CHAPTER 4

SCALING AND DYNAMIC THINNING

4.1 ISSUES CONCERNING SCALING AND DYNAMIC THINNING

The study conducted till now has established the methodology of applying NBSA technique to thinning of an Electromagnetic Antenna Array. Though this is theoretically perfect, there is a need to look at the practical problems one will face on its implementation in order to realize its benefits.

There are two situations which warrant serious thinking in this regard.

- Systems where the antenna size is too large (of the order of few thousands of antenna elements)
- Systems where the antenna has to respond to varying needs on a dynamic basis

Specific questions which need to be addressed in this context are as follows:

- A large sized array would have a large solution space. How does the proposed approach navigate itself in such a large solution space?
- Are there any means by which solution space can be reduced?
- What are the implications on time taken to optimize for a solution in such large solution space?
- Would they be acceptable in a critical system (as say in an operational Radar system)?
- Looking at the specific problem in hand (thinning in large array) are there any mechanisms by which the computation time can be reduced?
• What is the sensitivity of the solution obtained through this approach? Would the solution be valid when conditions change? How can change management be done in such cases?

These are some of the issues which are being addressed in this chapter

4.2 SCALING PROBLEMS IN THINNED ARRAY DESIGN

A preliminary study on the above issues showed the need to concentrate on aspects related to solution space and computational complexity when the size of the antenna array is increased.

4.2.1 INCREASE IN SOLUTION SPACE

Scaling in the context of optimization problem relates closely to the solution space. Solution space is the total number of solutions, out of which one which meets the requirement is finally chosen as the solution. As the number of variables increases the solution space increases at a much larger scale.

Table 4.1 depicts the relationship between solution space and \( N_{\text{total}} \) for linear antenna arrays for different Thinning Factors (TF). For TF=0.25, for an array of 32 elements the solution space is about \( 1.05 \times 10^7 \) but increases to \( 2.4 \times 10^{23} \) when the total number of elements in the array is increased to 100. When the total number of elements in the array is doubled to 200, the solution space becomes too large for computation.

4.2.2 COMPLEXITY OF SOLUTION SPACE

When the solution space becomes large, the likelihood of landscape of the solution space becoming complex also increases. Though the type of exact landscape would depend to large extent on the parameters of the array and the objective function to be maximized / minimized, its broad nature would be of the type shown in Figure. 4.1.

For the purpose of complexity in optimization, they could be classified into at least eight different scenarios as shown in Figure 4.2. Extracting the optimum solution is a difficult proposition in at least six (Figure 4.2 c-h) out of the eight scenarios.
This does not mean the same situation cannot happen when the solution space is smaller. The only conclusion that can be drawn is that the likelihood of getting into the difficult situations is more likely when the solution space is large.

4.2.3 N-P COMPLETENESS

Optimization problems can generally be classified into two classes -- those that can be solved in polynomial time (Class P) and those that can only be solved in Non-Polynomial time (Class NP). Run-time required increase polynomially for Class P problems and exponentially for Class NP problems.

There are two key points to note about NP-complete problems:

- no polynomial time algorithm has ever been found for an NP-complete problem;
- if there is a polynomial time algorithm for any NP-complete problem, then there are polynomial time algorithms for all NP-complete problems.

These two points have led many people to conjecture that there can be no polynomial time algorithms for NP-complete problems, but as yet, no proof has been found for this conjecture [4.1]

Since the thinning problem is an N-P complete problem, it will be more difficult to solve it when the size of the array increases. Also, computer run time to solve the problem will increase exponentially as problem size increases.

4.3 DYNAMIC THINNING

‘Dynamic thinning’ refers to response of the thinning process to a dynamic situation. This can be treated as on-line or real time optimization process.

The optimization process should be able to respond quickly to the new demand. Sometimes the new demand may also arise after the optimization process has been initiated. This means that the requirements or constraints when the process of optimization started were different from the time, solution for optimization was finally
obtained. In such cases, sensitivity of the solution to the changed environment would decide the usefulness of the whole process. This is shown in Figure 4.3.

Two typical dynamic situations in respect of a practical antenna array are discussed below.

- Need to introduce a notch in a defined angular sector
- Need to switch the main beam of an antenna array to a different angle

### 4.3.1 NEED TO INTRODUCE A NOTCH IN A DEFINED ANGULAR SECTOR

Though ideally a radar receiver must be able to receive signals from only the intended direction, it does receive signals from other directions based on the characteristics of the radiation pattern outside its main beam. This vulnerability of the radar receiver can be made use of by external elements to send an unwanted interference signal (normally called a jammer signal) in order to corrupt or modify the radar receiver performance.

One way of avoiding this situation is to introduce a notch in the radiation pattern in an angular sector from where these unwanted signals are expected. Since these situations are not pre-planned, they need to be handled dynamically. This is shown in Figure 4.4

### 4.3.2 NEED TO SWITCH THE MAIN BEAM OF AN ANTENNA ARRAY TO A DIFFERENT ANGLE

On a regular basis, an electronic scanning array (ESA) shall be scanning from one end of its scan limit to the other end. This is mostly carried out by incrementing the differential phase shift among the array elements in a systematic manner. Such arrangements are useful for radar search operations.

However, while tracking multiple targets, the same radar antenna may be required to acquire different targets by switching its beams in different directions. Typical operational requirement may demand such switching to be done in a few milliseconds time which will not allow the beam to be scanned/ slewed from the first to the second direction. One such scenario is shown in Figure 4.5. In such operational situations, thinning which was applicable to the first direction may not be proper for the second direction. In other words, elements which were optimized to be inactive for the first direction would, in general, not suitable for the second direction.
In both the cases cited above, a thinned of an EAA needs to optimized dynamically based on situation. Handling thinning requirements in such situations are called Dynamic Thinning design.

4.3.3 APPLICABILITY OF DYNAMIC OPTIMIZATION TECHNIQUES

Many industrial optimization problems are solved in environments that undergo continual change. These problems are referred to as dynamic optimization problems and are characterized by an initial problem definition and a series of “events” that change it over time. An event defines some change either to the data of the problem or its structural definition while the problem is being solved. In comparison to static optimization problems, dynamic optimization problems often lack well defined objective functions, test data sets, criteria for comparing solutions and standard formulations. [4.2].

Generic forms of few dynamic optimization techniques described in literature are discussed below.

4.3.3.1 DECONSTRUCTION PROCESS

Here the optimization process works on the principle of deconstruction when an event takes place, rather than discarding the old solutions and restarting the construction process again.[4.3] Deconstruction removes the components from a solution that make it infeasible to the changed problem. Results for a dynamic version of the multidimensional knapsack problem showed that this process could quickly adapt to the changes in the problem. Further, work on dynamic aircraft landing problem (using a real time simulator) indicates that the approach is capable of producing good schedules in a timely fashion. [4.4]

4.3.3.2 REBALANCED EXPLORATION

Exploration and exploitation are two aspects of searching the solution space. Exploration refers exploring new areas in solution space and exploitation refers to exploiting already known information about the solution space. When a changed situation is anticipated, the algorithm rebalances its priorities between exploration and exploitation, based on its memory about the solution space. For this, the algorithm collects information about local
optima whenever they occur and record in memory. Some clouts are given this responsibility which help in re-balancing [4.5]

4.3.3.3 ALTERING COMPONENT FITNESS

In this method, objective function is associated with various components of the solution. When an event takes place, worst individual solution component is replaced by a random one. Over a number of iterations it is expected that the overall solution quality will improve for the changed condition. In one of its variant versions, each solution component is assigned a rank k based on its current fitness within the solution and mutates it according to a function based on the probability distribution of a function of k. This approach seems to have given good results for an image tracking problem where fast optimization was necessary to cluster the incoming reports according to the objects, before the relevant information on target objects can be processed by a tracker module. [4.6]

4.3.3.4 ADAPTING ABOVE TECHNIQUES FOR PRESENT PROBLEM

Adapting above techniques for dynamic thinning in antenna arrays was examined. It was observed that most of these techniques were developed for managing discrete event optimization for industrial applications such as shop floor management or scheduling. The techniques were not found to be much useful for the purpose of the present investigation.

4.3.4 DYNAMIC THINNING PROBLEMS

As discussed earlier, dynamic array thinning refers to varying the thinning pattern on real time basis to suit to varying conditions. Even though SGA is a well suited tool for solving array thinning problems, computational complexity figure prominently in applying the algorithm effectively for dynamic thinning.
Computational complexity involved in thinned array design for many large size antenna arrays is too time consuming and hence may not be directly suitable for on-line implementation involving dynamic thinning. The size of the array affects computational effort involved in thinned array design in two ways.

**Objective Function (OF) evaluation:** For large EAA, requirements of computer resources for OF evaluation far exceed the functional requirements for implementation of SGA. In most cases, the evaluation of OF is computationally intensive involving lengthy procedures. All efforts to reduce the intensity of computation would help reduce overall time, helping in real-time implementation.

**Exploring solution space:** A practical Electronic Scanning array antenna may consist of hundreds or thousands of elements. In such cases, the solution space would be very large and also rugged. Exploring such a large solution space using SGA would not only require time but also may result in getting trapped in a local optimum leading to premature convergence. As the solution space increases, speed of convergence of SGA procedure gets affected drastically, which affects usage of SGA for dynamic conditions.

### 4.4 RESOLVING DYNAMIC THINNING ISSUES

Since the major problem relates to computational complexity, efforts to reduce computational time and complexity was taken up as main issue. The approach involved

- **Bulk Array Calculation (BAC):** avoids computational redundancy in calculations related to thinning process.

- **Exploiting Geometric symmetry:** in the structures of most practical array antennas to reduce the solution space.

- **Zoning and Exploring Dynamic Symmetry:** helps in formulating solution space.

- **Concept of acceptable solution:** search for practically acceptable solution
4.4.1. BULK ARRAY COMPUTATION

This was the technique used to reduce computational time to evaluate OF of a thinned array.

Implementation of GA requires evaluation of OF for every member of the population in each iteration. Each evaluation is based on the array factor calculation $F_M(\theta)$ for a $M$-element linear array given by

$$F_M(\theta) = \sum_{i=1}^{M} R_i e^{i k d_i \cos(\theta)} \quad (4.1)$$

For a 2-D planar array the array factor given by

$$F_M(\theta, \phi) = \sum_{n=1}^{N} \sum_{m=1}^{M} F_{nm} * \cos[\pi(2n-1)\sin(\theta)\cos(\phi)] * \cos[\pi(2m-1)\sin(\theta)\sin(\phi)] \quad (4.2)$$

These equations are highly nonlinear and involve lengthy procedures. In order to reduce the computational burden, a method called as ‘Bulk Array computation’ (BAC) is introduced for thinning calculations.

It is based on creating a table to store the data of the radiated fields of all elements in all directions as is done by Brill [4.7]. But instead of adding contribution of radiations from the active elements, $F_M(\theta, \phi)$ is calculated by subtracting the contributions of radiations from the inactive elements. Based on this, radiation pattern of the thinned array corresponding to each member of the population is then computed. The flow graph for BAC is shown in Figure 4.6.

Major steps involved in the computation are,

a. Generate data for creating ‘Element Table’ which has all details about element location and its complex excitation coefficient.

b. Generate data for creating ‘Angle Table’ which contains details of each angular direction in ($\theta, \phi$) coordinates

c. Compute and store the radiated field due to each element of the array in each direction of interest

d. Initial population of ‘inactive elements’ is generated.
e. Effect of ‘inactive elements’ is then coalesced on the stored data of the radiated field of the array; radiation pattern of the thinned array corresponding to each member of the population over the required angular sector is then computed.

f. Feedback parameter is extracted from the set of radiated patterns of the thinned arrays and is used in an iterative manner to generate successive populations, using GA procedures.

g. This is continued iteratively till the terminating criterion is obtained.

Table 4.2 compares the mathematical operations required between conventional procedures and BAC for calculating the OF based on array factor as indicated in equation 4.1 and 4.2 for different array sizes and dimensions. The reduction in computational operations range from about $2.8 \times 10^8$ in case of a simple 200 element linear array to about $2.32 \times 10^{12}$ in case of a planar array of 4096 elements.

4.4.2. GEOMETRIC SYMMETRY

Several types of symmetry are common in practice in practical antenna arrays. [4.8] Some of these symmetries can be used to obtain reduction in the computational complexity and solution space.

In the present investigative study a linear array is considered as symmetric from center of the array i.e., the element excitations in the second half of the array is mirror image of the excitations of the elements in the first half. For thinning of planar arrays, quadrature symmetry is used. Table 4.3 shows the reduction in the solution space for symmetrical arrays compared to Table 4.1.

4.4.3 ZONING TECHNIQUE

Zoning refers to partitioning the antenna array into convenient zones, so that the solution space can be usefully explored. Though there is no restriction in the total number of zones $N_z$, it is practical to have only 2 or 3 zones. Each zone is expected to consist of at least 2 elements. Figure 4.7 (a) shows a linear array that has been partitioned into $N_z = n$ zones. Figure 4.7(b) and (c) show typical zoning of linear and planar array for $N_z=3$;
Dynamic symmetry considerations were also explored while zoning the array antenna to formulate the solution space.

Dynamic symmetry is a proportioning system conceived by Jay Hambidge an American artist born in Canada. Based on careful examination of many classical buildings in Greece he formulated theory "dynamic symmetry" [4.9-4.11]. Dynamic Symmetry system uses ratios such as 1.618…, 1.272..., 2.618....etc. These numbers are some special numbers derived from the ‘golden ratio’ number 1.618. 1.272 and 2.618 are the square root and squared of the golden ration number. These numbers give rise to Fibonacci and related series.

The 1.618 proportion appears in the curve-cross configuration of the pineapple, pine cone, and sunflower, and in the animal world for example, in the spiral of snails and the chambered nautilus and also in the proportions of the human body. This proportion gives rise to equiangular or logarithmic spiral which concept is used in designing antennas with very large bandwidth.

For antenna thinning purposes, the proportion 1.618 has been used in this investigation for partitioning the complete array into zones. The partition is done such that the ratio between numbers of antenna elements in adjoining zones is near equal to the golden ratio value of 1.618.

After determining the size of the zones, the number of inactive elements is distributed based on the defined thinning factor. Total number of inactive elements $N_{inactive}$ is first computed based on TF and $N_{total}$ of the array under consideration. This is then distributed among the zones so that

$$N_{inactive}(n) \leq N_{inactive}(n+1) ;$$

$$\sum_n N_{inactive}(n) = N_{inactive}$$

where $N_{inactive(n)}$ refers to the number of inactive elements within zone $Z_n$. Based on this, initial population is generated and SGA is carried out.
Generally, it is expected that number of inactive elements in the central portion shown as zone 1 in Figure 4.7 (b and c) of a thinned array would be much less than in other zones. Such a-priori knowledge can help in partitioning of the array into zones and better exploitation of the solution space. In general, zoning is based on dividend or return likely to yield while exploiting the zone [4.12, 4.13]. Though it is possible to partition into any number of zones, it may not be advisable to consider more than 2 or 3 zones, as shown later.

Zoning technique provides ample scope for using any a-priori or intuitive information about the antenna array. By this approach considerable reduction in solution space occurs, resulting fast convergence as demonstrated in the next chapter.

4.4.4. CONCEPT OF ACCEPTABLE SOLUTION

Generally an optimization problem would strive at getting at the globally optimized solution within the set constraints. Thinning problem is an optimization problem, where we would like to minimize the number of active elements, $N_{active}$ out of a total available $N_{total}$ elements so as to meet the OF.

In a real-time scenario as in the case of dynamic thinning where time is at premium, a solution which gives a near-optimum value would be a more useful input than the absolute minimum value of $N_{active}$, as far as the OF is satisfied. The penalty paid in terms of extra time spent for obtaining the exact minimum solution may defeat the very purpose of the mission in such cases. Also, for practically applying the result, knowing all the elements of the set \{N_{active}\} is more important than merely knowing the minimum value of $N_{active}$.

Thus, dynamic thinning should look for acceptable solution rather than the absolute minimum value of $N_{active}$. However, what is an acceptable solution would depend on the operational scenario. For example, obtaining a thinning solution which can meet the objectives fully with $N_{active} =3000$, within a fraction of time would be much more valuable than getting a similar solution with $N_{active} =2998$ (where $N_{total}=4096$). Similarly, obtaining a solution which can offer a side lobe value of say -39.8 db within a fraction of time would be much more valuable than getting a similar solution which
provides a side lobe value of -40 dB. Dynamic thinning can benefit by recognizing such acceptable solutions relevant to the operational scenario.

In most conventional cases of optimization, there can be only one solution which would get qualified to be called as “the optimum solution”. But on the notion of the ‘acceptable solution’, there may be many solutions meeting the requirement and thus the optimization process which looks for an ‘acceptable solution’ can take advantage of this to cut short time for computation.

4.5 THINNING ALGORITHM BASED ON GENETIC SEARCH (TAGS)

BAC and Zoning technique are useful in reducing the overall computation time and help in achieving fast convergence required for dynamic thinning. Optimization algorithm based on genetic search can be used for real-time thinning design by combining these techniques with a criterion based on an acceptable solution relevant to the dynamic requirement. An integrated approach combining all the above aspects called ‘Thinning Algorithm based on Genetic Search’ (TAGS) is used for dynamic thinning of large antenna arrays. This involves,

- Search using Simple GA
- Zoning based on Dynamic symmetry
- Cost based Objective function
- Concept of acceptable solution
- Bulk array Computation
- Geometric symmetry

A typical flow graph of TAGS is shown in Figure 4.8.

The major steps involved in TAGS are,

a) Accept the Array inputs, Radiation inputs, GA inputs.

b) Formulate either the cost function (CF)/ objective function (OF) based on the inputs.

c) Apply solution space reduction techniques – Based on Array symmetry, zoning and thinning factor.

d) Compute the Bulk Array radiation field values using BAC
e) Generate initial population
f) Evaluate OF/CF for each member of population
g) Test for objective convergence; if yes {end the process}; else {Apply SGA} and Go to step e.

4.6 DYNAMIC THINNING PROGRAMMER

In case of larger operational arrays involving very large number of elements when the time response based on the above approach may not be adequate due to operational constraints, a Dynamic Thinning Programmer (DTP) can be used as a system integrator.

The concept of DTP as a system integrator is shown in Figure-4.9. It consists of a Pre-stored Data Set, a Dynamic Thinning Logic Unit (DTLU), and a Dynamic Control Circuit (DCC). The Pre-stored Data Set contains information about the elements of the set relevant for various conditions in the form of look-up tables. Based on operational requirements, appropriate trigger signals would be sent to the DTLU, which would retrieve information about the relevant \( N_{\text{active}} \) set. For instance, in case of switching the beam to a new direction in a phased array, the trigger signal would be the information about the new direction. This will enable DTLU to retrieve information about the on/off requirements of the array elements relevant for the new beam conditions. These requirements would then be translated to appropriate control signals by the DCC and sent to the RF manifold for optimum thinning. By this process dynamic thinning can be achieved based on pre-stored data.

4.7 SUMMARY OF THE STUDY

The study carried out on scaling and dynamic thinning can be summarized as follows

- The need for a study on scaling and dynamic thinning was first established for realizing the benefits of the present investigation and using it for practical implementation.
- Implications of scaling in terms of solution space and complexity of optimization technique were studied.
• It was also concluded that the major factors affecting dynamic thinning are the computational complexity and the nature of Objective function
• Techniques available in open literature for dynamic optimization were investigated, but were found to be not of much use for the present application.
• Three approaches were found promising
  • Bulk array computation
  • Zoning Technique
  • Concept of ‘acceptable solution’
• Combining these techniques, an integrated approach called ‘Thinning Algorithm based on Genetic Search’ (TAGS) was formulated for thinning of EAA. Results on the simulation studies are presented in the next chapter.
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<thead>
<tr>
<th>$N_{\text{total}}$</th>
<th>$TF$</th>
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Table 4.1 Solution space --linear antenna arrays

Figure 4.1 Typical landscape for optimization
Figure 4.2  Possible landscape scenarios of solution space
(a) before start of optimization  (b) at end of optimization

Figure 4.3 Sensitivity of an optimized solution to changed situation
Figure 4.4 Introducing a notch in Radiation pattern
(a) Antenna pattern before introducing the notch
(b) Antenna Pattern after introducing the notch

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<th>Type of array</th>
<th>Total no. of operations by conventional method</th>
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**Table 4.2 Reduction in computational operations due to BAC**
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<td>$4.6 \times 10^{12}$</td>
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**Table 4.3 Solution Space for a symmetrical Linear antenna array**
(a) Linear Array partitioned into ‘n’ zones

(b) Zoning of linear array for \( N_z = 3 \)

(c) Zoning of planar array for \( N_z = 3 \)

Figure-4.7  Zoning for linear and Planar arrays
Figure 4.8 - A flow graph for TAGS

Figure 4.9 Dynamic Thinning Programmer
REFERENCES


