CHAPTER 3

MATHEMATICAL MODELING OF SHE EQUATIONS AND FITNESS FUNCTION

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Chapter - 3. Mathematical Modeling of SHE Equations and Fitness Function

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MATHEMATICAL MODELING OF SHE EQUATIONS AND FITNESS FUNCTION

3.1 INTRODUCTION

Application of Fourier series to per phase output voltage waveform of three-phase cascade H-bridge 11-level inverter and mathematical modeling of non linear transcendental trigonometric SHE equations set has been discussed in this chapter. In addition to that, formation of objective function which has to be minimized and the development of fitness/cost function by considering penalty function is also presented.

3.2 DEVELOPMENT OF SHE EQUATIONS

As mentioned in chapter 2, among all commercially existing multilevel inverter topologies, cascade H-bridge multilevel inverter is one of the preferred topology in high-power medium voltage drive applications. The inverter is mainly made up of a number of identical H-bridge power cells connected in cascade. In general, the number of H-bridge cells in a CHB inverter is primarily determined by the order of the harmonics to be eliminated, inverter operating voltage, power and manufacturing cost. In high-power medium voltage (MV) drives applications, cascade H-bridge inverter with seven to eleven voltage levels have been increasingly used [7]. Hence, three-phase cascade H-bridge 11-level inverter has been chosen for analysis.
As mentioned earlier, cascade H-bridge multilevel inverter is simply a cascaded connection of multiple H-bridge inverters with isolated dc sources. Fig. 3.1 represents the chosen configuration for analysis i.e. three-phase CHB 11-level inverter and Fig. 3.2 represents the per phase output voltage waveform of chosen configuration which contains eleven steps in output voltage waveform. The eleven-steps are +5V\textsubscript{dc}, +4V\textsubscript{dc}, +3V\textsubscript{dc}, +2V\textsubscript{dc}, +V\textsubscript{dc}, 0, -V\textsubscript{dc}, -3V\textsubscript{dc}, -4V\textsubscript{dc}, -5V\textsubscript{dc} and -5V\textsubscript{dc} respectively. For an eleven-level inverter which contains five separate sources, The per phase voltage is given by

$$V_{an} = V_{H1} + V_{H2} + V_{H3} + V_{H4} + V_{H5}$$ \hspace{1cm} (3.1)
3.2.1 Application of Fourier series and formation of SHE equations

This section deals with the application of Fourier Series to the per phase output voltage waveform of three-phase cascade H-bridge inverter to derive the non linear transcendental harmonic equations corresponding to the selective harmonic elimination or fundamental switching scheme.

Jean Baptiste Joseph Fourier has first proposed Fourier series to solve heat equation in metal plate and first explained that periodic function can be represented by an infinite sum of periodic sine or cosine functions which contains the harmonically related terms. In other words, each trigonometric term in this infinite series has a
frequency equal to an integral multiple of the fundamental frequency of the original periodic function [12].

For a periodic integrable function in the interval \([0,2\pi]\), as mentioned by Fourier, \(f(\omega t)\) can be expressed as a sum of sine and cosine terms such that \(f(\omega t)\) approximates \(f(x)\) as the number of coefficients \(N\) tends to infinity, above statement can be expressed as

\[
f(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{N} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]
\] (3.2)

The Fourier coefficients \(a_0\), \(a_n\) \& \(b_n\) are defined as follows

\[
a_0 = \frac{1}{\pi} \int_{0}^{2\pi} f(\omega t) d(\omega t)
\] (3.3)

\[
a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(\omega t) \cos(n\omega t) d(\omega t) \ (n \geq 0)
\] (3.4)

\[
b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(\omega t) \sin(n\omega t) d(\omega t) \ (n \geq 1)
\] (3.5)

From equation (3.2) represents a periodic function which can be divided into an infinite number of trigonometric components at different frequencies (or) integral multiples of \(\omega\). It can be further understood that, the waveform \(f(x)\) can be understood as composed of a dc component \((a_0)\) and harmonic components \((n \geq 2)\).

Fig. 3.2 represents the per phase output voltage waveform of cascade H-bridge 11-level inverter at fundamental switching scheme or selective harmonic elimination technique. By applying Fourier series for the output voltage waveform in Fig. 3.2 and considering odd quarter–wave function symmetry in to account. The Fourier
coefficients $a_0 \& a_n$ becomes zero and only the odd sine harmonics can be non zero.

\[ a_0 = 0 \text{ and } a_n = 0 \] (3.6)

\[ b_n \neq 0 \text{ for all odd values of } n \] (3.7)

\[ b_n = 0 \text{ for all even values of } n \] (3.8)

In general, by considering odd-quarter wave symmetry conditions in to account, for a periodic waveform with time period $T$, \( b_n \) can be expressed as

\[ b_n = \frac{8}{T} \int_{0}^{T/4} f(\omega t) \sin(n\omega t) \, d(\omega t) \] (3.9)

Here, the time period is $2\pi$, hence, \( b_n \) can be expressed as

\[ b_n = \frac{8}{2\pi} \int_{0}^{\pi/2} f(\omega t) \sin(n\omega t) \, d(\omega t) \] (3.10)

By considering the switching angles in the output phase voltage waveform of chosen inverter as in Fig. 3.2

\[ b_n = \int_{\alpha_1}^{\alpha_2} \frac{4}{\pi} (V_{dc}) \sin(n\omega t) \, d(\omega t) + \int_{\alpha_2}^{\alpha_3} \frac{4}{\pi} (2V_{dc}) \sin(n\omega t) \, d(\omega t) + \]

\[ \int_{\alpha_3}^{\alpha_4} \frac{4}{\pi} (3V_{dc}) \sin(n\omega t) \, d(\omega t) + \int_{\alpha_4}^{\alpha_5} \frac{4}{\pi} (4V_{dc}) \sin(n\omega t) \, d(\omega t) + \]

\[ \int_{\alpha_5}^{\pi} \frac{4}{\pi} (5V_{dc}) \sin(n\omega t) \, d(\omega t) \] (3.11)

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \& \alpha_5$ are switching angles or variables to be find out to eliminate fifth, seventh, eleventh and thirteenth order of harmonics and ‘$V_{dc}$’ is the voltage of independent dc source used in CHB topology.
Performing integration on equation (3.11), it can be simplified into

\[ b_n = \frac{4V_{dc}}{\pi n} \left[ \cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3) + \cos(n\alpha_4) + \cos(n\alpha_5) \right] \] (3.12)

Equation (3.12) represents the peak values of these odd harmonics in terms of the switching angle \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \).

Substituting equation (3.12) in equation (3.2)

Fourier expansion for per phase output voltage wave form of cascade H-bridge 11-level inverter is given by

\[ V_{an}(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{\pi n} \left[ \cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3) + \cos(n\alpha_4) + \cos(n\alpha_5) \right] \cos n\omega_0 \sin \omega t \] (3.13)

Main objective of this work is to control the peak value of fundamental output voltage \( V_1 \) by eliminating or minimizing the harmonics of order 5th, 7th, 11th and 13th so that %THD produced, complies with IEEE 519-1992 harmonic guidelines.

The fundamental voltage in terms of switching angles is given by

\[ \frac{4V_{dc}}{\pi} (\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) + \cos(\alpha_4) + \cos(\alpha_5)) = V_1 \] (3.14)

Modulation index \( M_f \) is defined as the ratio of fundamental voltage \( V_1 \) to the maximum obtainable fundamental voltage \( (V_{1\text{max}}) \). Maximum fundamental voltage can be obtained by keeping all switching angles in zero degrees i.e

\[ V_{1\text{max}} = \frac{4V_{dc}}{\pi} \] (3.15)

Where ‘\( S \)’ is the number of switching angles or number of independent sources.
Modulation index \((M)\) = \(\frac{V_1}{V_{imax}}\) \hspace{1cm} (3.16)

Substituting equation (3.14) in equation (3.15)

\[
\text{Modulation index } (M_I) = \frac{\pi V_1}{4SV_{dc}}
\hspace{1cm} (3.17)
\]

If one wants to control the peak value of the fundamental output voltage \(V_1\) by eliminating fifth, seventh, eleventh and thirteenth order of harmonics, the resulting harmonic equations are:

\[
\begin{align*}
[\cos(\alpha_1) + \cos(\alpha_2) + \cdots + \cos(\alpha_s)]/5 &= M_I \\
\cos(5\alpha_1) + \cos(5\alpha_2) + \cdots + \cos(5\alpha_s) &= 0 \\
\cos(7\alpha_1) + \cos(7\alpha_2) + \cdots + \cos(7\alpha_s) &= 0 \\
\cos(11\alpha_1) + \cos(11\alpha_2) + \cdots + \cos(11\alpha_s) &= 0 \\
\cos(13\alpha_1) + \cos(13\alpha_2) + \cdots + \cos(13\alpha_s) &= 0
\end{align*}
\hspace{1cm} (3.18)
\]

Equation set (3.18) consists of system of five non linear transcendental trigonometric equations which can also be called as selective harmonic equations. Here, it is required to obtain a desired fundamental peak voltage \(V_1\), by eliminating desired lower order the harmonics such that

\[
0 \leq \alpha_1 \leq \alpha_2 \leq \ldots \ldots \leq \alpha_s \leq \frac{\pi}{2}
\hspace{1cm} (3.19)
\]

In the process of elimination of harmonics, triplen harmonics are not considered because for balanced three-phase systems, each phase voltage will contain triplen harmonics equal in both magnitude and phase to the triplen harmonics present in other two phases. Hence, all
triplen harmonics will be cancelled out in the line to line output voltage waveforms.

The developed SHE equations set (3.18) are highly nonlinear transcendental in nature, there may exist single, multiple or even no solutions for a particular modulation index [87]. In some drive applications, it is required to operate the drive over a range of modulation index. Hence, providing feasible solutions for SHE equations during full range of modulation index from 0 to 1, with less computational burden, less complexity which in turn controls the multilevel inverter with less %THD to comply with IEEE 519-1992 harmonic guidelines has been a challenging task for the researchers over a decades.

3.3 FORMATION OF OBJECTIVE AND FITNESS FUNCTION

In order to overcome the drawbacks of iterative techniques and to find feasible solutions for SHE equations set during complete range of modulation index, stochastic optimization techniques may provide feasible solutions with less computational effort and mathematical calculations like without using gradients and determinants. While developing the algorithms in stochastic optimization techniques, it is required to formulate an objective function and fitness function.

An objective describes the measure of effectiveness of minimizing selected order of harmonics such as 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th} and 13\textsuperscript{th} while maintaining fundamental voltage at a specified value. Later, it can be
converted into optimization problem by considering the constraints into account.

The main aim is to minimize the objective function \( F(\alpha) \)

\[
F(\alpha_1, \alpha_2, \ldots, \alpha_s) \quad (3.20)
\]

Subject to

\[
0 \leq \alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_s \leq \frac{\pi}{2} \quad (3.21)
\]

In stochastic approaches, another parameter called “fitness value” has been introduced. It is a measure of the appropriateness of a solution with respect to the original objective and the “amount of infeasibility”. The fitness function is formed by adding the original objective function to penalty function [88,89]. Since, finding analytical solution during full range of modulation index is a major problem, a penalty approach is used to decrease the fitness of infeasible solutions towards the feasible region. Where, \( V_1^* \) is the desired fundamental harmonic, ‘\( S \)’ is the number of switching angles, and \( h_s \) is the order of the \( s^{th} \) viable harmonic at the output of a three phase multilevel inverter, e.g. \( h_2 = 5 \) and \( h_5 = 13 \).

Here it is required to find a set of switching angles such that the magnitude of the fundamental harmonic reaches a desired value, i.e. \( V_1^* \).
Fitness function can be expressed as

\[ f = \alpha_i^{min}\left(\left(100 \frac{v_i^1-v_1^1}{v_i^1}\right)^4 + \sum_{s=2}^{S} \frac{1}{h_s} \left(50 \frac{v_h^s}{v_1^1}\right)^2\right); \ i = 1,2,\ldots,S \]  \tag{3.22}

Subject to

\[ 0 \leq \alpha_i \leq \pi/2 \]  \tag{3.23}

Whenever the fundamental harmonic violates its set point by more than 1\%, the first term of equation 3.22, fines it by a power of ‘4’ which is a very heavy penalty. As a result of using a power of ‘4’, corresponding penalties for any deviation less than 1\% gets a negligible value. Each harmonic ratio is weighted by the reciprocal of its harmonic order, i.e. \(\frac{1}{h_s}\)  [86].

As a result of this weighting method, reducing the low order harmonics gets a higher importance. It should be noted that, low order harmonics are more harmful and their filtering is more troublesome. Hence, in this work minimization lower order harmonics in CHB eleven-level inverter has been considered.

This developed fitness function has been used for developing the algorithms in the Stochastic optimization techniques such as Continuous-Genetic algorithm and Modified Species based Particle Swarm Optimization techniques.
3.4 CONCLUSIONS

In this chapter, selective harmonic elimination technique has been applied to the chosen inverter configuration, Fourier series has been applied and formation of non linear transcendental trigonometric equations or SHE equations set (3.18) has been presented. In addition to that in order to overcome the limitations of iterative method, to obtain the feasible solutions during entire region of modulation index from 0 to 1, objective function and fitness functions have been developed by adding penalty to original objective function. Minimization of lower order harmonics such as 5th, 7th, 11th and 13th gains higher importance, because of the penalty approach.