CHAPTER 1

INTRODUCTION

1.1 GENERAL

Mathematical analysis is expressed in a more concrete form of philosophical point of view. It is very significant to reveal its fundamental structures. Besides the deeper understanding of the specific features, the higher level of generalization is necessary for the rigorous treatment of the theoretical topics. The main purpose of a scientific discipline is to generate knowledge and to come closer to truth without making any value judgments while technologies normally try to generate tools for solving problems rather, very often by either accepting or being based on the given value Schemes. It seems to be a very close relationship between a model and a theory in scientific inquiry. Both are probably based on the theoretical statements that can either be formal axioms or scientific laws. The analysis of scientific laws (theories) with observational evidences is the fundamentals of models. This proposed work mainly focuses on the analysis of scientific laws connected with fuzzy numbers, fuzzy number intervals, fuzzy number mapping etc..

Mathematical analysis is an aggregate of all branches of mathematics concerned with the study of functions and its associated variables. In particular mathematical analysis is the systematic study of the nature of real and complex valued functions. Moreover mathematical analysis is the branch of mathematics which claims to be associated with
almost all investigations. A fuzzy function is a generalization of the concept of a classical function. There can be a crisp mapping from a fuzzy set that carries along the fuzziness of the domain and therefore generates a fuzzy set. Ordinary functions can have fuzzy properties or be constrained by fuzzy constraints. This thesis provides a systematic study of fuzzy numbers and fuzzy number mappings over real and complex fields.

Fuzzy sets are separated from crisp set because of their membership grade. In this research, properties of membership functions have also been focused. Fuzzy numbers and fuzzy number mappings play an important role in fuzzy set theory. This research work contributes a theoretical analysis of fuzzy numbers which is combined with fuzzy number mappings. In mathematical analysis, functions and their properties are defined within subsets of a main set especially in real line they are defined by considering intervals closed and open. In fuzzy Mathematics, analysis becomes very effective using closed fuzzy number interval \([\mu]^r, \mu \in E\), where \(E\) is a fuzzy number space and \(r \in [0,1]\). This research work concentrates on closed fuzzy intervals and the results are based on the partition of their membership values. Moreover motivation of this research work lies in the applications of fuzzy numbers to various fuzzy systems by partitioning fuzzy intervals together with their membership values.

1.2 OBJECTIVES OF THE RESEARCH

The Objectives of this research work is

- Analyze the characteristics of Fuzzy number mappings with illustrations.
• Analyze the bounded variation and total variation of convex fuzzy number mappings over fuzzy number space with a new defined fuzzy metric.

• Analyze the integrability of fuzzy number mappings by defining Oscillatory Fuzzy Sum (OFS).

• Analyze the fuzzy number inner product over fuzzy number space and extend the projection theorem to fuzzy Hilbert space.

• Analyze the complex fuzzy numbers over complex fuzzy number space.

1.3 ORGANISATION OF THE THESIS

Chapter 1 deals with the general introduction, objectives and organization of the research work. The second chapter gives a literature review that is helpful to identify other works related to this research. The third chapter of this work is devoted to the fundamental structures of fuzzy numbers, fuzzy number mappings and fuzzy set theory.

Chapter 4 focuses on the comparison between fuzzy number mappings on different fuzzy number intervals especially the most commonly used fuzzy numbers triangular, Gaussian and trapezoidal with detailed numerical illustrations. These illustrations reveal the characteristics of fuzzy number mappings over fuzzy number space.

The fifth chapter provides some important definitions and theorems of fuzzy bounded variation and total variation functions by introducing a new fuzzy metric defined on fuzzy number interval. Chapter 6 deals with the integrability of fuzzy number mappings by introducing upper and lower fuzzy sums on a fuzzy number interval in connection
with fuzzy number mapping and then bring a new definition for upper and lower fuzzy integrals over fuzzy number space. These fuzzy sums have been illustrated through an example. The definition of upper and lower fuzzy sums leads to another important definition - Oscillatory fuzzy sum (OFS). On the basis of this OFS, Riemann Integrability of fuzzy number mapping has been explained in the later part. Moreover some definitions and theorems connecting integration with continuity and monotonicity of fuzzy number mappings are established.

Chapter 7 extends fuzzy numbers to fuzzy number vectors and brings a new approach for inner product and norm using fuzzy number vectors over fuzzy number space. This chapter has also been developed a connection between fuzzy Hilbert space and fuzzy number mappings. Moreover the orthogonal property of fuzzy number vectors defined in this section helps to prove the projection theorem of fuzzy Hilbert space on fuzzy number space. In addition to this, establishes some results for fuzzy number inner product spaces, orthogonality, orthonormality etc..

Chapter 8 forwards a new approach for complex fuzzy numbers with respect to fuzzy numbers. In the later part of this chapter the attention turns to complex fuzzy number mappings corresponding to fuzzy number mappings. In this chapter the properties of complex fuzzy numbers - normality, convexity convergence etc., partition of complex fuzzy number intervals and complex fuzzy number mappings, its differentiability, continuity all are explained. The last chapter deals with the conclusion and scope for future work.