Appendix A

Appendix A title ...

General form for Mayer Problem of Optimal Control

Let $A$ be a subset of the $tx$–space $\mathbb{R}^{1+n}$, let $U$ be a given subset of the $u$–space $\mathbb{R}^m$. Let $f(t, x, u) = (f_1, \cdots, f_2)$ be a given function on $A \times U$. For every $(t, x) \in A$ let $Q(t, x) = f(t, x, U) \subset \mathbb{R}^n$ be the set of all $z = (z_1, \cdots, z_n)$ with $z = f(t, x, u)$ for some $u \in U$. Let $B$ be a given subset of $t_1 x_1 t_2 x_2$– space $\mathbb{R}^{2n+2}$. The Mayer problem of optimal control is finding minima of the functional

$$I[x, u] = g(t_1, x(t_1), t_2, x(t_2)) \quad (A.1)$$

for pairs of functions $x(t) = (x_1, \cdots, x_n)$, $u(t) = (u_1, \cdots, u_m)$, $t_1 \leq t \leq t_2$, $x$ absolutely continuous, $u$ measurable, satisfying

$$\frac{dx}{dt} = f(t, x(t), u(t)), \quad t_1 \leq t \leq t_2, \quad (A.2)$$

boundary conditions

$$e[x] = (t_1, x(t_1), t_2, x(t_2)) \in B, \quad (A.3)$$

and constraints

$$(t, x(t)) \in A, \quad t_1 \leq t \leq t_2, \quad (A.4)$$

$$u(t) \in U, \quad t_1 \leq t \leq t_2. \quad (A.5)$$

in the class $\Omega$ of all admissible pairs $(x, u)$. By an admissible pair for the problem $(A.1$-$A.5)$ we mean a pair $(x(t), u(t)), t_1 \leq t \leq t_2$, $x$ absolutely continuous, $u$ measurable, satisfying all requirements $(A.2$-$A.5)$. Here $x$ and $u$ are said to be an
admissible trajectory and an admissible control respectively.

**Theorem A.0.1** (The Filippov Existence Theorem for Mayer Problem of Optimal Control). If $A$ and $U$ are compact, $B$ is closed, $f$ is continuous on $A \times U$, $g$ is continuous on $B$, $\Omega$ is not empty, and for every $(t, x) \in A$ the set $Q(t, x) = f(t, x, U) \subset \mathbb{R}^n$ is convex, then $I[x, u]$ has an absolute minimum in $\Omega$. 