CHAPTER 3

PERFORMANCE ANALYSIS OF OFDM BASED
BIDIRECTIONAL RELAY NETWORK IN THE
PRESENCE OF PHASE NOISE

3.1 PREAMBLE

The PLNC based OFDM system transmits high data streams over numerous narrow band flat fading channel, thereby achieving high spectral efficiency. In practice, phase noise is introduced with the information symbols transmitted on all subcarriers in OFDM transceiver. In OFDM systems, the degradation with orthogonality due to the phase noise is the major issue. The effect of Phase Noise (PHN) on the OFDM signal is modeled in (Tomba 1998), where PHN causes a rotation of each time-domain OFDM sample by a random phase drift. The sensitivity of the phase noise of the oscillator used for frequency down-conversion is discussed in (Armada AG 1998 & Armada AG 2001). The effect of phase noise in the presence of multipath fading environment is discussed in (Wu and Bar-Ness 2004). The influence of phase noise on OFDM is analyzed in (Petrovic et al 2007 & Rabiei et al 2008).

Though much work has been done on the performance analysis of PLNC based wireless relay network in the literature, in most of the works, it is assumed that both carrier and phase are perfectly synchronized between the transmitter and at the receiver. It is customary to characterize the achievable performance of the OFDM based PLNC relay network due to practical constraints. The first objective of this chapter is to model the OFDM based bidirectional relay network in the presence of phase noise. The second
objective is to analyze the effect of phase noise in the outage and BER performance at relay node in first time slot and source nodes in second time slot. The third objective is to study the overall end to end outage and BER performance of the OFDM based bidirectional relay network in the presence of phase noise.

3.2 SYSTEM MODEL

Consider a bidirectional relay network consisting of two source nodes $S_1, S_2$ and a relay node $R$ as shown in Figure 3.1. Each node has single antenna and the network operates in half duplex mode. The baseband equivalent of quasi static channel impulse response between $m^{th}$ source node $S_m$ and relay node $R$ is denoted by $g_m = [g_0, g_1, \ldots, g_{L_m-1}]^T, m = 1, 2$. In quasi-static environment, the channel statistics does not change within a transmission duration of one frame but can vary from one frame to another. The frequency response of the $L_m \times 1$ channel impulse response vector $g_m$ is defined as

$$
\tilde{g}_m[k] = \sum_{l=0}^{L_m-1} g_m[l] \exp \left( \frac{-j2\pi kl}{N} \right), \quad k = 0, 1, 2, \ldots, N - 1, \quad m = 1, 2
$$

(3.1)

where $N$ is the number of subcarriers in OFDM symbol.

During time slot I, both the source nodes $S_1$ and $S_2$ transmit OFDM signals with $N$ information symbols and $L_{CP}$ Cyclic Prefix (CP) symbols to the relay node $R$. The CP length $L_{CP}$ satisfies the condition that $L_{CP} \geq \max \{L_1-1, L_2-1\}$ to avoid Inter-Symbol Interference (ISI). In an ideal case, the transmit symbols on all subcarriers $k = 0, 1, 2, \ldots, N - 1$ can be ideally recovered from the receive signal samples at relay node $R$ using the concept of
PLNC. But, in practice, the presence of phase noise in the network reduces the receive SNR and hence increases BER.

![Diagram of a bidirectional relay network](image)

**Figure 3.1 Bidirectional relay network**

The time domain phase noise component at $n^{th}$ symbol duration in source node $s_m$ is modeled as $\phi_m(n) = \phi_m(n-1) + \varepsilon$, with the initial condition $\phi_m(0) = 0$ and $\varepsilon$ is zero mean Gaussian distributed variable with variance $\sigma_{\varepsilon}^2$.

The phase noise at $k^{th}$ subcarrier

$$\tilde{p}_m[k]$$ is defined as

$$\tilde{p}_m[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \exp(j\phi_m(n))\exp(-j2\pi nk/N) \quad k = 0,1,...N-1$$

(3.2)

The phase noise variance $\sigma_{\varepsilon}^2$ is defined as $\sigma_{\varepsilon}^2 = 2\pi\beta_m T$, where $\beta_m$ is 3dB phase noise bandwidth at source node and $T$ is OFDM signal duration. When the phase noise bandwidth $\beta$ is greater than the subcarrier spacing $\Delta f$ of OFDM symbol, Inter-Carrier Interference is introduced in the receive signal. Assuming that perfect timing and frequency offset corrections are performed, the receive signal at the relay node $R$ after taking DFT, is given by

$$\tilde{y}_r[k] = \sum_{m=3}^{2} \sqrt{E_m} \tilde{\chi}_m[k] \tilde{\gamma}_m[k] \tilde{p}_m[0] + \sum_{q=0}^{N-1} \sum_{m=1}^{2} \sqrt{E_m} \tilde{\chi}_m[q] \tilde{\gamma}_m[q] \tilde{p}_m[k-q] - \tilde{n}_r[k]$$

$$k = 0,1,...N-1$$

(3.3)
$E_m$ represents the transmit power at source node $S_m$. $\tilde{n}_r[k]$ is circularly symmetric complex Gaussian noise with zero mean and variance $\sigma^2_r$, $\tilde{p}_{mr}[0]$ is the Common Phase Error (CPE) at $m^{th}$ source node and $\tilde{p}_{mr}[k-q]$ represents ICI due to the phase noise. $\tilde{p}_{mr}[k]$ is effective phase noise at $k^{th}$ subcarrier in the relay node $R$ during the first time slot. The bandwidth of the phase noise sequence $\phi_{mr}$ is $\beta_m + \beta_r$. Where $\beta_r$ is the phase noise bandwidth at relay node $R$. The transmit symbol sequences $\hat{x}_1$ and $\hat{x}_2$ from source nodes $S_1$ and $S_2$ respectively are jointly estimated using Maximum Likelihood (ML) principle at relay node $R$. The estimate of the $k^{th}$ symbols of both the sequences is given by

$$\{\hat{x}_1[k], \hat{x}_2[k]\} = \arg\min_{\{\hat{x}_1[k], \hat{x}_2[k]\} \in \{1, \ldots, N-1\}^2} |\tilde{y}_r[k] - \sqrt{E_r} \tilde{x}_1[k] \hat{g}_1[k] - \sqrt{E_r} \tilde{x}_2[k] \hat{g}_2[k]|^2 \quad k = 0, 1, \ldots, N-1$$

(3.4)

Using the joint estimation of the symbols of $\hat{x}_1$ and $\hat{x}_2$, a new sequence $\hat{x}_r[k]$ is formed as $\hat{x}_r[k] = \hat{x}_1[k] \hat{x}_2[k]$, $k = 0, 1, \ldots, N-1$ at the relay node $R$. Then, an OFDM signal corresponding to the signal vector $\hat{x}_r[k]$ is generated by taking IDFT and appending CP of length $L_{cp}$.

During the time slot II, the relay node $R$ broadcasts the OFDM signal to the nodes $S_1$ and $S_2$. Assuming perfect timing and frequency offsets, the receive signal at node $S_m$, perturbed by phase noise, is expressed as

$$\tilde{y}_m[k] = \left[ \sqrt{E_r} \tilde{x}_r[k] \hat{g}_m[k] \right] \tilde{p}_{rm}[0] + \left[ \sum_{q=0}^{N-1} \sqrt{E_r} \tilde{x}_r[q] \hat{g}_m[q] \tilde{p}_{rm}[k-q] \right] + \tilde{n}_m[k]$$

$$k = 0, 1, \ldots, N-1, \quad m = 1, 2$$

(3.5)
where $E_r$ is the transmit power at relay node $R$, $\tilde{n}_m[k]$ is circularly symmetric complex Gaussian noise with zero mean and variances $\sigma_m^2$ at source node $S_m$. The ML estimate of $k^{th}$ symbol of $\tilde{x}_r$ at the source node is given by

$$\hat{x}_r^{(m)}[k] = \arg \min_{\tilde{x}[k] \in \{1, 1\}, \ldots, N-1, \ldots, 1, 1} \left| \tilde{y}_m[k] - \sqrt{E_r} \tilde{g}_m[k] \tilde{x}_r[k] \right|^2$$

$$k = 0, 1, ..., N-1, \quad m = 1, 2$$

(3.6)

Then, source node $S_1$ estimates the data from $S_2$ by multiplying the symbol $\hat{x}_r^{(1)}[k]$ with its own symbol $\hat{x}_1[k]$ and $S_2$ estimates the data from node $S_1$ by multiplying the symbol $\hat{x}_r^{(2)}[k]$ with its own symbol $\hat{x}_2[k]$ as follows

$$\hat{x}_2[k] = \hat{x}_r^{(1)}[k] \hat{x}_1[k], \quad k = 0, 1, ..., N-1$$

(3.7)

$$\hat{x}_1[k] = \hat{x}_r^{(2)}[k] \hat{x}_2[k], \quad k = 0, 1, ..., N-1$$

(3.8)

3.3 SINR ANALYSIS

In this section, the expressions for Signal-to-Interference Noise Ratio (SINR) are derived for the receive signal at relay node $R$ in time slot I and the source node $S_m$ in time slot II.

3.3.1 SINR Analysis at Time Slot – I

It is assumed that the channel frequency response vectors $\tilde{g}_m, m = 1, 2$ are known at the relay node $R$. Using the Equation (3.3), the expressions for instantaneous SINR of the receive signal in relay node $R$ at $k^{th}$ subcarrier, considering the signal from the source node $S_m$, is derived as,
\[ \gamma_{m,\text{relay}}^{PN}[k] = \frac{E \left[ E_m \left| \tilde{x}_m[k] \right|^2 \right] \cdot \left| \tilde{g}_m[k] \right|^2 \cdot \left| \tilde{p}_{mr}(0) \right|^2}{E \left[ \sum_{q=0,q \neq k}^{N-1} \sqrt{E_m \tilde{x}_m[q] \tilde{g}_m[q] \tilde{p}_{mr}[k-q]} \right]^2 + E \left[ \left| n_r[k] \right|^2 \right]}, \quad m = 1, 2 \]

(3.9)

Using Cauchy’s Schwarz inequality, the first term in the denominator of the Equation (3.9) is written as

\[ E \left[ \sum_{q=0,q \neq k}^{N-1} \sqrt{E_m \tilde{x}_m[q] \tilde{g}_m[q] \tilde{p}_{mr}[k-q]} \right]^2 \leq E \left[ \sum_{q=0,q \neq k}^{N-1} E_m \tilde{x}_m[q]^2 \right] \cdot \sum_{q=0,q \neq k}^{N-1} \left| \tilde{p}_{mr}[k-q] \right|^2 \cdot \sum_{q=0,q \neq k}^{N-1} \left| \tilde{g}_m[q] \right|^2 \]

(3.10)

Since \( E \left[ \tilde{x}_m[k]^2 \right] = 1 \) and \( E \left[ \sum_{q=k} \tilde{x}_m(q)^2 \right] = 1 \), the expression for lower bound SINR in Equation (3.9) is expressed as

\[ \gamma_{m,\text{relay}}^{PN}[k] \geq \frac{E_m \left| \tilde{g}_m[k] \right|^2 \cdot \left| \tilde{p}_{mr}(0) \right|^2}{E_m \sum_{q=0,q \neq k}^{N-1} \left| \tilde{p}_{mr}[k-q] \right|^2 \cdot \sum_{q=0,q \neq k}^{N-1} \left| \tilde{g}_m[q] \right|^2 + \sigma_r^2}, \quad m = 1, 2 \]

(3.11)

Assuming that the CPE term \( \tilde{p}_{mr}(0) \) is compensated, it can be further simplified as

\[ \gamma_{m,\text{relay}}^{PN}[k] \geq \frac{\left| \tilde{g}_m[k] \right|^2}{\sum_{q=0,q \neq k}^{N-1} \left| \tilde{p}_{mr}[k-q] \right|^2 \cdot \sum_{q=0,q \neq k}^{N-1} \left| \tilde{g}_m[q] \right|^2 + \frac{\sigma_r^2}{E_m}}, \quad m = 1, 2 \]

(3.12)
Since the phase noise terms are orthonormal, it can be written as

\[ \sum_{k=0}^{N-1} \left| \tilde{P}_{mr}[k] \right|^2 = 1. \]

Hence, the first term in the denominator is written as

\[ \sum_{q=0,q \neq k}^{N-1} \left| \tilde{P}_{mr}[q] \right|^2 = 1 - \left| \tilde{P}_{mr}(0) \right|^2. \]

The CPE term \( \tilde{P}_{mr}(0) \) is defined as

\[ \tilde{P}_{mr}(0) = \frac{1}{N} \sum_{n=0}^{N-1} \exp(j\phi_m(n)) \exp(j\phi_r(n)). \]

Then the variance of \( \tilde{P}_{mr}(0) \) is expressed as

\[ \sigma_p^2 = \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{n_2=0}^{N-1} \int \int \exp(jx) \exp(jy) f_x(x) f_y(y) dx dy \]  

(3.13)

where \( x = (\phi_m(n_1 - n_2)) \) and \( y = \phi_r(n_1 - n_2) \). Substituting the PDFs of \( x \) and \( y \) and evaluating the integrals, the variance of \( \tilde{P}_{mr}(0) \) is determined as

\[ \sigma_p^2 = 1 - \frac{\pi(\beta_m + \beta_r)NT_s}{3} \]

(Songping Wu and Yeheskel Bar-Ness 2004). Hence, the term

\[ \sum_{q=0,q \neq k}^{N-1} \left| \tilde{P}_{mr}[k - q] \right|^2 \]

is equal to \( \frac{\pi(\beta_m + \beta_r)NT_s}{3} \). Substituting this value in Equation (3.12) \( r_{m,relay}^{PN}[k] \) is written as

\[ r_{m,relay}^{PN}[k] \geq \frac{\left| \tilde{g}_m[k] \right|^2}{\frac{\pi(\beta_m + \beta_r)NT_s}{3} \sum_{q=0,q \neq k}^{N-1} \left| \tilde{g}_m[q] \right|^2 + \frac{\sigma_r^2}{E_m}}, \quad m = 1, 2 \]  

(3.14)

3.3.2 SINR Analysis at Time Slot – II

The instantaneous SINR of the receive signal at source nodes \( S_1 \) and \( S_2 \) at \( k^{th} \) subcarrier, is given by
\[
\gamma_{m,\text{dest}}^{PN}[k] = \frac{E\left[\sqrt{E, \hat{x}_r[k]} \right]^2 \hat{g}_m[k] \left| \hat{p}_m(0) \right|^2}{E \left[ \sum_{q=0,q \neq k}^{N-1} \sqrt{E, \hat{x}_r[q]} \hat{g}_m[q] \hat{p}_m[k-q] \right]^2 + E \left[ n_m[k] \right]^2}, \quad m = 1, 2
\]

(3.15)

Using Cauchy’s Schwartz inequality, the interference term of (3.15) is written as

\[
E \left[ \sum_{q=0,q \neq k}^{N-1} \sqrt{E, \hat{x}_r[q]} \hat{g}_m[q] \hat{p}_m[k-q] \right]^2 \leq E \left[ E_0 \sum_{q=0,q \neq k}^{N-1} x_r[q] \right]^2 \sum_{q=0,q \neq k}^{N-1} \hat{p}_m[k-q] \sum_{q=0,q \neq k}^{N-1} \hat{g}_m[q] \left| \hat{p}_m(0) \right|^2
\]

Substituting this the SINR expression is written as

\[
\gamma_{m,\text{dest}}^{PN}[k] \geq \frac{E_0 \left[ E_0 \sum_{q=0,q \neq k}^{N-1} x_r[q] \right]^2 \sum_{q=0,q \neq k}^{N-1} \hat{p}_m[k-q] \sum_{q=0,q \neq k}^{N-1} \hat{g}_m[q] \left| \hat{p}_m(0) \right|^2}{E \left[ n_m[k] \right]^2}, \quad m = 1, 2
\]

(3.16)

Since \( E \left[ \hat{x}_r[k] \right]^2 = 1 \) and \( E \left[ \sum_{q=0,q \neq k}^{N-1} x_r[q] \right]^2 = 1 \), Equation (3.16) is simplified as

\[
\gamma_{m,\text{dest}}^{PN}[k] \geq \frac{E \left[ \hat{g}_m[k] \right]^2 \left| \hat{p}_m(0) \right|^2}{E \sum_{q=0,q \neq k}^{N-1} \hat{p}_m[k-q] \sum_{q=0,q \neq k}^{N-1} \hat{g}_m[q] + \sigma_m^2}, \quad m = 1, 2
\]

(3.17)

Assuming that the CPE term \( \hat{p}_m(0) \) during the time slot II has been compensated, the SINR expression can be further simplified as

\[
\gamma_{m,\text{dest}}^{PN}[k] \geq \frac{\left| \hat{g}_m[k] \right|^2}{\sum_{q=0,q \neq k}^{N-1} \left| \hat{p}_m[k-q] \right|^2 \sum_{q=0,q \neq k}^{N-1} \left| \hat{g}_m[q] \right|^2 + \frac{\sigma_m^2}{E}}, \quad m = 1, 2
\]

(3.18)
The SINR expression is written as

\[
\gamma_{m,det}^{PN}[k] \geq \frac{\mu}{3 \frac{(B_r + B_w) N T}} \sum_{q=0, q \neq k}^{N-1} \vert \tilde{g}_m[q] \vert^2 + \frac{\sigma_m^2}{E_r}, \quad m = 1, 2
\]

(3.19)

3.4. OUTAGE ANALYSIS

In this section, the effect of phase noise on the outage performance at the relay node \( R \) and destination node \( S_m \) are characterized. To derive an exact closed form expression for outage probability at relay node \( R \) is difficult, hence a tight upper bound and lower bound results are obtained.

3.4.1 Outage Analysis at Time Slot – 1

The upper bound outage probability at the relay node \( R \) is given by

\[
P_{out,relay}^{PN(Upper)}(\gamma) = \sum_{m=1}^{2} P_{out,\gamma_m}^{PN}(\gamma)
\]

(3.20)

where \( P_{out,\gamma_m}^{PN}(\gamma) \) represents the outage probability at the link between the source node \( S_m \) and relay node \( R \) in \( k^{th} \) subcarrier, \( \gamma = 2^{R_d} - 1 \) is the threshold SINR with data rate \( R_d = 1 \) bits/s/Hz and it is given by

\[
P_{out,\gamma_m}^{PN}(\gamma) = \Pr \left[ \gamma_{m,relay}^{PN}[k] < \gamma \right], \quad m = 1, 2
\]

(3.21)

Substituting for \( \gamma_{m,relay}^{PN}[k] \) from the Equation (3.14) in (3.21), the upper bound outage probability is expressed as
\[ P_{\text{out}, \text{relay}}^{\text{PN(Upper)}}(\gamma) = \sum_{m=1}^{2} \Pr \left[ \frac{\sum_{q=0}^{N-1} |\tilde{g}_m[q]|^2}{\pi (\beta_m + \beta_r) N T_s} \leq \gamma \right] \]

(3.22)

Let \( X_m = |\tilde{g}_m[k]|^2 \) is circularly symmetric complex Gaussian random variable with zero mean and unit variance, the random variable \( X_m \) is chi–square distributed with 2 degrees of freedom, \( Y_m = \sum_{q=0}^{N-1} |\tilde{g}_m[q]|^2 \) is chi-square distributed with \( 2(N-1) \) degrees of freedom, \( U_m = \frac{\pi (\beta_m + \beta_r) N T_s}{3} \) and \( V_m = \frac{\sigma_r^2}{E_m} = \frac{1}{\text{SNR}} \). Then, Equation (3.22) can be rewritten as

\[ P_{\text{out}, \text{relay}}^{\text{PN(Upper)}}(\gamma) = \sum_{m=1}^{2} \Pr \left[ \frac{X_m}{U_m Y_m + V_m} \leq \gamma \right] \]

(3.23)

The outage probability of \( X_m \) conditioned on \( Y_m \), Equation (3.23) can be written as

\[ P_{\text{out}, \text{relay}}^{\text{PN(Upper)}}(\gamma) = \sum_{m=1}^{2} \left[ \int_0^{N-1} \int_0^{\gamma (U_m Y_m + V_m)} f_{X_m}(x_m) f_{Y_m}(y_m) dx_m dy_m \right] \]

(3.24)

The Probability Density Function (PDF) of \( X_m \) and \( Y_m \) are expressed as

\[ f_{X_m}(x_m) = \exp(-x_m) \]

(3.25)

\[ f_{Y_m}(y_m) = \frac{1}{\Gamma(N-1) \left( \frac{1}{N-1} \right)^{N-2}} y^{N-2} \exp\left( \frac{-y}{\left( \frac{1}{N-1} \right)} \right) \]

(3.26)
Substituting Equation (3.25) and Equation (3.26) in Equation (3.24) and evaluating the integrals, the outage probability \( P_{out, \text{relay}}^{(\text{PN UPPER})} (\gamma) \) at relay node \( R \) during the time slot \( I \) is determined as

\[
P_{out, \text{relay}}^{(\text{PN UPPER})} (\gamma) = \sum_{m=1}^{2} 1 - \exp\left( -\gamma (U_m + V_m) \right)
\]

(3.27)

Substituting the parameter \( U_m \) and \( V_m \), it is written as

\[
P_{out, \text{relay}}^{(\text{PN UPPER})} (\gamma) = \sum_{m=1}^{2} 1 - \exp\left[ -\gamma \left( \frac{\pi \left( \beta_m + \beta_r \right) N T_s (\text{SNR}) + 3}{3\text{SNR}} \right) \right]
\]

(3.28)

The upper bound outage probability at the relay node \( R \) is determined as

\[
P_{out, \text{relay}}^{(\text{PN UPPER})} (\gamma) = 2 - \exp\left[ -\gamma \left( \frac{\pi \left( \beta_1 + \beta_r \right) N T_s (\text{SNR}) + 3}{3\text{SNR}} \right) \right]
\]

\[-\exp\left[ -\gamma \left( \frac{\pi \left( \beta_2 + \beta_r \right) N T_s (\text{SNR}) + 3}{3\text{SNR}} \right) \right]
\]

(3.29)

The lower bound outage probability at the relay node \( R \) is defined as

\[
P_{out, \text{relay}}^{(\text{PN LOWER})} (\gamma) = \Pr \left[ \min \left( \gamma_{1, \text{relay}}^{PN} [k], \gamma_{2, \text{relay}}^{PN} [k] \right) \leq \gamma \right]
\]

(3.30)

Now the equation (3.30) is written in terms of CDF of \( \gamma_{1, \text{relay}}^{PN} [k] \) and \( \gamma_{2, \text{relay}}^{PN} [k] \) as,

\[
P_{out, \text{relay}}^{(\text{PN LOWER})} (\gamma) = 1 - \left( 1 - F_{\gamma_1}^{R} (\gamma) \right) \left( 1 - F_{\gamma_2}^{R} (\gamma) \right)
\]

(3.31)

Since \( F_{\gamma_m}^{R} (\gamma) = P_{out, \gamma_m}^{PN} (\gamma), m = 1, 2 \), substitute Equation (3.21) in (3.31), the lower bound outage probability at relay node \( R \) can be determined as
\[ P_{out,relay}^{PN}(\gamma) = 1 - \exp\left(-\gamma\left(\sum_{m=1}^{2}(U_m + V_m)\right)\right) \]  

(3.32)

Substituting for \( U_m \) and \( V_m \), the lower bound outage probability at relay node \( R \) is given by

\[ P_{out,relay}^{PN}(\gamma) = 1 - \exp\left(-\gamma\left(\frac{\pi NT_s (\beta_1 + \beta_2 + 2\beta_r) + 2}{3 \cdot SNR}\right)\right) \]  

(3.33)

3.4.2 Outage Analysis at Time Slot – II

The outage probability at the link between the relay node \( R \) and source node \( S_m \) at \( k^{th} \) subcarrier in time slot II can be defined as

\[ P_{out,m}^{PN}(\gamma) = \Pr\left[ \gamma_{m,dest}^{PN} \leq \gamma \right] \]  

(3.34)

Substituting for \( \gamma_{m,dest}^{PN} \) from the Equation (3.19) in (3.34), it is written as

\[ P_{out,m}^{PN}(\gamma) = \Pr\left[ \frac{\left| \tilde{g}_m[k] \right|^2}{\frac{\pi (\beta_x + \beta_y) NT_s}{3} \sum_{q=0,q\neq k}^{K-1} \left| \tilde{g}_m[q] \right|^2 + \frac{\sigma_r^2}{E_r}} \leq \gamma \right], m = 1, 2 \]  

(3.35)

Equation (3.35) can be rewritten as,

\[ P_{out,m}^{PN}(\gamma) = \Pr\left[ \frac{X_m}{U_m Y_m + V_m} \leq \gamma \right], m = 1, 2 \]  

(3.36)

The outage probability of \( X_m \) conditioned on \( Y_m \) can be written as

\[ P_{out,m}^{PN}(\gamma) = \int_0^\gamma \int_0^{U_m Y_m + V_m} f_{x_m}(x_m) f_{y_m}(y_m) dx_m dy_m \]  

(3.37)
Substituting Equation (3.25) and Equation (3.26) in Equation (3.37) and then evaluating the integrals, the outage probability \( P_{out,m}^{PN}(\gamma) \) at destination node \( S_m \) during the time slot II is determined as

\[
P_{out,m}^{PN}(\gamma) = 1 - \exp\left(-\gamma(U_m + V_m)\right)
\]  

(3.38)

Substituting the parameters \( U_m \) and \( V_m \) in (3.38), the outage probability at the destination node \( S_m \) in time slot II is given by

\[
P_{out,m}^{PN}(\gamma) = 1 - \exp\left[-\gamma \left( \frac{\pi NT_x (\beta_r + \beta_m) SNR + 3}{3SNR} \right) \right]
\]  

(3.39)

3.4.3 End-to-End Outage Analysis

The overall upper bound end – to – end outage probability is expressed as

\[
P_{out,E \rightarrow E}^{PN(Upper)}(\gamma) = P_{out,relay}^{PN(Upper)}(\gamma) + \left[1 - P_{out,relay}^{PN(Upper)}(\gamma)\right] P_{out,\gamma_{out}(dest)}^{PN}(\gamma)
\]  

(3.40)

The overall lower bound end – to – end outage probability is expressed as

\[
P_{out,E \rightarrow E}^{PN(Lower)}(\gamma) = P_{out,relay}^{PN(Lower)}(\gamma) + \left[1 - P_{out,relay}^{PN(Lower)}(\gamma)\right] P_{out,\gamma_{out}(dest)}^{PN}(\gamma)
\]  

(3.41)

Substituting Equation (3.28) and Equation (3.39) in Equation (3.40), the upper bound end – to – end outage probability from source node \( S_1 \) to \( S_2 \) is determined as
\[ P_{\text{out},E-E(S_1-S_2)}^{PN (\text{Upper})}[k] = 1 - \exp \left( -\gamma \left( \frac{\pi (\beta_1 + 2\beta_r + \beta_2) NT_s SNR + 3}{3\text{SNR}} \right) \right) \]
\[ - \exp \left( -\gamma \left( \frac{\pi (2\beta_r + 2\beta_2) NT_s SNR - 3}{3\text{SNR}} \right) \right) \]
\[ + \exp \left( -\gamma \left( \frac{\pi (\beta_r + \beta_2) NT_s SNR + 3}{3\text{SNR}} \right) \right) \]  
\[ (3.42) \]

Substituting Equation (3.28) and Equation (3.39) in Equation (3.41), the lower bound end to end outage probability is determined as
\[ P_{\text{out},E-E(S_1-S_2)}^{PN (\text{Lower})}[k] = 1 - \exp \left( -\gamma \left( \frac{\pi (\beta_1 + 2\beta_r + \beta_2) NT_s SNR + 6}{3\text{SNR}} \right) \right) \]
\[ - \exp \left( -\gamma \left( \frac{\pi (2\beta_r + 2\beta_2) NT_s SNR + 6}{3\text{SNR}} \right) \right) \]
\[ + \exp \left( -\gamma \left( \frac{\pi (\beta_r + \beta_2) NT_s SNR - 3}{3\text{SNR}} \right) \right) \]  
\[ (3.43) \]

3.5 **AVERAGE BER ANALYSIS**

In this section, the effect of phase noise on the Bit Error Rate (BER) at the relay node \( R \) and at the destination nodes \( S_m \) in time slot I and II are characterized respectively. The tight upper and lower bound analytical expression for the average error probabilities at decode and forward relay node \( R \) employing PLNC concept and end to end average BER are derived.

3.5.1 **Average BER Analysis at Time Slot I**

The instantaneous upper and lower bound probability of error at the relay node \( R \) are expressed as,
\[ P_{e,\text{relay}}^{PN} < P_{\text{out},E-E(S_1-S_2)}^{PN (\text{Upper})} = \mathcal{O} \left( \sqrt{2 \gamma_1^{PN} [k]} \right) + \mathcal{O} \left( \sqrt{2 \gamma_2^{PN} [k]} \right) \]  
\[ (3.44) \]
\[
\Pr_{e, \text{relay}}^\text{PN} > \Pr_{e, \text{relay}}^\text{PN (Lower)} = Q\left(\sqrt{2 \min\left(\gamma_{1, \text{relay}}^\text{PN}[k], \gamma_{2, \text{relay}}^\text{PN}[k]\right)}\right) \tag{3.45}
\]

Hence, the average upper bound probability of error at the relay node \( R \) can be expressed as,
\[
\Pr_{e, \text{relay}}^\text{PN (Upper)} = \sum_{m=1}^{2} Q\left(\sqrt{2 \gamma_{m, \text{relay}}^\text{PN}[k]}\right) f_{\gamma_{m, \text{relay}}}(\gamma) d\gamma \tag{3.46}
\]
where \( f_{\gamma_{m, \text{relay}}}(\gamma) \) is the PDF of \( \gamma_{m, \text{relay}}^\text{PN}[k] \). It is determined by computing the derivative of CDF \( F_{\gamma_{m, \text{relay}}}(\gamma) \). From Equation the CDF of \( \gamma_{m, \text{relay}}^\text{PN}[k] \) is written as
\[
F_{\gamma_{m, \text{relay}}}(\gamma) = 1 - \exp\left[-\gamma\left(\frac{\pi N T_s (\beta_r + \beta_m) \text{SNR} + 3}{3 \text{SNR}}\right)\right], \quad m = 1, 2 \tag{3.47}
\]
Now the PDF is determined as,
\[
f_{\gamma_{m, \text{relay}}}(\gamma) = \left(\frac{\pi N T_s (\beta_r + \beta_m) \text{SNR} + 3}{3 \text{SNR}}\right) \exp\left(-\gamma\left(\frac{\pi N T_s (\beta_r + \beta_m) \text{SNR} + 3}{3 \text{SNR}}\right)\right), \quad m = 1, 2 \tag{3.48}
\]
Substituting Equation (3.48) in (3.46), the upper bound average probability of error at relay node \( R \) is expressed as,
\[
\Pr_{e, \text{relay}}^\text{PN (Upper)} = \sum_{m=1}^{2} (U_m + V_m) \int_{0}^{\gamma} Q\left(\sqrt{2 \gamma_{m, \text{relay}}^\text{PN}[k]\right)} \exp\left(-\gamma (U_m + V_m)\right) d\gamma
\]
\[
\tag{3.49}
\]
Using the integral Q-function, \( \int_{0}^{\gamma} \exp(-x^2/2)Q(x/\sigma)dx = \frac{1}{2} \left[1 - \frac{1}{\sqrt{\sigma^2+1}}\right]\) the average upper bound probability of error at the relay node \( R \) is determined as
\[
\Pr_{e, \text{relay}}^\text{PN (Upper)} = \sum_{m=1}^{2} \frac{1}{2} \left[1 - \sqrt{\frac{3 \text{SNR}}{\pi (\beta_m + \beta_r) N T_s + 3 \text{SNR} + 3}}\right] \tag{3.50}
\]
The upper bound probability of error at the relay node $R$ is further simplified as

$$
P_{e,\text{relay}}^{\text{PN(Upper)}} = 1 - \frac{\sqrt{3\text{SNR}}}{2} \sum_{m=1}^{2} \frac{1}{\sqrt{\pi (\beta_m + \beta_r) NT_s + 3\text{SNR} + 3}}
$$

(3.51)

The average lower bound probability of error for the relay node $R$ can be expressed as,

$$
P_{e,\text{relay}}^{\text{PN(Lower)}} = \frac{\gamma}{0} \mathcal{O}(\sqrt{2\gamma_{\text{min}}^{\text{PN}}[k]}) f_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma) d\gamma
$$

(3.52)

where $\gamma_{\text{min}}^{\text{PN}}[k] = \min(\gamma_{1,\text{relay}}^{\text{PN}}[k], \gamma_{2,\text{relay}}^{\text{PN}}[k])$, $f_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$ is the PDF of $\gamma_{\text{min}}^{\text{PN}}[k]$. It is determined by computing the derivative of CDF function $F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$.

The CDF $F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$ is expressed as,

$$
F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma) = 1 - \left[1 - F_{\gamma_{1,\text{relay}}^{\text{PN}}}^{R}(\gamma)\right] \left[1 - F_{\gamma_{2,\text{relay}}^{\text{PN}}}^{R}(\gamma)\right]
$$

(3.53)

Substituting Equation (3.47) and simplifying the CDF $F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$ is determined as

$$
F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma) = 1 - \exp \left[-\gamma \left(\frac{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)}{3} + \frac{2}{\text{SNR}}\right)\right]
$$

(3.54)

By computing derivative of $F_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$, the PDF function $f_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma)$is written as,

$$
f_{\gamma_{\text{min}}^{\text{PN}}}^{R}(\gamma) = \left(\frac{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)}{3} + \frac{2}{\text{SNR}}\right) \exp \left[-\gamma \left(\frac{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)}{3} + \frac{2}{\text{SNR}}\right)\right]
$$

(3.55)

Substituting (3.55) in (3.52), the lower bound average probability of error at relay node $R$ is expressed as
\[
\bar{P}_{e, \text{relay}}^{PN(\text{Lower})} = \left( \frac{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)}{3} + \frac{2}{SNR} \right) \times \\
\frac{1}{\sqrt{2\gamma_{\text{min}}[k]}} \exp \left[ -\gamma \left( \frac{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)}{3} + \frac{2}{SNR} \right) \right] d\gamma
\]

(3.56)

After simplification, the average lower bound probability of error at the relay node \( R \) is determined as
\[
\bar{P}_{e, \text{relay}}^{PN(\text{Lower})} = \frac{1}{2} \left[ 1 - \sqrt{n \left( \frac{3SNR}{\pi NT_s (\beta_1 + \beta_2 + 2 \beta_r)SNR + 3SNR + 12} \right)} \right]
\]

(3.57)

3.5.2 Average BER Analysis at Time Slot II

The instantaneous probability of error at the destination node \( S_m \) in time slot II is expressed as
\[
P_{e, s, \text{dest}}^{PN} = Q\left( \sqrt{2\gamma_{m, \text{dest}}[k]} \right)
\]

(3.58)

The average probability of error at the destination node \( S_m \) can be expressed as
\[
\bar{P}_{e, s, \text{dest}}^{PN} = \int_0^\infty Q\left( \sqrt{2\gamma_{m, \text{dest}}[k]} \right) f_{\gamma_{m, \text{dest}}} (\gamma) d\gamma
\]

(3.59)

where \( f_{\gamma_{m, \text{dest}}} (\gamma) \) is the PDF of \( \gamma_{m, \text{dest}}[k] \) and it is determined by computing the derivative of the CDF \( F_{\gamma_{m, \text{dest}}} (\gamma) \). By computing the derivative of \( F_{\gamma_{m, \text{dest}}} (\gamma) \) the PDF \( f_{\gamma_{m, \text{dest}}} (\gamma) \) is written as
\[
f_{\gamma_{m, \text{dest}}} (\gamma) = \left( \frac{\pi NT_s (\beta_r + \beta_m) + 3}{3SNR} \right) \exp \left[ -\gamma \left( \frac{\pi NT_s (\beta_r + \beta_m) + 3}{3SNR} \right) \right], \quad m = 1, 2
\]

(3.60)
Substituting Equation (3.60) in Equation (3.59) the average probability of error at the destination node \( S_m \) is expressed as

\[
\overline{P}_{PN_{e,s_{\text{dest}}}}^{\text{PN}} = \left( \frac{\pi N T_s (\beta_r + \beta_m) \text{SNR} + 3}{3 \text{SNR}} \right) \times \\
\int_0^\infty Q \left( \sqrt{2 \gamma_{m,\text{dest}}^{PN} k} \right) \exp \left( -\gamma \left( \frac{\pi N T_s (\beta_r + \beta_m) \text{SNR} + 3}{3 \text{SNR}} \right) \right) d\gamma, \quad m = 1, 2
\]  

(3.61)

Using the integral Q-function, \( \int \exp \left( -x^2 / 2 \right) Q(x/\sigma) dx = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{\sigma^2 + 1}} \right] \), the average error probability at the destination node \( S_m \) in time slot II is determined as

\[
\overline{P}_{PN_{e,s_{\text{dest}}}}^{\text{PN}} = \frac{1}{2} \left[ 1 - \sqrt{\frac{3 \text{SNR}}{\pi N T_s (\beta_r + \beta_m) \text{SNR} + \text{SNR} + 6}} \right]
\]  

(3.62)

### 3.5.3 End-To-End Average BER Analysis

In this subsection, the upper bound end-to-end instantaneous probability of error at \( S_m \) is expressed as,

\[
P_{e, E \to E_{1/2}}^{\text{PN}(\text{Upper})} = P_{e, \text{relay}}^{\text{PN}(\text{Upper})} \left( 1 - P_{e,s_{\text{dest}}}^{\text{PN}(\text{Upper})} \right) + \left( 1 - P_{e, \text{relay}}^{\text{PN}(\text{Upper})} \right) P_{e,s_{\text{dest}}}^{\text{PN}}, \quad m = 1, 2
\]  

(3.63)

Averaging \( P_{e, E \to E_{1/2}}^{\text{PN}} \) and \( P_{e,s_{\text{dest}}}^{\text{PN}} \), the end-to-end instantaneous BER \( P_{e, E \to E}^{\text{PN}} \) is given by

\[
P_{e, E \to E}^{\text{PN}(\text{Upper})} = \frac{1}{2} \sum_{m=1}^{2} P_{e, E \to E_{1/2}}^{\text{PN}(\text{Upper})} = P_{e, \text{relay}}^{\text{PN}(\text{Upper})} + \frac{1}{2} \left( P_{e,s_{\text{dest}}}^{\text{PN}(\text{Upper})} + P_{e,s_{\text{dest}}}^{\text{PN}(\text{Upper})} \right) P_{e, \text{relay}}^{\text{PN}(\text{Upper})} \left( P_{e,s_{\text{dest}}}^{\text{PN}} + P_{e,s_{\text{dest}}}^{\text{PN}} \right)
\]  

(3.64)
The average upper bound end-to-end probability of error is given by,

\[ P_{EtoE}^{P1\text{(Upper)}} = E\left[ P_{e,\text{EtoE}}^{P1}\right] = E\left[ P_{e,\text{relay}}^{P1}\right] + \frac{1}{2} E\left[ P_{e,\text{s,kdest}}^{P1} + P_{e,\text{EtoE}}^{P1}\right] \\
- E\left[ P_{e,\text{relay}}^{P1} \left( P_{e,\text{s,kdest}}^{P1} - P_{e,\text{s,kdest}}^{P1}\right)\right] \]

(3.65)

where \( E\left[ P_{e,\text{relay}}^{P1}\right] \) and \( E\left[ P_{e,\text{s,kdest}}^{P1} + P_{e,\text{s,kdest}}^{P1}\right] \) are easily solved. Third term can be solved as

\[ E\left[ P_{e,\text{relay}}^{P1} P_{e,\text{s,kdest}}^{P1} \right] = E\left[ \{Q(\sqrt{2\gamma_1}) + Q(\sqrt{2\gamma_2})\} \{Q(\sqrt{2\gamma_3}) - Q(\sqrt{2\gamma_4})\} \right] \\
- E\left[ Q(\sqrt{2\gamma_1}) E\left[ Q(\sqrt{2\gamma_4}) \right] + E\left[ Q(\sqrt{2\gamma_2}) E\left[ Q(\sqrt{2\gamma_3}) \right] \right.\right] \\
+ E\left[ Q(\sqrt{2\gamma_1}) Q(\sqrt{2\gamma_3}) \right] + E\left[ Q(\sqrt{2\gamma_2}) Q(\sqrt{2\gamma_4}) \right] \]

(3.66)

where \( \gamma_1 \) and \( \gamma_2 \) are the instantaneous SINR combined at the relay node \( R \), \( \gamma_3 \) and \( \gamma_4 \) are the instantaneous SINR at destination node \( S_1 \) and \( S_2 \) respectively. The characteristics of the instantaneous SINR \( \gamma_1, \gamma_3 \) are identical and \( \gamma_2, \gamma_4 \) are identical. Hence the average probability of error for the first and second term of Equation (3.66) is determined as

\[ E\left( Q(\sqrt{2\gamma_m}) \right) = \frac{1}{2} \left[ 1 - \sqrt{\frac{3\text{SNR}}{\pi NT_i (\beta_r + \beta_m) \text{SNR} + \text{SNR} + 6}} \right], \quad m = 1, 2 \]

(3.67)

The average probability of error for the third and fourth term of Equation (3.66) (Min Chul & II-Min Kim 2010) is determined as
\[
E\left[ \mathcal{O}\left(\sqrt{2\gamma_2}\right) \mathcal{O}\left(\sqrt{2\gamma_4}\right) \right] = \frac{1}{2} \left[ \frac{\phi_1(\overline{\gamma}_2, \overline{\gamma}_4)}{\pi} \frac{\beta_1}{\pi} \left( \frac{\pi}{2} + \tan^{-1} \varepsilon_1 \right) \right] \\
+ \frac{1}{2} \left[ \frac{\phi_2(\overline{\gamma}_2, \overline{\gamma}_4)}{\pi} \frac{\beta_2}{\pi} \left( \frac{\pi}{2} + \tan^{-1} \varepsilon_2 \right) \right]
\]

(3.68)

where
\[
\phi_1(\overline{\gamma}_2, \overline{\gamma}_4) = \frac{\pi}{2} - \tan^{-1}\left( \frac{\overline{\gamma}_4}{\overline{\gamma}_2} \right), \quad \phi_2(\overline{\gamma}_2, \overline{\gamma}_4) = \tan^{-1}\left( \frac{\overline{\gamma}_4}{\overline{\gamma}_2} \right),
\]
\[
\beta_1 = \frac{\gamma_2}{\sqrt{2 + \gamma_2}} \text{sgn}\left( \phi_1(\overline{\gamma}_2, \overline{\gamma}_4) \right), \quad \beta_2 = \frac{\gamma_4}{\sqrt{2 + \gamma_4}} \text{sgn}\left( \phi_2(\overline{\gamma}_2, \overline{\gamma}_4) \right),
\]
\[
\varepsilon_1 = -\beta_1 \cot\left( \phi_1(\overline{\gamma}_2, \overline{\gamma}_4) \right) \quad \text{and} \quad \varepsilon_2 = -\beta_2 \cot\left( \phi_2(\overline{\gamma}_2, \overline{\gamma}_4) \right)
\]

\[
E\left[ \mathcal{O}\left(\sqrt{2\gamma_1}\right) \mathcal{O}\left(\sqrt{2\gamma_3}\right) \right] = \frac{1}{2} \left[ \frac{\phi_1(\overline{\gamma}_1, \overline{\gamma}_3)}{\pi} \frac{\beta_3}{\pi} \left( \frac{\pi}{2} + \tan^{-1} \varepsilon_3 \right) \right] \\
+ \frac{1}{2} \left[ \frac{\phi_2(\overline{\gamma}_1, \overline{\gamma}_3)}{\pi} \frac{\beta_4}{\pi} \left( \frac{\pi}{2} + \tan^{-1} \varepsilon_4 \right) \right]
\]

(3.69)

where
\[
\phi_1(\overline{\gamma}_1, \overline{\gamma}_3) = \frac{\pi}{2} - \tan^{-1}\left( \frac{\overline{\gamma}_3}{\overline{\gamma}_1} \right), \quad \phi_2(\overline{\gamma}_1, \overline{\gamma}_3) = \tan^{-1}\left( \frac{\overline{\gamma}_3}{\overline{\gamma}_1} \right),
\]
\[
\beta_3 = \frac{\gamma_1}{\sqrt{2 + \gamma_1}} \text{sgn}\left( \phi_1(\overline{\gamma}_1, \overline{\gamma}_3) \right), \quad \beta_4 = \frac{\gamma_3}{\sqrt{2 + \gamma_3}} \text{sgn}\left( \phi_2(\overline{\gamma}_1, \overline{\gamma}_3) \right),
\]
\[
\varepsilon_3 = -\beta_3 \cot\left( \phi_1(\overline{\gamma}_1, \overline{\gamma}_3) \right) \quad \text{and} \quad \varepsilon_4 = -\beta_4 \cot\left( \phi_2(\overline{\gamma}_1, \overline{\gamma}_3) \right)
\]

Substituting the Equation (3.67), (3.68) and (3.69) in Equation (3.66), it can be simplified as
\[
E \left( P_{e, Relay}^{PN} \left| P_{e, r-s_a}^{PN} \right. \right) = \prod_{m=1}^{2} \left[ 1 - \frac{3SNR}{\sqrt{\pi NT_s (\beta_r + \beta_m) SNR + SNR + 6}} \right]
+ \frac{1}{2} \left[ \frac{\phi_1 (\gamma_2, \gamma_4)}{\pi} - \frac{\beta_1 \left( \frac{\pi}{2} + \tan^{-1} \mathcal{E}_3 \right)}{\pi} + \frac{1}{2} \left[ \frac{\phi_3 (\gamma_2, \gamma_4)}{\pi} - \frac{\beta_3 \left( \frac{\pi}{2} + \tan^{-1} \mathcal{E}_2 \right)}{\pi} \right] \right]
+ \frac{1}{2} \left[ \frac{\phi_1 (\gamma_1, \gamma_3)}{\pi} - \frac{\beta_1 \left( \frac{\pi}{2} + \tan^{-1} \mathcal{E}_3 \right)}{\pi} + \frac{1}{2} \left[ \frac{\phi_3 (\gamma_1, \gamma_3)}{\pi} - \frac{\beta_3 \left( \frac{\pi}{2} + \tan^{-1} \mathcal{E}_4 \right)}{\pi} \right] \right]
\]
(3.70)

Let \( E \left( P_{e, Relay}^{PN} \left| P_{e, r-s_a}^{PN} \right. \right) = \Xi (\gamma_1, \gamma_2, \gamma_3, \gamma_4) \). By substituting Equation (3.51), (3.62) and (3.70) in (3.65) the end to end average probability of error is determined as

\[
P_{E \rightarrow E}^{PN (Upper)} = \frac{1}{2} \left[ 1 - \frac{3SNR}{\sqrt{\pi NT_s (\beta_r + \beta_m) SNR + SNR + 6}} \right] - \Xi (\gamma_1, \gamma_2, \gamma_3, \gamma_4)
\]
(3.71)

Using Equation (3.55), (3.62) and (3.70), the average lower bound end-to-end probability of error is given by,

\[
P_{E \rightarrow E}^{PN (Upper)} = \frac{1}{2} \left[ 1 - \sqrt{\frac{3SNR}{\pi NT_s (\beta_r + \beta_m) SNR + SNR + 6}} \right] - \Xi (\gamma_1, \gamma_2, \gamma_3, \gamma_4)
\]
(3.72)

### 3.6 RESULTS AND DISCUSSION

In this section, the outage and BER performances of the OFDM based bidirectional relay network in the presence of phase noise are analyzed.
The list of parameters which are used in the numerical analysis is given in Table 3.1.

**Table 3.1  Numerical parameters of the bidirectional relay network with phase noise**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>$T_s$</td>
<td>OFDM Symbol duration</td>
<td>50 ns</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phase noise Bandwidth</td>
<td>100,500,1000,1500,3000 (Hz)</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Data rate</td>
<td>1 bits/s/Hz</td>
</tr>
</tbody>
</table>

Figure 3.2 shows the Outage performance of OFDM based bidirectional relay network. Assume that the network has perfect synchronization. At 20 dB SNR, the upper bound and lower bound outage probability are 0.04 and 0.01 respectively. Although the exact outage probability at relay node is not measured, the result shows that the deviation between the lower and upper bound outage probability is very less. It proves that the derived outage probability of the bidirectional relay network meets the actual measurements.
Figure 3.2 Outage performance of OFDM based bidirectional relay network

Figure 3.3 shows the upper bound outage probability of OFDM based bidirectional relay network at the relay node in the presence of phase noise. It is observed that when the phase noise bandwidth increases from 500 Hz to 1000 Hz, the outage probability also increases from 0.01 to 0.03 at 25 dB SNR and further increase in the phase noise bandwidth, the system reaches deep fade.
Figure 3.3  Upper bound outage performance at the relay node in OFDM based bidirectional relay network in the presence of phase noise

Figure 3.4 shows the lower bound outage probability of OFDM based bidirectional relay network at the relay node in the presence of phase noise. It is observed that when the phase noise bandwidth increases from 100 Hz to 500 Hz, the outage probability also increases from 0.006 to 0.003 at 25 dB SNR and further increase in the phase noise bandwidth, the system performance becomes worse.
Figure 3.4 Lower bound outage performance at the relay node in OFDM based bidirectional relay network in the presence of phase noise

Figure 3.5 shows the end-to-end upper bound outage performance of OFDM based bidirectional relay network in the presence of phase noise. The phase noise is present in relay node and destination node transceiver in two time slots. The combined effect of phase noise in the bidirectional relay network drastically increases the outage of the network. It is observed that when the phase noise bandwidth increases from 100 Hz to 500 Hz, the outage probability increases to 0.01 from 0.07 at 25 dB SNR. Comparing with Figure 3.3 and Figure 3.4, it is noted that the outage is 0.067 more in this case at 500 Hz phase noise bandwidth. It is evident that the combined effect of phase noise in end-to-end outage performance is dramatically increased.
Figure 3.5 End-to-end upper bound outage performance of OFDM based bidirectional relay network in the presence of phase noise

Figure 3.6 shows the end-to-end lower bound outage performance of OFDM based bidirectional relay network in the presence of phase noise. The phase noise is present in relay node and destination node in two time slots. Comparing with Figure 3.5, it is noted that the outage probability in lower bound end-to-end is lower. At 25 dB SNR, when the phase noise bandwidth increases from 100 Hz to 500 Hz, the outage probability increases from 0.005 to 0.03.
Figure 3.6 End-to-end lower bound outage performance of OFDM based bidirectional relay network in the presence of phase noise

Figure 3.7 shows the average BER performance of OFDM based bidirectional relay network. Assume that the network has the perfect synchronization. It is observed that two bounds for relay node are tight. At 25 dB SNR, the upper bound and lower bound average probability of error are 0.001 and 0.004 respectively. Although the exact average BER performance is not determined at relay node, the deviation between the lower and upper bound average BER is very less. It proves that the derived average BER of the bidirectional relay network meets the actual BER derivation at relay node.
Figure 3.7 Upper and lower bound average BER performance of OFDM based bidirectional relay network

Figure 3.8 shows the upper bound average BER performance at the relay node in the presence of phase noise. The phase noise is added at the RF front end of the source and the relay receiver. It is observed that when the phase noise bandwidth increases from 100 Hz to 100 Hz, the average BER also increases from 0.003 to 0.005 at 25 dB SNR and further increase in the phase noise bandwidth up to 3000 Hz, the relay node not exceed the target average BER value 0.01. It conclude that the effect of the phase noise in the average BER performance at relay is less.
Figure 3.8 Upper bound average BER performance at Relay node in OFDM based bidirectional relay network in the presence of the phase noise

Figure 3.9 shows the lower bound average BER performance at the relay node in the presence of phase noise. The phase noise is added at the RF front end of the source and the relay receiver. Comparing with Figure 3.8, it is observed that the lower bound average BER is less than the upper bound. At 25 dB SNR, when the phase noise bandwidth increases from 100 Hz to 500 Hz, the average BER increases from 0.002 to .003 and further increase in the phase noise bandwidth up to 3000 Hz, the relay node not exceed the target average BER.
Figure 3.9 Lower bound average BER performance at relay node in OFDM based bidirectional relay network in the presence of the phase noise

Figure 3.10 shows the end-to-end average BER performance in the presence of phase noise. The phase noise occurs at the RF front end of the source transmitter, the relay receiver and transmitter and the destination receiver. Comparing with Figure 3.9, it is evident that the combined effect of the phase noise in end-to-end average BER is higher than the relay average BER. At 25 dB SNR, when the phase noise bandwidth increases from 100 Hz to 500 Hz, the average BER increases from 0.01 to 0.03 which indicates the network exceed the target average BER for phase noise bandwidth 500 Hz and above.
3.7 SUMMARY

In this thesis, PLNC and OFDM based bidirectional relay network is modeled in the presence of phase noise. The analytical expressions for SINR, upper and lower bound outage probabilities and average probability of errors are derived at the relay node in time slot I and destination nodes in time slot II. The impact of phase noise on OFDM based bidirectional relay network is extensively studied through the numerical results. It is observed that the presence of phase noise causes severe performance degradation in the OFDM based bidirectional relay system. At the phase noise bandwidth of 500 Hz, the minimum per subcarrier SNR requirement for the end-to-end outage probability of 0.01 is 25dB compared to 23dB in ideal case for 64 subcarrier based bidirectional relay network at the bandwidth of 20 MHz. At the phase noise bandwidth of 500 Hz, the minimum per subcarrier SNR requirement for the end-to-end average BER of $10^{-2}$ is 28dB compared to 25dB in ideal case.