CHAPTER 0

INTRODUCTION

1. INTRODUCTION :-

The idea of continuity is not only important to calculus, it can rather be considered as one of the central notions in Mathematics. It was towards the close of eighteenth century that a clear picture of continuity of a function emerged. It was Louis Arbogast who got the credit through one of his memoir in 1791. Bernard Bolzano was the first mathematician who discussed the concept of continuity in the modern sense. Cauchy in 1821 also defined continuity in the way similar to Bernard Bolzano. It was Louis Arbogast who in this prize winning memoir singled out intermediate value property of continuous functions, that plays an important role in calculus.

In 1922, Blumberg [Tans. Amer. Math. Soc. 24( 1922) ] first introduced the notion of almost continuity under the name “densely approaching ” and was studied intensively by Ptak [ Bull. Soc. Math. France 86 (1958)] under the name “near continuity ”. Almost continuous functions have been used in literature for different type of non-continuous functions.

Functions and of course continuous functions stand among the most important and most researched points in whole of the mathematical science. Many different form of continuous functions have been over that years. Continuity have played a major role in the topological spaces, bitopological spaces, ordered topological spaces, fuzzy topological spaces, fuzzy bitopological spaces etc.
2. CONTINUITY IN BITOPOLOGICAL SPACES

Our objective in this work will be a study of some weaker forms of continuous functions in bitopological spaces. The literature on bitopological spaces has been growing incessantly. Within a short span of nineteen years, since its very emergence in 1960, up to this bright day of 2009. In 1963, the subject of bitopological spaces a brain-child of Kelly [8] has become a cynosure for many a mathematician of many a country and there after a large number of papers have been done to generalize the topological concepts to bitopological setting. Strictly speaking, there has been no attempt to generalized continuity to bitopological setting.

In 1967, Pervin [14] has been the first to speak of continuity in bitopological spaces. According to him, a mapping from one bitopological space to another is continuous if it is continuous w. r. to first topologies and w. r. to second topologies, taken for both spaces. Some authors call such a mapping pairwise continuous, a term we too shall adhere to in our thesis whenever we have on occasion to speak of such mappings Singal and Singal [24] appear to be the first to talk of a pairwise open mapping, a pairwise closed mapping and a pairwise homeomorphism u. s. c. and l.s.c. mapping on a bitopological spaces.

In 1981, Bose [2] introduced the concept of semi open sets, semi continuity and semi open mappings in bitopological spaces, and study the conditions under which the various properties. In 1981, Bose and Sinha [3] introduced almost open, almost closed, $\theta$-continuous and almost quasi compact mappings in bitopological spaces and have studied their properties.

In 1982, Bose and Sinha [4] introduced almost continuous mapping and weakly continuous mapping in bitopological spaces. It has been shown that continuity implies almost continuity and almost
continuity implies weak continty. But the converses are not always true. Bose and Sinha investigated some other important properties of these two mappings in bitopological spaces.

In 1991, M. Jelic [6] introduced and study three different variations of pairwise continuity i.e. pairwise LC-continuous, strongly pairwise LC-continuous, pairwise LC-irresolute. All three notions are defined by using the concept of an (i, j) locally closed sets.

In 1991, Khedr and Alshibani [9] is to define and study super continuous mappings and other forms of continuity such as strong continuity, perfect continuity and complete continuity in bitopological spaces and investigate the relations between these kinds of continuity and their effects on some kinds of spaces.

In 1992, Kar and Bhattacharya [7] introduced the concepts of preopen sets, precontinuity and preopen mappings in bitopological spaces. The conditions under which the various properties enjoyed by the above concepts in single topological space can be generalized into a bitopological space are investigated.

In 1992, Khedr and AL. Areefi [10] generalized the notion of semi-preopen sets to bitopological spaces and define semi-precontinuity in bitopological spaces and obtain many properties of pairwise precontinuous functions and pairwise sp-continuous functions.

In 1993, Sampath Kumar [19] introduced the concepts of pairwise $\alpha$-open, $\alpha$-closed and $\alpha$-irresolute functions in bitopological spaces and discuss some of the basic properties of them. Several examples are provided to illustrate the behaviour of these new classes of functions.

In 1994, Sampath Kumar [20] gives the idea of A - sets and A - continuity has been extended to the bitopological situation and
consequently, it gives rise to the notion of \((i, j)\) \(A\)-sets and pairwise \(A\)-continuous functions.

In 1995, Sampath Kumar [21] introduced the idea of pairwise faintly continuous functions by using \((i, j)\) \(\theta\)-open sets. Several basic properties of pairwise faintly continuity is obtained.

In 1996, Sampath Kumar [22] introduced the concepts of \(\beta\)-open (\(\beta\)-closed) sets, \(\beta\)-continuity, \(\beta\)-open (\(\beta\)-closed) and \(\beta\)-irresolute mappings for bitopological spaces. Their relations with pairwise \(\alpha\)-continuous [5], pairwise semi continuous [2] and pairwise precontinuous [10] mappings are investigated in detail.

In 1997, Sampath Kumar [23] introduced the concepts of \((\tau_i, \tau_j)\) weak \(\beta\)-sets and pairwise weak \(\beta\)-continuity in bitopological spaces and a decomposition of pairwise continuity is obtained.

In 1998, Nasef and Noiri [12] introduced the concept of feebly open sets and feebly continuity in bitopological spaces and generalized some properties.


In 2007, Noiri and Popa [13] introduced the concept of weakly precontinuous functions in bitopological spaces as a generalization of precontinuous functions and obtain several characterizations and some properties of weakly precontinuous functions.

In 2007, Khedr and Noiri [11] introduced the notion of \((i, j)\)-almost \(s\)-continuous functions and some other forms of continuity in bitopological spaces and their properties. Also they investigated the relations between
several kinds of continuity and their effects on some kinds of spaces.

In 2008, Rajesh [16] introduced the various types of g-continuous functions in bitopological space by using bitopological g-closed sets and study their properties. In 2008, Ravi, Lellis and Ekici [18] introduced a new type of sets called (1, 2)*-A set, (1, 2)*-t set, (1, 2)*-B set, (1, 2)*-h set and (1, 2)*-C set and a new class of mappings called (1, 2)*-A continuous, (1, 2)*-B continuous and (1, 2)*-C continuous. Ravi, Lellis and Ekici [17] obtained several characterizations of this class and study its bitopological properties and investigate the relationships with other mappings like (1, 2)*-α continuous.

3. PRELIMINARIES :-

Throughout the present thesis the spaces X, Y, and Z always represent bitopological spaces \((X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2)\) and \((Z, \eta_1, \eta_2)\) on which no separation axioms are assumed unless explicitly mentioned and the indices \(i\) and \(j\) take value \(i, j = (1, 2)\) and \(i \neq j\). By \(\tau_i\)-Int\((A)\) and \(\tau_i\)-Cl\((A)\), we shall mean respectively the interior and the closure of subset \(A \subset X\) with respect to the topology \(\tau_i\).

**Definition [20].** A function \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is said to be **pairwise A - continuous** if for each \(\sigma_i\) - open set \(V \subset Y\), \(f^{-1}\(V\)\) is an \((i, j)\) A - set in X.

**Definition [4].** Let \(f : (X, \tau_i, \tau_2) \rightarrow (Y, \tau'_1, \tau'_2)\) be a mapping. Then \(f\) is said to be **\(\tau_i\tau'_1\) almost continuous** w. r. to \(\tau_2\) at a point \(x \in X\) if and only if for every \(\tau'_1\)-neighbourhood \(M\) of \(f(\ x)\) there exists a \(\tau_1\) - neighbourhood \(N\) of \(x\) such that \(f(\ N) \subset \tau'_1\text{Int}(\ tau'_2\text{Cl}(\ M))\).

**Definition [4].** Let \(f\) be a mapping from \((X, \tau_i, \tau_2)\) to \((Y, \tau'_1, \tau'_2)\). Then \(f\) is said to be **\(\tau_i\tau'_1\) weakly continuous** w. r. to \(\tau_2\) if for each point \(x \in X\)
and each $\tau'_1$ neighbourhood $V$ of $f(x)$ there exists a $\tau_1$ neighbourhood $U$ of $x$ such that $f(U) \subseteq \tau'_2 \text{Cl}(V)$.

**Definition [3].** A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \tau'_1, \tau'_2)$ is said to be $\tau_1\tau'_1\theta$-continuous w. r. to $\tau_2$ if for each $x \in X$ and each $\tau'_1$ neighbourhood $U$ of $f(x)$ there exists a $\tau_1$ neighbourhood $V$ of $x$ such that $f(\tau_2 \text{Cl}(V)) \subseteq \tau'_2 \text{Cl}(U)$.

**Definition [2].** A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise semi-continuous if for each $\sigma_1$-open set $V \subseteq Y$, $f^{-1}(V)$ is an $(i, j)$ semi-open in $X$.

**Definition [20].** A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise weak* continuous if for each $\sigma_i$-open set $V$ in $Y$, $f^{-1}((i, j) \text{-Fr}(V))$ is $\tau_i$-closed in $X$ where $(i, j) \text{Fr}(V) = \sigma_j \text{-Cl} \setminus \sigma_i \text{-Int}(V)$. \ denotes the complement.

**Definition [20].** A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise locally weak* continuous if and only if there is an open basis $B$ for the topology $\sigma_i$ on $Y$ such that $f^{-1}((i, j) \text{-Fr}(V))$ is $\tau_i$-closed in $X$ for each $V$ in $B$.

**Definition [22].** A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is said to be pairwise $\beta$-continuous if $f^{-1}(V)$ is $(j, i)\beta$-open set in $X$ for each $\eta_i$-open set $V$ of $Y$.

**Definition [10].** A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is said to be pairwise precontinuous if the inverse image of each $\eta_i$-open set of $Y$ is $(i, j)$ preopen set in $X$.

**Definition [14].** A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ is said to be pairwise continuous if the inverse image of each $\eta_i$-open set of $Y$ is a $\tau_i$-open set in $X$. 
Definition [21]. A function \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **pairwise faintly continuous** if for each \( x \in X \) and each \((i, j)\) \(\theta\)-open set \(V\) containing \(f(x)\), there exists a \(\tau_i\)-open set \(U\) containing \(x\) such that \(f(U) \subseteq V\).

Definition [10]. A function \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **pairwise sp- continuous** if the inverse image of each \(\sigma_i\)-open set of \(Y\) is an \((i, j)\) semi-preopen in \(X\).

Definition [5]. A function \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **pairwise \(\alpha\)- continuous** if the inverse image of each \(\sigma_i\)-open set of \(Y\) is an \((i, j)\) \(\alpha\)-set in \(X\).

Definition [6]. A mapping \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is called **\(p\)-LC continuous** if \(f^{-1}(V) \in (i, j)\) LC(\(X\)), for every \(\sigma_i\)-open set \(V\) in \(Y\).

Definition [6]. A mapping \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **\(pS\)-almost continuous** if \(f^{-1}(V) \in \tau_i\), for each \((i, j)\) regular open set \(V\) in \(Y\).

Definition [13]. A function \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **pairwise \((i, j)\) weakly precontinuous** if for each \(x \in X\) and each \(\sigma_i\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists an \((i, j)\) preopen set \(U\) containing \(x\) such that \(f(U) \subseteq \sigma_j\)-Cl(\(V\)).

Definition [13]. A function \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **(i, j)-weakly* quasicontinuous** if for every \(\sigma_i\)-open set \(V\) of \(Y\), \(f^{-1}(j\text{ Cl}(V) - V)\) is biclosed in \(X\).

Definition [1]. A mapping \( f : ( X, \tau_1, \tau_2) \to ( Y, \sigma_1, \sigma_2) \) is said to be **(i, j)-\(\delta\) continuous** if for each \(x \in X\) and each \(i\)-neighbourhood \(V\) of \(f(x)\) there exists an \(i\)-neighbourhood \(U\) of \(x\) such that \(f(i\text{-Int}(j\text{-Cl}(U))) \subseteq i\text{-Int}(j\text{-Cl}(V))\).
**Definition [9].** A mapping \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is said to be (i, j)-completely continuous if \( f^{-1}(V) \) is an (i, j) regular open in \( X \) for each \( i \)-open subset \( V \) of \( Y \).

**Definition [9].** A mapping \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is said to be (i, j)-perfectly continuous if \( f^{-1}(V) \) is both j-closed and j-open in \( X \) for each \( i \)-open subset \( V \) of \( Y \).

**Definition [16].** A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called \( G(i, j)\)-\( \sigma_k \)-continuous (or \( G(\tau_1, \tau_2)\)-\( \sigma_k \)-continuous) if the inverse image of every \( \sigma_k \)-closed set in \( (Y, \sigma_1, \sigma_2) \) is an (i, j)-g-closed in \( (X, \tau_1, \tau_2) \).

**Definition [23].** A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called pairwise weakly \( \beta \)-continuous if the inverse image under \( f \) of each \( \sigma_i \)-open set in \( Y \) is an (i, j) weak \( \beta \)-set in \( X \).

**Definition [18].** Let \( f : X \rightarrow Y \) be a mapping. Then \( f \) is said to be

1. **(1, 2)*-A continuous** if \( f^{-1}(V) \) is \( (1, 2)*\)-A set in \( X \) for every \( \sigma_{1,2} \)-open set \( V \) in \( Y \).

2. **(1, 2)*-B continuous** if \( f^{-1}(V) \) is \( (1, 2)*\)-B set in \( X \) for every \( \sigma_{1,2} \)-open set \( V \) in \( Y \).

3. **(1, 2)*-C continuous** if \( f^{-1}(V) \) is \( (1, 2)*\)-C-set in \( X \) for every \( \sigma_{1,2} \)-open set \( V \) in \( Y \).

**Definition [17].** Let \( f : X \rightarrow Y \) be a mapping. Then \( f \) is said to be

1. **(1, 2)* continuous** if \( f^{-1}(V) \) is \( \tau_{1,2} \)-open set in \( X \) for every \( \sigma_{1,2} \)-open set \( V \) in \( Y \).

2. **(1, 2)*-\( \alpha \) continuous** if \( f^{-1}(V) \) is \( (1, 2)*\)-\( \alpha \)-open set in \( X \) for every \( \sigma_{1,2} \)-open set \( V \) in \( Y \).
3. ABSTRACT OF WORK DONE :-

The first chapter is devoted to the study of almost continuous, almost $\beta$-continuous, almost precontinuous and almost semi-continuous functions in bitopological spaces. The concept of almost continuous mappings in topological spaces was introduced by Singal and Singal [The Yokohama Math. J. 16 (1968)]. This class contains the class of continuous mappings and is contained in the class of weakly continuous mappings. The concept of $\beta$-open sets and $\beta$-continuous functions in topological spaces was introduced and investigated by Abd El-Monsef et al. [Bull. Fac. Sci. Assiut Univ. 12 (1983)]. Nasef and Noiri [Acta Math. Hungar. 74 (1997)] was introduced a new class of functions called almost precontinuous functions in topological spaces and studied fundamental properties of almost $\beta$-continuous functions and Noiri and Popa [Acta Math. Hungar. 79(4) (1998)] investigated further properties of almost $\beta$-continuous functions. Jafari and Noiri [Internat. J. Math and Math. Sci. 24 (3) (2000)] investigated some more properties of almost precontinuous functions. Munshi and Bassan [The Math. Student 49 (3) (1981)] introduced a new class of mappings called almost semi-continuous mappings in topological spaces. This class contains the class of almost continuous mappings Singal and Singal [The Yokohama Math. J. 16 (1968)] and that of semi-continuous mappings Levine [Amer. Math. Monthly 70 (1963)].

Bose and Sinha [Bull. Cal. Math. Soc. 74 (1982)] introduced the concepts of pairwise almost continuous by using $(i, j)$ regular open set in place of $\sigma_i$-open sets in the range of space $(Y, \sigma_1, \sigma_2)$. We give further properties of almost continuity and closure continuity in bitopological spaces and we give almost continuity imply closure continuity in bitopological spaces. We introduced pairwise almost $\beta$-continuous functions and investigate further properties of pairwise almost $\beta$-continuous functions. We give several properties concerning pairwise $\beta$-continuity, pairwise almost $\beta$-continuity and pairwise weak $\beta$-continuity.
Noiri and Popa [ Soochow J. Math. 33(1) (2007)] introduced the concepts of almost precontinuous functions in bitopological spaces. We investigated some more properties of pairwise almost precontinuous functions. It turns out that pairwise almost precontinuity is stronger than pairwise almost weak continuity. Khedr and Noiri [ J. Egypt. Math. Soc. 15(1)(2007)] introduced the concepts of almost s-continuous functions in bitopological spaces. We investigated some more properties of pairwise almost semi-continuous functions in bitopological spaces. This class contains the class of pairwise almost continuous mappings and that of pairwise semi-continuous mappings. We discuss the strength of pairwise almost semi-continuous vis-a-vis several other pairwise continuities such as pairwise feebly continuity, pairwise semi-continuity, pairwise weak continuity Bose and Sinha [ Bull. Cal. Math. Soc. 74 (1982)], pairwise almost continuity and pairwise θ-continuity Bose and Sinha [ Bull. Cal. Math. Soc. 73 (1981)].

We give the composition and product of two almost semi continuous functions in bitopological spaces. We also introduced pairwise pre semi-open and pairwise pre-semi δ-open mapping and their properties. Some of the results of pairwise almost semi-continuous mappings is published paper Sharma and Solanki [ The Mathematics Education Vol. XLII, No.3 (2008)]. Some results of this chapter are in various stage of publications. The main results is.

**Theorem.** For a function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \), the following are equivalent:

1. \( f \) is pairwise almost continuous.

2. \( \tau_i - \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\sigma_j - \text{Cl}(V)) \) for every \( V \in (i, j) \text{ SPO}(Y) \).

3. \( \tau_j - \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\sigma_j - \text{Cl}(V)) \) for every \( V \in (i, j) \text{ SO}(Y) \).

4. \( f^{-1}(V) \subseteq \tau_i - \text{Int}(f^{-1}(\sigma_i - \text{Int}(\sigma_j - \text{Cl}(V)))) \) for every \( V \in (i, j) \text{ PO}(Y) \).

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be pairwise almost
**Theorem.** Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be pairwise almost continuous. Then \( f \) is pairwise closure continuous.

**Theorem.** For a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \), the following are equivalent:

1. \( f \) is pairwise almost \( \beta \)-continuous at \( x \in X \).
2. For each \( \sigma_i \)-neighbourhood \( V \) of \( f(x), x \in \tau_j-\text{Cl}(\tau_i-\text{Int}(\tau_j-\text{Cl}(f^{-1}((i, j)s\text{Cl}(V)))))) \).
3. For each \( \sigma_i \)-neighbourhood \( V \) of \( f(x) \), and each \( \tau_i \)-neighbourhood \( U \) of \( x \), there exists a nonempty \( \tau_i \)-open set \( G \subseteq U \) such that \( G \subseteq \tau_j-\text{Cl}(f^{-1}((i, j)s\text{Cl}(V)))) \).
4. For each \( \sigma_i \)-neighbourhood \( V \) of \( f(x) \), there exists \( U \in (i, j) \text{SO}(X, x) \) such that \( U \subseteq \tau_j-\text{Cl}(f^{-1}((i, j)s\text{Cl}(V)))) \).

**Theorem.** For a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \), the following are equivalent:

1. \( f \) is pairwise almost \( \beta \)-continuous.
2. \( f((i, j)\beta \text{Cl}(A)) \subseteq (i, j) \delta \text{Cl}(f(A)) \) for every subset \( A \) of \( X \).
3. \((i, j) \beta \text{Cl}(f^{-1}(B)) \subseteq f^{-1}((i, j)\delta \text{Cl}(B)) \) for every subset \( B \) of \( Y \).
4. \( f^{-1}(F) \in (i, j)\beta \text{C}(X) \) for every \((i, j)\delta\)-closed set \( F \) of \( Y \).
5. \( f^{-1}(V) \in (i, j)\beta \text{O}(X) \) for every \((i, j)\delta\)-open set \( V \) of \( Y \).

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be a function and \( g : (X, \tau_1, \tau_2) \to (X \times Y, \mathcal{I}_1, \mathcal{I}_2) \) the graph function defined by
g(x) = (x, f(x)) for every x ∈ X. Then g is pairwise almost β-continuous iff f is pairwise almost β-continuous.

**Theorem.** If f : (X, τ₁, τ₂) → (Y, σ₁, σ₂) is pairwise M-preopen pairwise almost weakly continuous. Then f is pairwise almost precontinuous.

**Theorem.** Let f, g : (X, τ₁, τ₂) → (Y, σ₁, σ₂) be functions and Y is pairwise Hausdorff. If f is pairwise weakly α-continuous and g is pairwise almost precontinuous, then the set E = {x ∈ X : f(x) = g(x)} is (i, j) preclosed in X.

**Theorem.** Let f : (X, τ₁, τ₂) → (Y, σ₁, σ₂) be pairwise semi-continuous and g : (Y, σ₁, σ₂) → (Z, η₁, η₂) be pairwise almost continuous then gof : (X, τ₁, τ₂) → (Z, η₁, η₂) is pairwise almost semi-continuous.

**Theorem.** Let f : (X, τ₁, τ₂) → (Y, σ₁, σ₂) be pairwise almost semi-continuous mapping. If A ⊂ X and A is τ₁-open, then f/ A : (X, τ₁/A, τ₂/A) → (Y, σ₁, σ₂) is pairwise almost semi-continuous.

**Theorem.** Let h : X → X₁ × X₂ be pairwise almost semi-continuous, where X, X₁ and X₂ are bitopological spaces. If fᵢ : X → Xᵢ be defined for each x ∈ X as fᵢ(x) = xᵢ where h(x) = (x₁, x₂), then fᵢ : X → Xᵢ is pairwise almost semi-continuous.

**Theorem.** If X = ∪ Gα where each Gα is an (i, j) semi open set and f : (X, τ₁, τ₂) → (Y, σ₁, σ₂) is a mapping such that f / Gα is pairwise almost semi-continuous for each α, then f is pairwise almost semi-continuous.

(1) (1994)] introduced (i, j)A-sets and pairwise A-continuous functions and he also prove that pairwise A-continuity different from some known forms of pairwise continuities in the weak sense. We generalized some results of A - continuous functions in bitopological setting and gives some decomposition of pairwise A-continuous functions and relate this pairwise continuities from other types of pairwise continuous functions. We introduced the concepts of B and C-continuous functions in bitopological setting and gives some properties of these functions. Some of the results of B-sets and B-continuity in bitopological spaces is forthcoming paper Sharma and Solanki [ The Mathematics Education, Vol. XLII, No.1 (2008)]. On pairwise C-continuous functions and pairwise A-continuity is forthcoming paper Solanki [ Applied Science Periodical Vol. XII, No.2 and Vol. XII, No.3 (2010)] respectively. The main results is.

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function. Then

1. \( f \) is pairwise A-continuous iff \( f \) is pairwise semi continuous and pairwise LC continuous.

2. \( f \) is continuous iff \( f \) is pairwise \( \alpha \)-continuous and pairwise LC continuous.

3. \( f \) is pairwise \( \alpha \)-continuous iff \( f \) is pairwise precontinuous and pairwise semi-continuous.

4. \( f \) is pairwise continuous iff \( f \) is pairwise precontinuous and pairwise LC continuous.

5. \( f \) is pairwise continuous iff \( f \) is pairwise precontinuous and pairwise A- continuous.

**Theorem.** A subset \( S \) in a bitopological space \((X, \tau_1, \tau_2)\) is a \( \tau_1 \)-open iff it is an \((i, j)\) pre-open and an \((i, j)\) B-set.
**Theorem.** A subset $S$ is a $\tau_i$-open in a bitopological space $(X, \tau_1, \tau_2)$ iff it is an $(i, j)$ $\alpha$-set and an $(i, j)$ $C$-set.

In third chapter, we introduced slightly continuous and slightly semi-continuous functions in bitopological spaces. The concepts of slightly continuous functions in topological spaces was introduced by Jain [Ph.D. Thesis, Meerut University, Institute of Advanced Studies Meerut, India, 1980] and slightly semi-continuous functions in topological spaces was introduced by Nour [Bull. Cal. Math. Soc. **87** (1995)] and Noiri and Chae [Bull. Cal. Math. Soc. **92** (2) (2000)]. We generalizes some results of slightly continuous and slightly semi-continuous functions in bitopological setting and give some characterizations of these spaces. This class contains the class of pairwise slightly continuous functions and that of pairwise semi-continuous functions. Some of the results of slightly continuous functions in bitopological spaces is forthcoming paper Solanki [The Mathematics Education Vol. XLIV, No.2 June (2010)].

The main results is.

**Theorem.** For a mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following are equivalent:

1. $f$ is pairwise slightly continuous.
2. Inverse image of every $(i, j)$ clopen subset of $Y$ is a $\tau_i$-open subset of $X$.
3. Inverse image of every $(i, j)$ clopen subset of $Y$ is an $(i, j)$ clopen subset of $X$.
4. For each $x \in X$ and each $(i, j)$ clopen neighbourhood $M$ of $f(x)$, there exists a $\tau_i$-open neighbourhood $N$ of $x$ such that $f(N) \subset M$.
5. For each $x \in X$ and for every net $\langle X_\lambda \rangle_{\lambda \in D}$ which converges to $x$, the net $\langle f(x_\lambda) \rangle_{\lambda \in D}$ is eventually in every $(i, j)$ clopen set containing $f(x)$. 
Theorem. If \( f \) is a pairwise slightly continuous mapping of a bitopological space \((X, \tau_1, \tau_2)\) into a pairwise 0-dimensional space \((Y, \sigma_1, \sigma_2)\), then \( f \) is pairwise continuous.

Theorem. Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be a mapping and \( g : (X, \tau_1, \tau_2) \to (X \times Y, \bar{\tau}_1, \bar{\tau}_2) \) (where \( \bar{\tau}_k \) \((k = 1, 2)\) is the product topologies generated by \( \tau_k \) and \( \sigma_k \)) given by \( g(x) = (x, f(x)) \) be its graph map. Then \( f : X \to Y \) is pairwise slightly continuous iff \( g : X \to X \times Y \) is pairwise slightly continuous.

Theorem. Let \((Y, \sigma_1, \sigma_2)\) be a pairwise extremally disconnected, pairwise C-compact pairwise Hausdorff space. If a mapping \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is pairwise slightly continuous, then \( f \) is pairwise continuous.

Theorem. Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be a mapping of \( X \) into \( Y \) and \( X = X_1 \cup X_2 \) where \( X_1 \) and \( X_2 \) are \( \tau_i \)-closed and \( f / X_1 \) and \( f / X_2 \) are pairwise slightly continuous, then \( f \) is pairwise slightly continuous.

Theorem. For a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \), the following are equivalent:

(1) \( f \) is pairwise slightly semi-continuous.

(2) Inverse image of every \((i, j)\)clopen subset of \( Y \) is an \((i, j)\) semi-open subset of \( X \).

(3) Inverse image of every \((i, j)\)clopen subset of \( Y \) is an \((i, j)\) semi-clopen subset of \( X \).

(4) For each \( x \in X \) and for every net \( \langle x_\lambda \rangle_{\lambda \in \mathcal{D}} \) which pairwise s-converges to \( x \), the net \( \langle f(x_\lambda) \rangle_{\lambda \in \mathcal{D}} \) is eventually in every \((i, j)\)clopen set containing \( f(x) \).

Theorem. If \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is pairwise slightly semi-continuous and \( Y \) is pairwise UT, then the graph \( G(f) \) of \( f \) is an
(i, j) semi \( \theta \)-closed in the bitopological product space \((X \times Y, \mathcal{J}_1, \mathcal{J}_2)\).

**Theorem.** If \( f \) is a pairwise slightly semi-continuous function of a space \((X, \tau_1, \tau_2)\) into a pairwise extremally disconnected space \((Y, \sigma_1, \sigma_2)\), then \( f \) is pairwise weakly semi-continuous.

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be pairwise irresolute and pairwise pre-semi-open and \( g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2) \) be any function. Then \( gof : X \to Z \) is pairwise slightly semi-continuous iff \( g \) is pairwise slightly semi-continuous.

In fourth chapter, we introduced the concept of slightly \( \beta \), \( \gamma \) continuous functions. The concept of slightly \( \beta \)-continuous functions in topological spaces was introduced by Noiri [Indian J. Math. and Math. Sci. 28 (8) (2001)] and slightly \( \gamma \)-continuous functions in topological spaces was introduced by Ekici and Caldas [Bal. Soc. Paran. Mat. (3s) 22(2) (2004)]. We generalizes some results of slightly \( \beta \)-continuous and slightly \( \gamma \)-continuous functions in bitopological setting. We give some characterizations of pairwise slightly \( \beta \)-continuity, pairwise slightly semi continuity, pairwise faintly pre continuity and pairwise slightly continuity. Some preservation theorems, composite of functions and graphs of pairwise slightly \( \gamma \)-continuous functions are investigated. Some of the results of this chapter are to appear for publication in the Journal of Mathematics and Mathematical Sciences Jahangirnagar University, Savar, Dhaka Bangladesh. The main results is.

**Theorem.** For a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \), the following statements are equivalent:

1. \( f \) is pairwise slightly \( \beta \)-continuous.
2. \( f^{-1}(V) \in (i, j)\ SPO(X) \) for each \( V \in (i, j)\ CO(Y) \).
(3) $f^{-1}(V) \in (i, j)\text{SPR}(X)$ for each $V \in (i, j)\text{CO}(Y)$.

(4) For each $x \in X$ and $V \in (i, j)\text{CO}(Y, f(x))$, there exists $U \in (i, j)\text{SPR}(X, x)$ such that $f(U) \subseteq V$.

(5) For each $x \in X$ and each $V \in (i, j)\text{CO}(Y, f(x))$, there exists $U \in (i, j)\text{SPO}(X, x)$ such that $f((i, j)\text{spCl}(U)) \subseteq V$.

**Theorem.** For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties hold:

1. If $f$ is pairwise slightly $\beta$-continuous and $X$ is pairwise extremally disconnected, then $f$ is pairwise faintly precontinuous.

2. If $f$ is pairwise slightly $\beta$-continuous and $X$ is an $(i, j)$ PS-space, then $f$ is pairwise slightly semi-continuous.

3. If $f$ is pairwise slightly $\beta$-continuous and $X$ is pairwise extremally disconnected and an $(i, j)$ PS-space, then $f$ is pairwise slightly continuous.

**Theorem.** Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be functions. Then

1. If $f$ is pairwise slightly $\beta$-continuous and $g$ is pairwise slightly continuous, then $g \circ f$ is pairwise slightly $\beta$-continuous.

2. If $f$ is pairwise $\beta$-irresolute and $g$ is pairwise slightly $\beta$-continuous, then $g \circ f$ is pairwise slightly $\beta$-continuous.

3. Let $f$ be a pairwise open continuous surjection. Then $g$ is pairwise slightly $\beta$-continuous iff $g \circ f$ is pairwise slightly $\beta$-continuous.
**Theorem.** If \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is pairwise slightly \( \beta \)-continuous surjection and \( X \) is pairwise \( \beta \)-connected, then \( Y \) is pairwise connected.

**Theorem.** For a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \), the following statements are equivalent:

1. \( f \) is pairwise slightly \( \gamma \)-continuous.
2. For every \((i, j)\) clopen set \( V \subset Y \), \( f^{-1}(V) \) is an \((i, j)\) \( \gamma \)-open.
3. For every \((i, j)\) clopen set \( V \subset Y \), \( f^{-1}(V) \) is an \((i, j)\) \( \gamma \)-closed.
4. For every \((i, j)\) clopen set \( V \subset Y \), \( f^{-1}(V) \) is an \((i, j)\) \( \gamma \)-clopen.

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be a function and \( x \in X \). If there exists \( U \in (i, j)\alpha O(X) \) such that \( x \in U \) and the restriction of \( f \) to \( U \) is a pairwise slightly \( \gamma \)-continuous function at \( x \), then \( f \) is pairwise slightly \( \gamma \)-continuous at \( x \).

**Theorem.** Let \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) and \( g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2) \) be functions. If \( f \) is pairwise \( \gamma \)-irresolute and \( g \) is pairwise slightly continuous, then \( g \circ f : X \to Z \) is pairwise slightly \( \gamma \)-continuous.

**Theorem.** If a function \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is pairwise slightly \( \gamma \)-continuous and \( K \) is pairwise \( \gamma \)-compact relative to \( X \), then \( f(K) \) is pairwise mildly compact in \( Y \).

**Theorem.** If \( f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is pairwise slightly \( \gamma \)-continuous and \( Y \) is pairwise clopen \( T_1 \), then \( G(f) \) is pairwise strongly \( \gamma \)-co-closed in \((X \times Y, \mathcal{I}_1, \mathcal{I}_2)\).
Theorem. for a function \( f : ( X, \tau_1, \tau_2) \rightarrow ( Y, \sigma_1, \sigma_2) \). If \( X \) is a pairwise submaximal pairwise extremally disconnected space and \( Y \) is pairwise 0-dimensional space, then the following are equivalent:

(1) \( f \) is pairwise slightly \( \gamma \)-continuous.

(2) \( f \) is pairwise \( \alpha \)-continuous.

In fifth chapter, we introduced the concept of pairwise \( z \)-continuous functions. The concept of \( z \)-continuous functions in topological spaces was introduced Singal and Nimse [ The Math. Student 66 (1- 4) (1997)]. We generalized some results of \( z \)-continuous functions in bitopological setting and their properties. The class of pairwise \( z \)-continuous function properly contains the class of pairwise weakly continuous functions of Bose and Sinha [ Bull. Cal. Math. Soc. 74 (1982)]. Some results of the last chapter are in various stages of publications. The main results is.

Theorem. For a function \( f : ( X, \tau_1, \tau_2) \rightarrow ( Y, \sigma_1, \sigma_2) \), then the following are equivalent :

(1) \( f \) is pairwise \( z \)-continuous.

(2) Inverse image of every \((i, j)\) cozero-set in \( Y \) is a \( \tau_i \)-open in \( X \).

(3) Inverse image of every \((i, j)\) zero-set in \( Y \) is a \( \tau_i \)-closed in \( X \).

Theorem. For a function \( f : ( X, \tau_1, \tau_2) \rightarrow ( Y, \sigma_1, \sigma_2) \), the following are equivalent :

(1) \( f \) is pairwise \( z \)-continuous.

(2) Inverse image of every \((i, j)\) \( z \)-open subset of \( Y \) is a \( \tau_i \)-open subset of \( X \).
(3) Inverse image of every (i, j) z-open subset of Y is an (i, j) z-open subset of X.

(4) Inverse image of every (i, j) z-closed subset of Y is a \( \tau_i \)-closed subset of X.

(5) Inverse image of every (i, j) z-closed subset of Y is an (i, j) z-closed subset of X.

(6) For each \( E \subseteq X \), \( f( [E] )_z \subseteq [ f(E) ]_z \).

**Theorem.** If \( f \) is a function of \( (X, \tau_1, \tau_2) \) into \( (Y, \sigma_1, \sigma_2) \), if \( X = X_1 \cup X_2 \), and if \( f / X_1 \) and \( f / X_2 \) are both pairwise z-continuous at a point \( x \) belonging to \( X_1 \cap X_2 \), then \( f \) is pairwise z-continuous at \( x \).

**Theorem.** Every pairwise weakly continuous function is pairwise z-continuous.

**Theorem.** Every pairwise z-continuous onto mapping is pairwise set connected.

**Theorem.** If \( f \) is a pairwise z-continuous function of a bitopological space \( (X, \tau_1, \tau_2) \) into a pairwise completely Hausdorff space \( (Y, \sigma_1, \sigma_2) \), then \( \{(x_1, x_2) : f(x_1) = f(x_2)\} \) is an (i, j) z-closed in the product space \( (X \times X, \mathcal{J}_1, \mathcal{J}_2) \).
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