis and variable properties on convective heat and mass transfer flow over a vertical wavy surface in a fluid saturated porous medium. To our best knowledge, no one has investigated the effect of thermophoresis on convective heat and mass transfer flow over a vertical wavy surface in a fluid saturated porous medium.

The study of thermophoresis plays a vital role in the species transport mechanism of several devices consists of small micron sized particles and large temperature gradient. In fact, the particles are moving from hot surfaces to cold one. As small such particles (example dust) are suspended in a gas with a temperature gradient, experience a force in the direction opposite to the temperature gradient. The effect of the thermophoresis is so widespread in many practical applications in removing small particles from gas streams, in studying particulate material deposition on turbine blades and in determining exhaust gas particle trajectories from combustion devices. This shows that the thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process which is utilized in germanium dioxide optical fiber performs and graded index silicon dioxide.
In view of these applications, in this chapter we investigate the influence of thermophoresis on double diffusive convection along a vertical wavy surface in a fluid saturated porous medium with variable properties.

4.2 Free Convection

An analysis is presented for the steady natural convection heat and mass transfer over a vertical wavy surface in a homogeneous Darcy porous medium with thermophoresis and variable properties. The governing boundary layer equations for momentum, energy and concentration are transformed into a system of ordinary differential equations by using an appropriate transformation and then solved by employing the shooting technique. A parametric study illustrating the influence of variable viscosity, variable thermal conductivity and thermophoresis parameter on the fluid velocity, temperature and concentration profiles as well as rate of heat and mass transfer is conducted. The computations in the absence of variable properties and thermophoresis effects correlate accurately with the earlier published results.

4.2.1 Basic Equations

We consider laminar, steady, viscous, incompressible two-dimensional flow of fluid on natural convective heat and mass transfer past a vertical wavy plate embedded in a Darcy saturated porous medium.
The configuration of the problem is shown in Fig. 2.1. The wavy configuration is defined as eq. (2.1)

Under the Boussinesq approximation the governing equations for flow momentum, energy and concentration are:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{4.1}
\]

\[
\frac{\partial}{\partial y} \left( \frac{\mu \bar{u}}{K} \right) = \frac{\partial}{\partial x} \left( \frac{\mu \bar{v}}{K} \right) + \rho g \left( \beta \frac{\partial T}{\partial y} + \beta \frac{\partial C}{\partial y} \right) \tag{4.2}
\]

\[
\bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( \alpha \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) \tag{4.3}
\]

\[
\bar{u} \frac{\partial C}{\partial x} + \bar{v} \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - \frac{\partial}{\partial x} (U_T C) - \frac{\partial}{\partial y} (V_T C) \tag{4.4}
\]

The corresponding boundary conditions are

\[
\bar{u} = 0; \bar{v} = 0; T = T_w; C = C_w \quad \text{at} \quad \bar{y} = \bar{g}(\bar{x}) = \bar{a} \sin \left( \frac{\pi \bar{x}}{l} \right) \tag{4.5}
\]

\[
\bar{u} \to 0; T \to T_w; C \to C_\infty \quad \text{as} \quad \bar{y} \to \infty
\]

where \( \bar{u} \) and \( \bar{v} \) are the volume averaged velocity components in \( \bar{x} \) and \( \bar{y} \) directions, respectively. In Eq. (4.4), \( U_T \) and \( V_T \) are thermophoretic velocities which can be written as (see Wu and Greif [66])

\[
U_T = -\frac{k \nu}{T} \frac{\partial T}{\partial x} \quad \text{and} \quad V_T = -\frac{k \nu}{T} \frac{\partial T}{\partial y} \tag{4.6}
\]

where \( k \) is thermophoretic coefficient which ranges in the values between 0.2 and 1.2 and is defined as (see Talbot et.al [67])
\[ k = \frac{2C_i \left( \frac{\lambda_g}{\lambda_p} + C_i \text{Kn} \right) \left[ 1 + \text{Kn} \left( C_1 + C_2 e^{-C_3/\text{Kn}} \right) \right]}{(1 + 3C_m \text{Kn}) \left( 1 + 2\frac{\lambda_g}{\lambda_p} + 2C_i \text{Kn} \right)} \]

where \( C_1, C_2, C_3, C_m, C_s, C_t \) are constants, \( \lambda_g \) and \( \lambda_p \) are thermal conductivities of fluid and diffused particles, respectively and \( \text{Kn} \) is the Knudsen number.

From eqns. (2.10) – (2.13), (4.6) and non-dimensional variables (2.14), the eqns. (4.1) – (4.5) reduces to

\[ \frac{1}{\theta - \theta_r} \left[ \frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial x} \right] + \frac{\partial^2 \theta^*}{\partial y^2} + \frac{\partial^2 \theta^*}{\partial x^2} = Ra \left( 1 - \frac{\theta}{\theta_r} \right) \left[ \frac{\partial \theta}{\partial y} + N \frac{\partial \phi}{\partial y} \right] \]  
(4.7)

\[ \frac{\partial \psi^*}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \theta}{\partial y} = \beta \left[ \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 \right] + (1 + \beta \theta) \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \]  
(4.8)

\[ Le \left( \frac{\partial \psi^*}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \phi}{\partial y} \right) = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - Sc \tau \left( \frac{\partial^2 \theta}{\partial x^2} \phi + \frac{\partial^2 \theta}{\partial y^2} \phi + \frac{\partial \theta}{\partial x} \frac{\partial \phi}{\partial y} + \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial x} \right) \]  
(4.9)

where \( \tau = -\frac{k}{T_r} (T_w - T_\infty) \) is the Thermophoretic parameter, and \( Sc = \frac{V}{D} \) is the Schmidt number. In eqn. (4.9), \( \tau = -\frac{k}{T_r} (T_w - T_\infty) \) represents thermophoresis parameter. From the practical point of view two possible cases exist, (i) heated surfaces \( (T_w - T_\infty > 0) \), which leads to \( \tau < 0 \), (ii) cold surfaces \( (T_w - T_\infty < 0) \), which gives rise to \( \tau > 0 \). For the second case thermophoresis is the special type of mechanism for capture of particles on cold surfaces, being especially important for submicron particles since thermophoretic velocity is relatively
independent of particle size, while the first one may be thought of suppress the particle deposition on the surface. It concludes that for heated surfaces a dust-free area is obtained adjacent to the surface due to particles moving away from the surface, thus fluid particle concentration is tending to zero near the surface. Hence, thermophoretic velocity is treated as particle deposition velocity when \( \tau < 0 \).

The associated boundary conditions are given by

\[
\begin{align*}
\psi^* &= 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on} \quad y = a \sin(x), \\
\psi^*_y &= 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad y \to \infty.
\end{align*}
\] (4.10)

### 4.2.2 Solution Methodology

We incorporate the effect of wavy surface and the usual boundary layer scaling into the governing equations (4.7) – (4.10) for free convection, using the transformations

\[
x = \xi, \quad \hat{y} = \frac{y - a \sin(x)}{\xi^{1/2} Ra^{-1/2}}, \quad \psi^* = Ra^{1/2} \psi.
\] (4.11)

along with the substitutions

\[
\eta = \frac{\hat{y}}{1 + a^2 \cos^3 \xi}, \quad \psi = \xi^{1/2} f(\eta), \quad \theta = \theta(\eta) \quad \text{and} \quad \phi = \phi(\eta).
\] (4.12)

Substituting eq. (4.11) and (4.12) into eqs. (4.7) – (4.10) and letting \( Ra \to \infty \), we obtain the following boundary layer equations:

\[
f'' + \frac{1}{\theta - \theta_\infty} \theta f' = \left(1 - \frac{\theta}{\theta_\infty}\right) \left(\theta' + N \phi'\right)
\] (4.13)
\[ \beta(\theta')^2 + (1 + \beta \theta')\theta' + \frac{1}{2} f \theta' = 0 \]  \hspace{1cm} (4.14)

\[ \phi'' + \left( \frac{1}{2} Lef - Sc \pi \theta \right) \phi' - Sc \phi\theta' = 0 \]  \hspace{1cm} (4.15)

where prime denotes differentiation with respect to \( \eta \).

The associated boundary conditions are

\[
\begin{array}{l}
\text{at } \eta = 0 & \quad f = 0, \quad \theta = 1, \quad \text{and } \phi = 1 \\
\text{at } \eta \to \infty & \quad f' \to 0, \quad \theta \to 0 \quad \text{and } \phi \to 0
\end{array}
\]  \hspace{1cm} (4.16)

The engineering design quantities of physical interest include Nusselt number and Sherwood numbers which are defined as

\[ Nu_\xi = -\theta' (0) Ra_\xi^{1/2} \frac{1}{(1 + a^2 \cos^2 (\xi))^{1/2}} \quad \text{and} \quad Sh_\xi = -\phi' (0) Ra_\xi^{1/2} \frac{1}{(1 + a^2 \cos^2 (\xi))^{1/2}} \]  \hspace{1cm} (4.17)

4.2.2.1 Numerical Method

Eqns. (4.13) – (4.16) are solved by employing shooting technique, which is given in chapter-2 in detail.

4.2.2.2 Grid Independence Test

In order to check the effects of step size \( (\Delta \eta) \), we found the Nusselt number and Sherwood number with four different step sizes as \( \Delta \eta = 0.1, \Delta \eta = 0.01, \Delta \eta = 0.001 \) and \( \Delta \eta = 0.0001 \). We observe from Table-4.1 that the results are independent with the step size \( (\Delta \eta) \). Hence a step size \( \Delta \eta = 0.01 \) is selected to be satisfactory for a convergence criterion of \( 10^{-6} \) in all cases.
Table-4.1: Grid-independence for $\beta=0.5$, $\alpha=0.5$, $\tau=1$, $N=1$, $Sc=0.6$, $Le=0.5$ and $\theta_r=3$. 

<table>
<thead>
<tr>
<th>$\Delta \eta$ (Step size)</th>
<th>$\nu_\zeta Ra_\zeta^{-1/2}$</th>
<th>$\sh_\zeta Ra_\zeta^{-1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.4673888944</td>
<td>0.3812042945</td>
</tr>
<tr>
<td>0.01</td>
<td>0.4673886097</td>
<td>0.3812047348</td>
</tr>
<tr>
<td>0.001</td>
<td>0.4673883736</td>
<td>0.3812042401</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.4673885689</td>
<td>0.3812046549</td>
</tr>
</tbody>
</table>

4.2.2.3 Accuracy of the results

As shown in the Table-4.2 we also checked absolute and relative error of the present results. It is observed that accuracy of the Nusselt number and Sherwood number results is obtained when reducing the errors. In this thesis we restrict the absolute error $10^{-7}$ and relative error $10^{-7}$ for obtaining the required accuracy ($10^{-6}$) of the flow characteristics.

Table-4.2: Absolute and Relative error of the results for $\beta=0.5$, $\alpha=0.5$, $\tau=1$, $N=1$, $Sc=0.6$, $Le=0.5$ and $\theta_r=3$. 

<table>
<thead>
<tr>
<th>$\nu_\zeta Ra_\zeta^{-1/2}$</th>
<th>$\sh_\zeta Ra_\zeta^{-1/2}$</th>
<th>Absolute Error</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4674956249</td>
<td>0.3812259106</td>
<td>1e-3</td>
<td>1e-3</td>
</tr>
<tr>
<td>0.4673894369</td>
<td>0.3812058383</td>
<td>1e-5</td>
<td>1e-5</td>
</tr>
<tr>
<td>0.4673886177</td>
<td>0.3812047062</td>
<td>1e-7</td>
<td>1e-7</td>
</tr>
<tr>
<td>0.4673886141</td>
<td>0.3812046996</td>
<td>1e-10</td>
<td>1e-10</td>
</tr>
</tbody>
</table>

4.2.2.4 Comparison with the earlier work

To validate the present method the results are compared with previously published results. Table-4.3 shows the comparison results
in the absence of variable properties for vertical flat plate (i.e., \( \alpha = 0 \)) and thermophoresis with Cheng [14]. The results are found to be in good agreement.

### 4.2.3 Results and Discussions

The results by the aforesaid method are expressed in terms of flow velocity, temperature and concentration profiles as well as rate of heat and mass transfer, and all the results are then explained through graphicas as shown in Figs. 4.1 – 4.17.

Fig. 4.1 represents the variation of variable viscosity \( \theta_r \) on velocity profile. It is observed that an increase in \( \theta_r \) results in a depreciation in the vicinity of the plate until it reaches a particular value, then increases until the value becomes constant, that is zero, at the outside the boundary layer. Fig. 4.2 shows the results of variable viscosity \( (\theta_r) \) on temperature \( (\theta) \) profile. It is noted that temperature profile increases with increase in \( \theta_r \). From fig. 4.3 it is clear that concentration profile increases with increase in \( \theta_r \). Thus, increase in \( \theta_r \) leads to an enhancement in the thermal and solutal boundary layer thickness.

Figs. 4.4 – 4.6 represents the variation of variable thermal conductivity \( \beta \) on flow velocity, temperature and concentration distributions respectively. From fig. 4.4 it is found that the velocity profile has similar behaviour as in fig. 4.1. However the change in velocity is larger as shown in fig. 4.4 with increase in \( \beta \). We assumed
that variable thermal conductivity ($\beta$) is to vary as a linear function of the temperature. It means that temperature profile significantly increased with increase in $\beta$ as shown in fig. 4.5. But the opposite results are noticed for concentration profile as mentioned in fig. 4.6 with increase in variable thermal conductivity $\beta$.

**Table-4.3:** Comparison the values of the rate of heat and mass transfer with values reported by Cheng [14] for $\beta=0$, $\tau=0$ and $\theta_\tau \to \infty$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\frac{Nu_z Ra_\tau^{-1/2}}{\xi \xi^{-1/2}}$</th>
<th>$\frac{Sh_z Ra_\tau^{-1/2}}{\xi \xi^{-1/2}}$</th>
<th>$\frac{Nu_z Ra_\tau^{-1/2}}{\xi \xi^{-1/2}}$</th>
<th>$\frac{Sh_z Ra_\tau^{-1/2}}{\xi \xi^{-1/2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.992</td>
<td>0.992</td>
<td>0.9923</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>0.681</td>
<td>3.290</td>
<td>0.6809</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>0.521</td>
<td>10.521</td>
<td>0.5208</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.559</td>
<td>1.358</td>
<td>0.5558</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.650</td>
<td>1.624</td>
<td>0.6510</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.728</td>
<td>1.852</td>
<td>0.7275</td>
</tr>
</tbody>
</table>

The variation of thermophoresis parameter ($\tau$) on flow velocity, temperature and concentration distributions has been displayed in figs. 4.7 – 4.9. From fig. 4.7 it is observed that increasing thermophoresis parameter($\tau$) tends to increase significantly i.e., increase the thickness of the velocity boundary layer. Fig. 4.8 reveals that the temperature profile decreased considerably with increase in $\tau$. From fig. 4.9 it is seen that increase in thermophoresis parameter
leads to raise the values of concentration. In other words increase in $\tau$ decelerates the thermal boundary layer thickness while accelerates the solutal boundary layer thickness.

The influence of variable viscosity ($\theta_r$) parameter on the rate of heat (Nusselt number) and mass (Sherwood number) transfer with stream wise coordinate $\xi$ is exhibited in Figs. 4.10 – 4.11. It is observed that as $\theta_r$ increases, both Nusselt number and Sherwood number decreases. The influence of variable thermal conductivity ($\beta$) on Nusselt number and Sherwood number with stream wise coordinate is highlighted in figs. 4.12 and 4.13 respectively. From these figures it is seen that increase in $\beta$ tends to decrease in Nusselt number but conversely enhances the Sherwood number. The variation of thermophoresis parameter ($\tau$) on Nusselt number and Sherwood number is depicted in figs. 4.14 and 4.15. It is observed that as $\tau$ increases, rate of heat transfer increases but rate of mass transfer reduces with increase in $\tau$. The variation of the amplitude of the wavy surface ($a$) on Nusselt number and Sherwood number are illustrated in 4.16 and 4.17. These results shows that with increase in $a$, Nusselt number and Sherwood number decreases and hence the amplitude of the Nusselt number and Sherwood decreased for large values of $a$. It is important to note that for $a=0$, the vertical wavy surface becomes flat surface.
4.2.4 Graphs

Fig-4.1: Velocity profile ($u$) for different values of variable viscosity ($\theta_r$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\beta = 0.5$, $Sc = 0.6$ and $\tau = 1$

Fig-4.2: Temperature profile ($\theta$) for different values of variable viscosity ($\theta_r$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\beta = 0.5$, $Sc = 0.6$ and $\tau = 1$
Fig-4.3: Concentration profile ($\phi$) for different values of variable viscosity ($\theta_r$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\beta = 0.5$, $Sc = 0.6$ and $\tau = 1$.

Fig-4.4: Velocity profile ($u$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\tau = 1$. 
Fig-4.5: Temperature profile ($\theta$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\tau = 1$

Fig-4.6: Concentration profile ($\phi$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\tau = 1$
Fig-4.7: Velocity profile ($u$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\beta = 0.5$.

Fig-4.8: Temperature profile ($\theta$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\beta = 0.5$. 


Fig-4.9: Concentration profile ($\phi$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $N = 1$, $Le = 0.5$, $\theta_r = 2$, $Sc = 0.6$ and $\beta = 0.5$

Fig-4.10: Axial distributions of the Nusselt number (Nu) for different values of variable viscosity ($\theta_r$) with stream wise coordinate ($\xi$) for $N = 1$, $a=0.5$, $Le = 0.5$, $\tau = 1$, $Sc = 0.6$ and $\beta = 0.5$
Fig-4.11: Axial distributions of the Sherwood number (Sh) for different values of variable viscosity ($\theta_r$) with stream wise coordinate ($\xi$) for $N = 1$, $a=0.5$, $Le = 0.5$, $\tau = 1$, $Sc = 0.6$ and $\beta = 0.5$

Fig-4.12: Axial distributions of the Nusselt number (Nu) for different values of variable thermal conductivity ($\beta$) with stream wise coordinate ($\xi$) for $N = 1$, $Le = 0.5$, $a=0.5$, $\tau = 1$, $Sc = 0.6$ and $\theta_r = 1.5$
Fig-4.13: Axial distributions of the Sherwood number (Sh) for different values of variable thermal conductivity ($\beta$) with stream wise coordinate ($\xi$) for $N = 1$, $\alpha=0.5$, $Le = 0.5$, $\tau = 1$, $Sc = 0.6$ and $\theta_r = 1.5$.

Fig-4.14: Axial distributions of the Nusselt number (Nu) for different values of thermophoresis parameter ($\tau$) with stream wise coordinate ($\xi$) for $N = 1$, $Le = 0.5$, $\alpha=0.5$, $\theta_r = 1.5$, $Sc = 0.6$ and $\beta= 0.5$. 
Fig-4.15: Axial distributions of the Sherwood number (Sh) for different values of thermophoresis parameter ($\tau$) with stream wise coordinate ($\xi$) for $N = 1$, $a=0.5$, $Le = 0.5$, $\theta_{r} = 1.5$, $Sc = 0.6$ and $\beta = 0.5$

Fig-4.16: Axial distributions of the Nusselt number (Nu) for different values of amplitude of the wavy surface (a) with stream wise coordinate ($\xi$) for $N = 1$, $Le = 0.5$, $\tau = 1$, $\theta_{r} = 1.5$, $Sc = 0.6$ and $\beta = 0.5$
Fig-4.17: Axial distributions of the Sherwood number (Sh) for different values of the amplitude of the wavy surface (a) with stream wise coordinate (\(\xi\)) for \(N = 1\), \(\tau = 1\), \(Le = 0.5\), \(\theta_r = 1.5\), \(Sc = 0.6\) and \(\beta = 0.5\)
4.3 Mixed convection

In this section, we investigate the influence of thermophoresis effect on mixed convective heat and mass transfer flow over a vertical wavy surface in a porous medium with variable properties, namely variable viscosity and variable thermal conductivity. The fluid flow is caused by density variations along with free stream flow. The governing equations pertaining to the present model were non-dimensionalised using appropriate transformations. The effect of wavy surface is incorporated into resultant equations by using suitable transformations and then transformed into non-linear ordinary differential equations by using the similarity transformations. The numerical method namely fourth order Runge-Kutta method and Newton-Raphson method with shooting technique is employed to find the solutions of the equations. The structure of flow, temperature and concentration fields in the Darcy porous media are more pronounced by complex interactions among variable viscosity, variable thermal conductivity, mixed convective parameter, thermophoresis and amplitude of the wavy surface. The transport process of flow, heat and mass transfer in the boundary layer for both cases of aiding and opposing flows has been discussed. The results are reported through comparison table and graphs for various physical parameters.

4.3.1 Basic Equations

\[
\frac{\partial \tau}{\partial x} + \frac{\partial \gamma}{\partial y} = 0 \quad (4.18)
\]
\[
\frac{\partial}{\partial y} \left( \frac{\mu}{K} \frac{u}{K} \right) = \frac{\partial}{\partial x} \left( \frac{\mu}{K} \frac{v}{K} \right) \pm \rho g \left( \beta \frac{\partial T}{\partial y} + \beta \frac{\partial C}{\partial y} \right) \tag{4.19}
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( \alpha \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) \tag{4.20}
\]

\[
\frac{\partial C}{\partial x} = \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - \frac{\partial}{\partial x} (U_i C) - \frac{\partial}{\partial y} (V_i C) \tag{4.21}
\]

The corresponding boundary conditions are

\[
\begin{align*}
\bar{u} &= 0, \bar{v} = 0, T = T_w, C = C_w, \quad \text{at} \quad \bar{y} = \sigma (\bar{x}) = \bar{a} \sin \left( \frac{\pi \bar{x}}{l} \right), \\
\bar{u} \rightarrow U_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \quad \bar{y} \rightarrow \infty
\end{align*} \tag{4.22}
\]

where \( U_\infty \) is the uniform free stream velocity. In right hand side of eq. (4.19), (+) sign represents aiding flow and (-) sign represents opposing flow.

Substituting Eqn. (2.10) - (2.13), (4.6) and non-dimensional variables (2.14) in Eqn. (4.18) and (4.22), we get

\[
\frac{1}{\theta - \theta} \left( \frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial x} \right) + \frac{\partial^2 \psi^*}{\partial y^2} + \frac{\partial^2 \psi^*}{\partial x^2} = \pm \frac{Ra}{Pe} \left( 1 - \theta \right) \left( \frac{\partial \theta}{\partial y} + N \frac{\partial \phi}{\partial y} \right) \tag{4.23}
\]

\[
\frac{\partial \psi^*}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \theta}{\partial y} = \beta \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 + \left( 1 + \beta \theta \right) \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \tag{4.24}
\]

\[
Le \left( \frac{\partial \psi^*}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial \phi}{\partial y} \right) = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - Sc \left( \frac{\partial^2 \theta}{\partial x^2} \phi + \frac{\partial^2 \theta}{\partial y^2} \phi + \frac{\partial \theta}{\partial x} \frac{\partial \phi}{\partial y} + \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial x} \right) \tag{4.25}
\]
where \( \tau = -\frac{k}{T_r} (T_w - T_\infty) \) is the Thermophoretic parameter, and \( Sc = \frac{V}{D} \) is the Schmidt number.

The associated boundary conditions are given by

\[
\begin{align*}
\psi^* = 0, \quad \theta = 1, \quad \phi = 1, \quad &\text{on} \quad y = a \sin(x), \\
\psi_y \rightarrow \frac{\alpha_0}{l} U_\infty, \quad &\theta \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty, \\
\end{align*}
\]

(4.26)

### 4.3.2 Solution Methodology

We incorporate the effect of wavy surface and the usual boundary layer scaling into the governing equations (4.23) – (4.26) for mixed convection, using the transformations

\[
x = \xi, \quad \dot{\xi} = \frac{y - a \sin(x)}{\xi^{1/2} Pe^{-1/2}}, \quad \psi^* = Pe^{1/2} \psi.
\]

(4.27)

along with the substitutions

\[
\eta = \frac{\dot{\xi}}{1 + a^2 \cos^2 \frac{\xi}{2}}, \quad \psi = \xi^{1/2} f(\eta), \quad \theta = \theta(\eta) \quad \text{and} \quad \phi = \phi(\eta).
\]

(4.28)

Substituting Eq. (4.27) and (4.28) into Eqs. (4.23) – (4.26) and letting \( Pe \to \infty \), we obtain the following boundary layer equations:

\[
f'' + \frac{1}{\theta - \theta_r} \theta f' = \Delta \left(1 - \frac{\theta}{\theta_r}\right) \left(\theta' + N\phi'\right)
\]

(4.29)

\[
\beta(\dot{\theta})^2 + (1 + \beta \theta) \dot{\theta} + \frac{1}{2} f \theta = 0
\]

(4.30)

\[
\phi'' + \left(\frac{1}{2} Lef - Sc \tau \theta\right) \phi' - Sc \tau \phi = 0
\]

(4.31)

where prime denotes differentiation with respect to \( \eta \).
The associated boundary conditions are

\[ f = 0, \quad \theta = 1, \quad \text{and} \quad \phi = 1 \quad \text{at} \quad \eta = 0 \]
\[ f' \rightarrow 1, \quad \theta \rightarrow 0 \quad \text{and} \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \]  

(4.32)

The engineering design quantities of physical interest include Nusselt number and Sherwood numbers which are defined as

\[ N_{\text{tu}} = \frac{-\theta'(0)Pe^{1/2}}{(1 + a^2 \cos^2(\xi))^{1/2}} \quad \text{and} \quad S_{\text{hu}} = \frac{-\phi'(0)Pe^{1/2}}{(1 + a^2 \cos^2(\xi))^{1/2}} \]  

(4.33)

**4.3.3 Results and Discussions**

The equations (4.29) – (4.32) have been solved by employing shooting technique as given in chapter 2. The results are reported for various physical parameters graphically. Figs.4.18 – 4.20 represent the variation of variable viscosity \((\theta_i)\) on velocity \((u)\), temperature \((\theta)\), and concentration \((\phi)\) distributions respectively. It is noted from these figures that increasing variable viscosity parameter leads to a depreciation in the flow velocity while enhances the temperature and the concentration for both cases of aiding and opposing flows. Thus, the hydrodynamic velocity boundary layer thickness decreases with increase in \(\theta_i\), but increases thermal and solutal boundary layer thickness.

Figs. 4.21 – 4.23 depicts the velocity, temperature and concentration distributions for different values of variable thermal conductivity \((\beta)\) respectively. It is observed that velocity and temperature distributions are more pronounced with increase in \(\beta\)
and hence these results are considerably increased within the boundary layer region for both cases of aiding and opposing flow. Conversely, the concentration decreases significantly with increase in $\beta$ for both cases of aiding and opposing flow.

Figs. 4.24 – 4.26 shows velocity, temperature and concentration distributions for different values of thermophoresis parameter ($\tau$) respectively. Increase in $\tau$ leads an enhancement in the flow velocity for opposing flow case but these results are reversed for aiding flow case. An increase $\tau$ results in an enhancement in temperature profile for aiding flow case while reduces the temperature profile for opposing flow case. The solutal boundary layer thickness reduces with increase in $\tau$ for both cases of aiding and opposing flow.

Figs. 4.27 – 4.28 represents the axial distributions of Nusselt number and Sherwood number for different values of $\theta$, respectively. We observed from these figs that the values of Nusselt number and Sherwood number are decreased with increase in $\theta$, i.e. amplitude of Nusselt number and Sherwood number are reduced for higher values of $\theta$, for both cases of aiding and opposing flow. Fig. 4.29 shows that increase in $\beta$ leads to depreciations in the amplitude of the Nusselt number for both aiding and opposing flow cases. An increase in $\beta$, reduces the Sherwood number for aiding flow case but the opposite results are noticed for opposing flow case as shown in fig. 4.30. Figs. 4.31 & 4.32 illustrate the variation of $\tau$ on Nusselt number and Sherwood number with stream wise coordinate ($\xi$) respectively. From
fig. 4.31 we noticed that increase in $\tau$ tends to decrease the Nusselt number considerably for aiding flow case but the opposite trend is observed for opposing flow case. Sherwood number values are increased more significantly with increase in $\tau$ for both cases of aiding and opposing flow as shown in fig. 4.32. Figs. 4.33 & 4.34 depict the variation of the amplitude of the wavy surface ($a$) on Nusselt number and Sherwood number respectively. An increase in $a$ show depreciation on the amplitude of the Nusselt number and Sherwood number for both cases of aiding and opposing flow. It is important to note that for $a = 0$ the waviness is negligible, i.e. the wavy surface becomes the flat surface.
4.3.4 Graphs

Fig-4.18: Velocity profile \( (u) \) for different values of variable viscosity \( (\theta_r) \) with transverse coordinate \( (\eta) \) for \( \beta = 0.5, N = 1, Le = 1, Sc = 0.6, \)
\[ \tau = 1 \]

Fig-4.19: Temperature profile \( (\theta) \) for different values of variable viscosity \( (\theta_r) \) with transverse coordinate \( (\eta) \) for \( \beta = 0.5, N = 1, Le = 1, Sc = 0.6, \)
\[ \tau = 1 \]
Fig-4.20: Concentration profile ($\phi$) for different values of variable viscosity ($\theta_r$) with transverse coordinate ($\eta$) for $\beta = 0.5$, $N = 1$, $Le = 1$, $Sc = 0.6$, $\tau = 1$

Fig-4.21: Velocity profile ($u$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $\theta_r = 1.5$, $N = 1$, $Le = 1$, $Sc = 0.6$, $\tau = 1$
Fig-4.22: Temperature profile ($\theta$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $\theta_r = 1.5$, $N = 1$, $Le = 1$, $Sc = 0.6$, $\tau = 1$.

Fig-4.23: Concentration profile ($\phi$) for different values of variable thermal conductivity ($\beta$) with transverse coordinate ($\eta$) for $\theta_r = 1.5$, $N = 1$, $Le = 1$, $Sc = 0.6$, $\tau = 1$.
Fig-4.24: Velocity profile ($u$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $\theta_0 = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $Sc = 0.6$

Fig-4.25: Temperature profile ($\theta$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $\theta_0 = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $Sc = 0.6$
Fig-4.26: Concentration profile ($\phi$) for different values of thermophoresis parameter ($\tau$) with transverse coordinate ($\eta$) for $\theta_r = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $Sc = 0.6$.

Fig-4.27: Axial distributions of the Nusselt number (Nu) for different values of variable viscosity ($\theta_r$) with stream wise coordinate ($\xi$) for $\beta = 0.5$, $N = 1$, Le = 1, $\tau = 1$, $a = 0.5$. 
Fig-4.28: Axial distributions of the Sherwood number (Sh) for different values of variable viscosity ($\theta_r$) with stream wise coordinate ($\xi$) for $\beta = 0.5$, $N = 1$, $Le = 1$, $\tau = 1$, $\alpha = 0.5$.

Fig-4.29: Axial distributions of the Nusselt number (Nu) for different values of variable thermal conductivity ($\beta$) with stream wise coordinate ($\xi$) for $\theta = 1.5$, $N = 1$, $Le = 1$, $\tau = 1$, $\alpha = 0.5$. 
Fig-4.30: Axial distributions of the Sherwood number (Sh) for different values of variable thermal conductivity ($\beta$) with stream wise coordinate ($\xi$) for $\theta_r = 1.5$, $N = 1$, $Le = 1$, $\tau = 1$, $a = 0.5$.

Fig-4.31: Axial distributions of the Nusselt number (Nu) for different values of thermophoresis parameter ($\tau$) with stream wise coordinate ($\xi$) for $\theta_r = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $a = 0.5$. 
Fig-4.32: Axial distributions of the Sherwood number (Sh) for different values of thermophoresis parameter ($\tau$) with stream wise coordinate ($\xi$) for $\theta = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $a = 0.5$.

Fig-4.33: Axial distributions of the Nusselt number (Nu) for different values of amplitude of the wavy surface ($a$) with stream wise coordinate ($\xi$) for $\theta = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $\tau = 1$. 
Fig-4.34: Axial distributions of the Sherwood number (Sh) for different values of the amplitude of the wavy surface (a) with stream wise coordinate ($\xi$) for $\theta_r = 1.5$, $\beta = 0.5$, $N = 1$, $Le = 1$, $\tau = 1$. 
4.4 Conclusions

The thermophoresis effect on the free convection heat and mass transfer along a vertical wavy surface embedded in a fluid saturated porous medium is analyzed numerically. Conservation equations for mass, momentum, energy and concentration are transformed into non-dimensional boundary layer equations by using a simple coordinate transformation and then solved by using shooting technique. The results for various physical parameters are reported graphically. The following conclusions can be made:

Case of Free Convection

- Increase in variable viscosity results depreciation in velocity profile, Nusselt number and Sherwood number while there is an enhancement in temperature and concentration profiles.
- Velocity and temperature profiles and Sherwood number are increased for large values of variable thermal conductivity parameter, but the opposite results are reported for concentration profile and Nusselt number.
- Increase in thermophoresis parameter leads to enhance the flow velocity and concentration and Nusselt number while depreciation in temperature profile and Sherwood number.
- Increasing the amplitude of the wavy surface decelerates the amplitude of the Nusselt number and Sherwood number.
Case of Mixed Convection

- Increase in variable viscosity results an enhancement in temperature and concentration distributions while depreciation in velocity profile and Nusselt number and Sherwood number.

- Velocity and temperature distributions are increased but concentration profile and Nusselt number results are decreased with increase in variable thermal conductivity for both cases of aiding and opposing flows. The Sherwood number increases for opposing flow case and reduced for aiding flow case for larger values of variable thermal conductivity.

- Increase in thermophoresis parameter causes to reduce concentration profile while enhance the Sherwood number results considerably for both cases of aiding and opposing flows. Velocity and Nusselt number are significantly reduced but enhance the temperature profile for aiding flow case but the opposite results are obtained for opposing flow cases as increase in thermophoresis parameter.

- For higher values of the amplitude of the wavy surface, the Nusselt number and Sherwood number results are reduced for both cases of aiding and opposing flows.

- It is noted that for aiding flow case Velocity profiles, Nusselt number and Sherwood number results are larger for aiding flow case but temperature and concentration profiles are smaller with compare in opposing flow case.