1.1 ACCEPTANCE SAMPLING AND RELATED CONCEPTS

Acceptance Sampling is a statistical procedure that specifies a method to accept or reject a lot, basing on the quality observed in the sample drawn from that lot. A sampling plan is a set of rules to execute acceptance sampling, which is known as lot-sentencing procedure. Several basic ideas of acceptance sampling can be found in Mittag & Rinne (1993),
Schilling(1989) etc. According to Dodge(1969), the following are the major types of acceptance sampling.

- Lot by lot sampling by attribute type inspection, in which each unit in the sample is inspected on a go-no-go basis.
- Lot-by-lot sampling by variable type inspection, in which each unit in the sample is measured for a single characteristic, such as weight or strength.
- Continuous sampling of a flow of units by the method of attributes
- Special purpose plans like chain Sampling, Skip-lot Sampling plans etc.

The test-lot from which a sample is drawn should have homogeneous material and should normally represent the true quality derived from the production process. The lot also has to consist of natural units and consequently its size is integer valued like rolls of yarn, electronic devices etc. This assumption excludes bulk goods like wire, paper, sand or fluids, which are not divided into separate sub units.

Acceptance inspection can also be performed in some special cases by 100% testing known as screening. When the quality characteristic to be tested is critical as in the case of shock in an electric iron and the testing is non-destructive then 100% inspection can be adopted and the lot is accepted or rejected basing on the observation. However, screening often leads to unaccountable errors in inspection owing to the fatigue or unstable performance by the inspector. However, screening is not meaningful when the test is either destructive or prohibitively expensive.

The quality observed in the sample usually matches with the quality in the remaining portion of the lot. Mood's theorem given below specifies basic understanding into this aspect.
1.2 MOOD’S THEOREM AND ITS IMPORTANCE

Suppose a lot of size \( N \) has been submitted for inspection. Let the sample size be \( n \) and the number of defectives in the sample is \( d \). If \( d \) is large, one can suspect that there will be a large number of defectives in the un-inspected portion of \((N-n)\) units and the lot should be rejected.

Mood (1943) has attempted this question and proposed a very important theorem that establishes correlation between defectives in the sample and defectives in the remaining lot (not the whole lot).

Let \( X_n \) be the number of defective units in the lot under consideration. Define the parameters \( \mu_N = E(X_N) \) and \( \sigma^2 = V(X_N) \) for whole lot. Now \( X_{N-n} = (X_N - X_n) \) denotes the number of defectives in the remainder lot.

Statement

A random sample of size \( n \) is taken without replacement from a lot of size \( N \). Define

\[
\mu^* = \mu_N (1 - \frac{n}{N})
\]

Then the correlation between the number of defective units \( X_n \) in the sample and the number of defective units \( X_{N-n} \) in the remainder lot is positive if \( \sigma^2_N > \mu^*_N \), zero if \( \sigma^2_N = \mu^*_N \) and negative if \( \sigma^2_N < \mu^*_N \).

The significance of Mood’s theorem helps in establishing the nature of relationship between \( X_n \) and \( X_{N-n} \).

(a) If the correlation is positive, it means that \( X_{N-n} \) will be small whenever \( X_n \) is small. Then lot can be accepted as long as \( X_n \) does not exceed a critical number \( c \) and this is an adequate strategy (Mittag and Rinne(1993)).

(b) If the correlation between \( X_n \) and \( X_{N-n} \) is negative, the rule will be reversed. We have to reject the lot, if \( X_n \) is small (\( \leq c \)) and accept otherwise. This is a rule, which is hard to explain to the producer.
(c) If $X_n$ and $X_{N-n}$ are uncorrelated, it is not possible to use quality in the sample to measure the quality in the lot. A good decision in such cases is either to inspect 100% or not to inspect the lot at all.

The importance of Mood’s theorem lies in the fact that when the product is statistically controlled, a population distribution can be postulated. Most of the acceptance sampling theory assumes that $X_n$ and $X_{N-n}$ are positively correlated. This theorem is very fundamental in the study of sampling plans but not discussed by many standard text books.

1.3 TYPES OF SAMPLING PLANS

Depending on whether the test characteristic is discrete or continuous sampling plans can be classified as follows.

- Attributive Sampling Plans or Sampling Plans for Attribute Type Inspection.
- Variable Sampling Plans or Sampling Plans for Variable Type Inspection.

A defect is a deviation from the specifications. Any item that does not match with the quality specifications is called non-conformity or a defective unit. In attributive acceptance sampling, the number of non-conforming units in the sample is counted. In Variable Sampling Plans, a measurement on the quality characteristics becomes the basis for sentencing the lot.

Regardless of whether the sampling is by attributes or variables, a sampling plan specifies the following parameters.

- Size of the sample $(n)$ to be drawn from the lot
- The statistic to be calculated from the sample data and
- Exact numerical condition for acceptance or rejection of the lot in the form of a critical number $(c)$

In addition to the two types, there are other ways of classifying sampling plans.
Plans with fixed sample size

In these plans the sample size is fixed before starting inspection. Some such plans are as follows.

- Single Sampling Plan in which lot sentencing is based only on a single sample from the lot.
- Double Sampling Plan in which a provision is made for examining a second sample whenever the first sample results in a state of indecision.
- Multiple Sampling Plan in which more than two samples are drawn before a decision is taken on the lot.

Plans with Variable sample size

Sampling plans of this category are based on Sequential Probability Ratio Test (SPRT) proposed by Wald (1943). One such plan is called Sequential Sampling Plan in which item by item inspection is carried out from the lot, until a decision to accept or reject is arrived at. This plan is suitable for both attribute and variable type inspection.

1.4 THE SINGLE SAMPLING PLAN AND ITS RAMIFICATIONS

The design and operation of sampling plan is based on the following concepts:

- **Acceptable Quality Level (AQL)** - This is the proportion of defectives (also called fraction defective) with which a lot can be accepted. It is based on the observation that in spite of all the efforts made to avoid defectives, a few occur in the lots and the consumer also agrees to accept such lots. It is usually expressed as a percent like 1% or 0.5% defectives being admitted. It is conventionally denoted by \( p_1 \).

- **Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD)** - This is the worst-case fraction defective at which the consumer is willing to accept the lot. If the observed fraction defective touches RQL, the lot is rejected. It is denoted by \( p_2 \) and \( p_2 > p_1 \). Beyond this level, the lot will be rejected.

- **Producer's Risk** - Since the decision on the lot is based on a random sample, there is every possibility that one sample may show a higher number defectives than another
drawn from the same lot. The producer, after inspection may reject a lot even though the lot really does not warrant rejection! This is called Type-I error and the probability of committing such an error is known as Producer's risk. This is denoted by \( \alpha \) and given by the conditional probability \( P(X \leq c | p \leq AQL) \), when \( p \) is the fraction defective in the sample.

- **Consumer's Risk** - It is the probability of accepting a lot, based on sample, given that the lot truly contains RQL. This error, known as Type-II error occurs because the sample might some times fail to reflect the real quality of the lot. The risk of committing this error is known as Consumer's risk, denoted by \( \beta \) and given by the conditional probability as \( P(X \leq c | p \geq RQL) \).

For obvious reasons, it is not possible to bring down these two risks to zero but their effect can be accounted for and kept at a minimum, while passing the decision on the lot.

Fixing the value of \( \alpha \) at same level, the procedure seeks the best 'acceptance rule' which minimizes \( \beta \).

Let \( N, n \) and \( c \) denote respectively the lot size, sample size and the acceptance number. The acceptance number denotes the maximum allowable number of defectives in the sample.

The procedure is as follows.

- Select a random sample of \( n \) from a lot of size \( N \).
- Inspect all the items included in the sample and let '\( d \)' be the number of defectives in the sample.
- If \( d \leq c \), accept the lot after replacing defective pieces found in the sample by non-defective ones.
- If \( d > c \), reject the lot or inspect the entire lot and replace all the defective items by the good one's.

**1.5 THE OPERATING CHARACTERISTIC (OC) FUNCTION**

Given a lot of size \( N \) with a sample of size \( n \) and critical number \( c \), the problem is to evaluate the probability of accepting the lot, denoted by \( P_a \), is called the Operating Characteristic or OC function of a sampling plan.
The OC function of the plan is specified as \( L(p) = P(X \leq c | p) \) where \( X \) denotes the random variable indicating the count of defective items in the sample. It gives the discriminatory power of the plan to choose between good lots and bad lots. The design of the plan is based on the values of AQL, RQL, \( \alpha \), \( \beta \), N and p. The OC curve can be found using either Hyper Geometric distribution (type-A) or its approximations like Binomial or Poisson (type-B).

The type-A OC function requires the use of Hyper Geometric distribution given by

\[
L(p | N; n; c) = \sum_{x=0}^{c} \binom{N}{x} \binom{N-M}{n-x} \left( \frac{M}{N} \right)^x \left( \frac{N-M}{N} \right)^{n-x}
\]  

(1.1)

One has to refer to statistical tables to evaluate (1.1) but it can be easily worked out using Spreadsheet Software like MS-Excel.

(More details about statistical functions available in MS-Excel are given in Appendix-1)

The type-B OC curve using Binomial distribution is given by

\[
L(p | n, c) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}
\]

(1.2)

and the OC function using Poisson distribution is given by

\[
L(p | n, c) = \sum_{x=0}^{c} \frac{(np)^x e^{-np}}{x!}
\]

(1.3)

These two forms can also be evaluated using Excel functions.
Designing an admissible Single Sampling Plan (SSP)

A Sampling Plan is said to be admissible if it satisfies the conditions

\[ L(AQL) \geq 1-\alpha \quad (1.4a) \]
\[ L(RQL) \leq \beta \quad (1.4b) \]

The classical method of designing a SSP is to search for the values of \( n \) and \( c \) so as to satisfy (1.4a) and (1.4b) as closely as possible. The situation warrants the use of type-B OC curve if \( n/N \leq 0.1 \) and \( p \geq 0.9 \). One method is to locate the Poisson parameter \( \lambda = np \) corresponding to the two percentiles at \((1-\alpha)\) and \( \beta \) of the Poisson distribution and thereby estimate \( n \). The search starts at \( c = 0 \) and terminates when the ratio \( R = np_0/np_1, \alpha \) crosses the threshold value \( R' = RQL/AQL \). This method often gives more than one plan, all of which satisfy one of the two risks exactly but not the other. It is however difficult to get a plan that meets both the risks exactly. This procedure requires cumulative Poisson tables, which are tabulated for specific values of \( \lambda \) and \( c \) only.

An interesting problem in the design of the plan is to estimate by an inverse procedure the Poisson parameter \( \lambda = np \) such that \( P(X \leq c|np) = AQL \). The same problem occurs when the Binomial distribution is used to define the OC function. In both cases it is difficult to locate the parameter from the cumulative probability distribution.

Let \( Pos(c|\lambda) = \sum_{i=0}^{c} \frac{\lambda^i}{i!} e^{-\lambda} = \omega \) be the cumulative Poisson distribution and let

\[ \text{Ch}[2\lambda, 2(c+1)] = \frac{1}{2^{(c+1)} \Gamma(c+1)} \int_{0}^{\infty} u^c e^{-u/\lambda} du \]

denote the cumulative Chi-square distribution with parameter \( 2\lambda \) and degrees of freedom \( 2(c+1) \).
The relationship between these distributions is given by (see Mittag and Rinne (1993))

\[ \text{Po}(c|\lambda) = 1 - \text{Ch} [2\lambda, 2(c+1)] \]

\[ \Rightarrow \text{Ch} [2\lambda, 2(c+1)] = 1-\omega \]

\[ \Rightarrow \chi^2_{2(c+1), 1-\omega} = 2\lambda \text{ from which we get} \]

\[ \lambda = 0.5 \{ \chi^2_{2(c+1), 1-\omega} \} \quad (1.5) \]

A similar relation between Binomial and F distributions exists given by

\[ B(c | n, p) = 1 - F \left( \frac{n-c}{c+1} \left( \frac{p}{1-p} \right) \right) \quad 2(c+1) ; 2(n-c) \]

where the LHS is the cumulative Binomial distribution up to \( c \) denoted by \( \omega \) and the RHS is the complementary F distribution up to \( c \) with \( \{2(c+1), 2(n-c)\} \) degrees of freedom.

Then the Binomial proportion \( p \) can be worked out with the formula for given values of \( n, c \) and \( \omega \).

\[ F_{2(c+1), 2(n-c), 1-\omega} = \left( \frac{n-c}{c+1} \right) \left( \frac{p}{1-p} \right) \quad (1.6) \]

Excel has built-in functions BINOMDIST and POISSON to find the OC values and CHINV and FINV to deal with inverse functions. These functions can be used in place of conventional statistical tables to determine \( n \) and \( c \). Details of these functions are discussed by Sarma (2001). Consider the following illustration.

Illustration-1.1

Let the plan parameters be \( n = 50, c = 2, AQL = 0.05 \) and \( RQL = 0.2 \). Then the Poisson parameter \( \lambda \) at AQL becomes \( \lambda_{AQL} = n^*\text{AQL} = 2.5 \).

The Excel function POISSON \((x, \text{Mean}, 1)\) gives \( P(X \leq c | \lambda) = \sum_{x=0}^c \frac{\lambda^x}{x!} e^{-\lambda} \).

The argument `1' indicates cumulative probability and the point probability mass can be obtained by taking this parameter as 0. With these values POISSON\((2, 2.5, 1)\) gives \( L(\text{AQL} | \lambda_{AQL}, c) = 0.5438 \). In the same way we get \( L(\text{RQL}) = 0.0028 \) with \( c = 2 \).
general we can evaluate L(p | λ,c) for different values of p by taking λ=np. Thus at AQL, the probability of accepting the lot becomes 0.5438.

Suppose α = 0.05 and β = 0.08 be the advertised risks. In the light of (1.4a) and (1.4b) this plan is not admissible as it violates (1.4a).

In the following illustration we obtain the value of n for a given value of c, α and AQL through appropriate inverse functions.

Illustration-1.2
Suppose α = 0.05, AQL = 0.10 and c = 0. We can find the Poisson parameter λ = np1-α from (1.5) with 1-α = 0.95 in the function CHINV. This gives λ = 2.996 so that an estimate of n at the AQL would be nAQL = np1-α/AQL = 29.96 ≈ 30. We can also get another estimate of n at the RQL by this method.

Thus the inverse functions available in Excel serve as quick tools to determine the parameters of a SSP instead of referring to tables, which were designed for fixed range of inputs.

1.6 DOUBLE AND MULTIPLE SAMPLING PLANS

Double Sampling Plan (DSP) is a procedure in which a second sample is required under certain circumstances before the lot is sentenced. The DSP contains four parameters

\[ n_1 = \text{size of the first sample}, \]
\[ c_1 = \text{acceptance number of the first sample}, \]
\[ n_2 = \text{size of the second sample and} \]
\[ c_2 = \text{acceptance number for both samples} \]
When the number of defectives in the first sample \( d_1 \) is greater than \( c_1 \) but less than or equal to \( c_2 \), a second sample of size \( n_2 \) is taken. If the number of defectives in combined sample \((d_1 + d_2) \geq c_2\) the lot is rejected. The probability of acceptance becomes

\[
P_a = p_a^1 + p_a^2,
\]

where \( p_a^1 \), \( p_a^2 \) are the acceptance probabilities of first and second samples.

An interesting aspect of DSP is that the inspector gets a second chance before sentencing lot in case of ambiguity. The question is on what do with the second sample, when the critical number is reached. One may opt for a complete inspection of \( n_2 \) items or curtail the inspection.

The Average Sample Number is a measure of effort required in judging the lots. It is given by

\[
\text{ASN} = n_1 + n_2 (1-p_1)
\]

where \( p_1 \) is the probability of lot dispositioning decision on the first sample.

Double Sampling can be designed in such a way that it get the same accuracy as if a single sampling plan of size \( n_1 + n_2 \) was adopted. The convenience lies in disposing the lots in the first sample itself when the quality is good (or bad). Double Sampling Plans with minimum ATI have been derived by Craig (1981).

The Multiple Sampling Plan (MSP) is an extension of DSP where the inspector takes several changes before disposing the lot. Though MSP has the advantage in recommending smaller samples at each stage it is a complex procedure to implement.

1.7 CONTINUOUS SAMPLING PLANS

In some cases production is done in such a way that lots are not formed at the time of inspection. Items that come out of a conveyor belt of a production-line or yarn coming out of a spinning section are some examples. Since there is no lot, the question of accepting
the lot does not arise. However, inspection is carried according to a procedure from the production line itself and the quality is assessed. Dodge (1943) has proposed such a plan and called it Continuous Sampling Plan (CSP). By its very nature, the CSP is not a sentencing procedure but acts as an audit that ensures the quality of the items, which leave the production line. It is a properly defined mixture of screening and sampling.

Dodge (1943), has proposed the CSP-1, which is a mixture of screening and sampling. At the start of the plan all units are inspected 100%. If 'i' consecutive non-defectives are found, screening is stopped and a fraction 'f' is inspected. If f = 1/10, it means one unit is selected at random out of 10 in the order and it is inspected. If the sampled unit is non-defective, we continue sampling from the next 10 units. If it is defective, we stop sampling and we resort to screening.

The objective of designing CSP-1 is to ensure that the outgoing fraction defective, average outgoing quality limit (AOQL) is kept at a minimum. The parameters (i, f) of the plan can be determined corresponding to an AOQL value.

There are many variations of CSP-1 resulting in CSP-2 and CSP-3. Wald and Wolfowitz (1945) have proposed another CSP type plan.

A Graphical Method of selecting the parameters for CSP-1, CSP-2, and CSP-R Under a Non replacement Assumption was developed by Abraham (1971), which is like a nomogram.

1.8 ALGORITHMS FOR DESIGNING A SINGLE SAMPLING PLAN

The conventional approach to design a sampling plan is based on the OC curve such that the plan has admissible risks at AQL and RQL. Theoretically there exists several
admissible plans for a given set of input parameters and we have to choose the one that minimizes the sample size \( n \). Analytical procedures leading to a closed form solution are not possible in case of Hypergeometric, Binomial or Poisson distributions and we have to use search methods. The Guenther's search algorithm (1959) and the Peach-Littauer's algorithm (1946) are two important search procedures that are popularly used. Hally (1980) has given a computerized approach for determining a SSP that yields the minimum sample size. The other methods include the procedure by Graf et al (1987) and the Phillips system. Mittag and Rinne (1993) have discussed these algorithms in detail. Chakraborty (1989) has formulated the single sampling plan problem as a non-linear mixed integer goal-programming problem.

In the following section we review and compare these methods.

(a) Guenther's search algorithm

This algorithm starts with \( c = 0 \) and \( n = 1 \). With the underlying distribution like Hyper Geometric, Binomial or Poisson, the OC value \( L(RQL) \) is evaluated at the RQL. In the outer loop by taking \( c = 0 \) we increase \( n \) stepwise by one, until \( L(RQL) \leq \beta \). Using these \( n \) and \( c \) if \( L(AQL) \geq (1-\alpha) \) is satisfied, we have found the plan with minimal sample size. Otherwise take \( c = c+1 \) in the outer loop and repeat the inner loop with \( n = c \). The entire procedure should be repeated until the admissibility conditions are satisfied. The plan derived by this method will have the property that among all admissible plans it has the minimum difference between effective and specified producer's risk and maximum difference between effective and specified consumer's risk. This algorithm can be implemented in any programming language or even in the Excel spreadsheet with Solver module.
(h) Peach-Littauer’s search algorithm

This is another procedure proposed by Peach and Littauer (1946) to determine n and c, when the underlying distribution is Poisson. Given the values of AQL, RQL, α, and β this method initially starts with c = 0. The OC percentiles using Chi Square approximation, given by

\[ q_1(c) = \frac{X^2_2(c+1), 1-\beta}{2.\text{RQL}} \]

and

\[ q_2(c) = \frac{X^2_2(c+1), \alpha}{2.\text{AQL}} \]

are calculated iteratively for different values of c = 0, 1, 2, 3..., until q_1(c) ≤ q_2(c). The value of n corresponding to this c is the smallest integer, which lies between q_1(c) and q_2(c).

(c) Graf, Henning, Stange, Wilrich algorithm (Graf et al algorithm)

This is a one step formula to determine n and c, which is an alternative to Guenther’s search algorithm when the statistic follows Binomial distribution. The approximation formula proposed by Graf et al is

\[ n = \frac{1}{4}\left( \frac{Z_{1-\alpha} + Z_{1-\beta}}{\phi_2 - \phi_1} \right)^2 - \frac{1}{4\phi_1 \phi_2} \]  

(1.7)

where \( \phi_1 = \text{Arc sin} \left( \sqrt{\text{AQL}} \right) \), \( \phi_2 = \text{Arc sin} \left( \sqrt{\text{RQL}} \right) \) and \( Z_\alpha \) is defined to be the percentile of order α from N(0,1). With this n, one obtains the acceptance number c as

\[ c \equiv n \{ \sin^2 \phi^* \} - 0.5 \]  

(1.8)

where \( \phi^* = \frac{1}{2} (\phi_1 + \phi_2) \left( 1 - \frac{1}{8n \phi_1 \phi_2} \right) + \left[ \frac{Z_{1-\alpha} + Z_{1-\beta}}{4\sqrt{n}} \right] \)

This formula is easy to implement but the admissibility conditions are not always satisfied by the plan. In chapter-2 we modify this plan by attaching a search procedure around the solution space.
(a) Philips system

This is also an approximation procedure proposed by the Dutch electro corporation Philips to construct sampling plans \( (N ; n , c) \) with given lot size \( N \). This formula is based on the elasticity of the OC function at the indifference quality level (IQL). When the test statistic follows Poisson distribution the following approximation formulae for determining \( n \) and \( c \) are used by the Philips system.

\[
c = \frac{K}{2} \left[ e \left( b_{0.5}^p \right) \right]^2 - 0.73 \quad \text{and} \quad n \approx \frac{c + \frac{2}{3}}{p_{0.5}^p},
\]

where \( e \left( P_{0.5}^p \right) = -2 \left( \frac{\ln \left( \frac{P_{0.5}^p}{c!} \right)^{c+1}}{c!} \right) e^{-\ln P_{0.5}^p} \)

or

\[
c \left( P_{0.5}^p \right) \approx -\sqrt{\frac{2}{\pi}} (c + 0.73) \quad \text{where} \quad P_{0.5}^p \approx \frac{c + \frac{2}{3}}{n}.
\]

Other methods include mathematical programming like the one given by Chakraborty (1989).

In the following section we discuss a spreadsheet solution to derive the plan using the method given by Graf et al.

1.9 COMPUTER PROGRAMS FOR SAMPLING PLANS

As an alternative to reference to sampling plan tables, efforts were made by researchers to develop a computer program that gives the optimum sampling plan for the given inputs.

Snyder and Storer (1972) have prepared a FORTRAN program to determine Single Sampling Plans with given AQL, RQL, \( \alpha \) and \( \beta \). Their approach is based on Poisson distribution and the program gives four different plans. Plan-1 gives minimum value of \( n \)
and will be close to $\beta$ risk. Plan-2 give maximum $n$ and will be close to $\alpha$ risk. The next two samples will have sample sizes, which will minimize the weighted sum and ratio of two risks respectively.

Haley (1980) developed a FORTRAN program to determine the minimum size sampling plan using Guenther's search procedure. This program is based on Binomial and Poisson OC functions and yields an admissible plan. Separate subroutines for Binomial and Poisson have been used for this purpose.

Appendix-2 contains a C-program to determine the single sampling plan using Guenther's algorithm.

Spreadsheet programs are convenient to deal with statistical functions and hence sampling plans can also be studied in on spreadsheets instead of writing programs. Further graphs relating to the sampling plans can be directly prepared and simulated on spreadsheet.

1.10 SKIP LOT SAMPLING PLAN

Dodge (1955) has originally developed the Skip Lot Sampling Plan (SkSP) as an extension of the Continuous Sampling Plan (CSP) for lots, which he called SkSP-1. Under this method each lot or unit of production is inspected by a single determination (called screening). When $t$ consecutive lots are found free of defects, the method of inspection is relaxed and a fraction $f$ of the incoming lots is only inspected (called sampling). The idea is to reduce the effort of inspection when the quality history is found to be good in a series of lots. Interestingly in this method, certain percent of defective lots are likely to escape inspection and reach the customer. One measure of performance of this plan is the percentage of lots accepted by the procedure, denoted by $L_{sk}(p)$ where $p$ is the probability of accepting a lot by the sampling procedure.
In SkSP-2 the decision to accept or reject a lot depends on the outcome of a reference plan, which is usually taken as Single Sampling Plan (SSP). The inspection procedure is similar to that of SkSP-1 except that lot sentencing is based on the outcome of the reference plan. The following are the steps to implement SkSP-2.

a) Start inspection with *screening* in which each incoming lot is inspected by SSP until 'i' successive lots are accepted (i ≥ 2).

b) Switch to *skip mode* in which only one out of 'k' lots is randomly selected and inspected. If this lot is accepted, the remaining (k-1) lots are also accepted and out of the next k lots, another one is selected and inspected. This method is continued until the single lot under skipping inspection is rejected, in which case we switch back to screening.

It is assumed that every rejected lot is replaced with a good one. Perry (1973) and Dodge and Perry (1971) have given a basic treatment of SkSP by using Markov Chain approach to determine the percentage of lots accepted under the plan. Soundararajan and Vijayaghavan (1989) have proposed a modification, called SkSP-3 in which the idea is to inspect next k consecutive lots whenever a lot under skipping inspection is rejected (instead of immediately switching to screening). If any of these lots is rejected then screening inspection is repeated.

Here are some basic results of interest.

1. The proportion of lots accepted by the plan is given by

\[ L_{sk}(p) = \frac{(1 - f)L(p)^i + L(p)}{(1 - f)L(p)^i + f} \]

where \( p \) = incoming quality level
1.\( (p) \) - acceptance probability based on the reference plan

\( \pi = \) sampling fraction

This function is considered as the OC function of SkSP-2.

2. The Average Fraction Inspected is given by

\[ \text{AFI}_{sk} = \frac{f}{(1-f)L(p) + f} \]

3. If the reference plan is a single sampling plan with sample size \( n \), then the Average Sample Number of the SkSP-2 is \( \text{ASN}_{sk}(P) = n \cdot \text{AFI}_{sk}(P) \).

4. The Average Outgoing Quality (AOQ) is the proportion of the outgoing nonconforming lots.

It is clear that given the values of \( \pi \) and \( f \), the performance of SkSP depends on the characteristics of the reference plan and the incoming lot quality.

Not many research articles are reported in literature on SkSP in the recent years. However, many interesting modifications have been made in the study of Continuous Sampling Plans which can be directly extended to SkSP procedures.

There are two important aspects of working with these plans. The first one is related to the development of a theoretical basis for the inspection procedure and the other one is the numerical evaluation of the plan.

In chapter-3 these aspects are discussed.

1.11 INSPECTION ERRORS AND THEIR INFLUENCE ON SAMPLING PLANS

One of the basic assumptions in the attribute type inspection is that the inspection process is free of errors. In practice it is not so and errors occur in inspection. This leads to misclassification of good items into bad and vice versa.
a) Problem of misclassification

An attribute gage is used to compare a dimension with standards and the inspector classifies the item into good or bad basing on the result of gage test. Over a period of time the gage itself becomes faulty, leading to wrong acceptance or rejection. It is also possible that the operators may not perform consistently even though they use the same gage. The concept of measurement system analysis focuses on two aspects of errors of a measurement system namely (a) Repeatability and (b) Reproducibility.

- **Repeatability:** It is the variation in the measurements obtained by one measuring instrument when used several times by one appraiser while measuring the identical characteristic on the same part.

- **Reproducibility:** It is the variation in average of measurements made by different appraisers using the same measuring instrument when measuring the identical characteristic on the same part.

The gage is acceptable, if all the measurement decisions agree. In the above case, each part is measured four times. If the measurements do not agree the gage should be improved or reevaluated.

b) Probabilistic aspects of inspection errors

There are two types of inspection errors similar to those, committed in tests of hypothesis. These are as follows.

**Type I Inspection error:** It is the error of classifying a good item as bad item.

**Type II Inspection error:** It is the error of classifying a bad item as good item.

Mittag and Rinne (1993) used the following notation to explain inspection errors.
Define $I = 0$ if sample unit is "good"
   - $I = 1$ if sample unit is "bad"

$I$ is unobservable stochastic variable that represents the true state of the unit.

Define $K = 0$ if sample unit is classified as "good"
   - $K = 1$ if sample unit is classified as "bad"

$K$ represents the outcome of inspection of a sample unit. The risks associated with type I and type II errors are basically conditional probabilities defined as follows.

**Type I risk:** ($\varepsilon$)

$$
\varepsilon = P(K = 1 | T = 0) = P(\text{classifying a good item as bad})
= \frac{P(K = 1, T = 0)}{P(T = 0)}
$$

**Type II risk:** ($\phi$)

$$
\phi = P(K = 0 | T = 1) = P(\text{classifying a bad item as good})
= \frac{P(K = 0, T = 1)}{P(T = 1)}
$$

It is practically difficult to estimate these risks exactly. Bauer (1987) has observed that they can take values as high as $\varepsilon = 0.4$ and $\phi = 0.9$.

c) **Apparent Fraction Defective**

Let the inspection process be carried out with one inspector using go-no-go gage and let $p$ be the true fraction defective of the incoming lots. Since $p$ is usually not known we can investigate the expected value or the observed fraction defective for a lot, in case of 100% inspection. The probability that a defective unit is correctly classified as defective becomes

$$
P(K = 1 | T = 1) = 1 - \phi.
$$
Since \( 0 \leq \phi \leq 1 \), only when type-I error is present the true fraction defective is underestimated as \( p(1-p)\phi \). Again when type-II error is present certain defective will be classified as good. Since \( 0 \leq \epsilon \leq 1 \) and \((1-p)\) is the proportion of good items and it is over-estimated as \( p(1-\phi) \).

The inspector may commit one of these two errors but only one error at a time. The two events will be mutually exclusive and the expected fraction defective becomes

\[
\pi = p(1-\phi) + (1-p)\epsilon
\]  (1.11)

This is called the observed fraction defective or apparent fraction defective.

It is also possible that an inspector may consistently commit only type I error or type II error over a period of time. It is a chronic bias. Thus when inspection errors are present, the true fraction defective of a lot will distorted. It will show its effect on the performance measure of the sampling plan, namely the OC function.

d) Behaviour of OC function in presence of errors

Consider the case of a single sampling plan with parameters \( n \) and \( c \). Assume that a type B OC function is used with Binomial distribution. When the lot fraction defective is \( \pi \), the OC function in presence of errors becomes

\[
L_c(p, n, c) = \sum_{i=0}^{c} \binom{n}{i} \pi^i (1-\pi)^{n-i} = \sum_{i=0}^{c} \binom{n}{i} [(p(1-\phi)+(1-p)\epsilon)\phi + (1-p)(1-\epsilon)]^{n-i} \]  (1.12)

Since, the lot sentencing (decision making) is based on apparent fraction defective \( \pi \) instead of \( p \), the ability of the plan to distinguish between good and bad lots depends on the empirical OC function given in (1.12). In fact, the results of the plan do not match with the specifications for which the plan was originally constructed.
When only type I error occurs we get $\phi = 0$ and $c > 0$. Then $\pi = p + (1-p)c$, which is an under estimate of $p$. This leads to $\pi > p$ and results in a decrease in the acceptance probability of the lot.

It means

$$L_e(p, n, c) < L(p), \quad \text{for} \quad p \in [0,1]$$

When only type II error is present, we get $\pi = p (1- \phi)$, which is less than $p$. This leads to

$$L_e(p, n, c) > L(p), \quad \text{for} \quad p \in [0,1]$$

More details on these effects can be found in Mittag and Rinne (1993).

c) ATI, AOQ and other measures

With reference to the single sampling plan under rectifying inspection it is known that

$$\text{ATI} (p) = \frac{n + (N-n)[1 - L(p)]}{1 - p} \quad (1.13)$$

In the presence of inspection errors, the empirical ATI function becomes

$$\text{ATI}_e(p) = \frac{n + (N-n)[1 - L_e(p)]}{\varphi + (1-p)(1-c)} \quad (1.14)$$

The Average outgoing quality is also affected by the apparent fraction defective and given by

$$\text{AOQ}_e = \left\{ \frac{p}{N} \right\} \frac{n \varphi + (N-n)[p \varphi + (1-p)(1-c)L_e(p) + \varphi (1-L_e(p))]}{\varphi + (1-p)(1-c)}$$

The effect of these errors on the AOQ function has been discussed by Mittag and Rinne (1993). It is observed that the effect of errors on the AOQ is similar to the effect on the OC curve.

In chapter 4 a theoretical model to estimate the apparent fraction defective using truncated Beta distribution is developed.
1.12 REVIEW OF RECENT LITERATURE

The new results and procedures proposed in this thesis are derived from fundamental concepts of sampling inspection. There are very few papers for which the results of this thesis can be viewed as extensions. However, some interesting aspects discussed by various researchers are reviewed here.

Brush and Bernard (1990) have developed a procedure to estimate the outgoing quality in the context of quality assurance program. They have proposed an estimator called QMP (Quality Measurement Plan) estimator. It is claimed to be robust and provides an interval estimate for quality.

Chun and Rinks (1998) have proposed three types of producer's and consumer's risks and called them classical, modified and Bayes risks. This classification is related to inspection errors. They have demonstrated these results using Binomial-Beta combination for Bayes estimators.

A comprehensive survey of literature on inspection errors is given by Dorris and Foote (1978).

Greenberg and Stokes (1992) have studied the problem of estimating non-conformance rates using zero defect sampling with rectification. They have proposed a non-parametric estimator which is very much similar to Horvitz-Thompson estimator (Cochran(1977)).

Greenberg and Stokes (1995) have studied the effect of inspection errors on repetitive testing which is a common procedure when rejections are costly. They have constructed a model from which they expected benefit of retests could be calculated and maximized. This study has specific application to semiconductor devices.
Sufficient focus was given on estimating the non-conformity rate in zero-defect sampling by Hahn (1986). Zaslavsky (1988) has extended the estimation procedure to c-defect sampling and derived confidence intervals. The objective is to estimate the total number of defective units in accepted lots.

A different type of inspection plan based on Maximum Allowable Percent Defective (MAPD) has been developed by Soundararajan (1975) which was further explored by Suresh & Ram Kumar (1996) and Suresh, Radhakrishnan and Alamedu (2002). These plans are of the type CSP-1 (c = 2) and the idea of Maximum Allowable AOQ has been used to derive matching sampling plans. Suresh, Radhakrishnan and Kavitha (2002) have used the idea MAAOQ to derive a TNT - Plan and constructed tables. These plans are matched with plans indexed through MAPD and AOQL.

Beainy and Case (1981) have developed performance measures of the AOQ and ATI type when inspection is imperfect. Using the idea of signal detection theory, they have demonstrated that these measures are influenced by the magnitude of inspection errors.

Johnson, Kotz and Rodriguez (1986) gave a detailed narration on the effects of imperfect inspection on Double Sampling and link sampling. Harichandra and Srivenkataramanana (1982) have developed theory on link sampling for attributes.

In the recent years user-friendly software has been developed to work with sampling plans for instance stat-graphics, QM for windows, Indostat etc have specific modules to deal with sampling plans.

Ghosh (1988,1989,1990) has developed optimum continuous sampling plans under different condition.
Chakraborty and Umesh kumar (1989) have used GERT (Graphic Evaluation and Review Technique) approach to analyze inspection errors, which is a non-conventional approach. Martz and Zimmer (1990) have used non-parametric approach to estimate the percent defectives in accepted lots. Their approach is called Bayes-Empirical-Bayes (BEB) approach.

1.13 FOCUS OF THE THESIS AND LAYOUT

The focus of the thesis is on two aspects

1) Computational aspects in determining the parameters of a plan. The use of spreadsheet solutions in place of reference to tables is highlighted.

2) Misclassification and inspection errors, their characterization, computational facilities and their impact on the sampling plan.

The following is the tentative layout of the thesis.

Chapter-2 deals with the determination of the parameters of SSP by using the Modified Graf et al Method (MGM). It is a spreadsheet procedure and the solution obtained by MGM is compared with that of conventional algorithm given by Guenther. It is found that the MGM gives a better plan quicker than the existing methods.

Chapter-3 is concerned with some computational experiences regarding SkSP in which the reference plan is generated with the help of MGM and the parameters of SkSP are automatically derived on the Excel Sheet. The OC, ASN and the AOQ are obtained with this method and compared with the SkSP plan when the reference plan is based on Guenther(1969) and Peacji & Littauer(1946).

Chapter-4 is concerned with characterization of inspection errors. The theoretical behavior of type-1 and type-2 risks of misclassification have been modeled by using a
truncated Beta distribution. The effect of this distribution on SSP and on the resulting SkSP are examined and analyzed.

Chapter-5 is concerned with a special type of inspection in which from each lot three samples of equal size are taken from top, middle and bottom of the lot. The samples are mixed and the quality is estimated from it. A skip lot-sampling plan based on this type of estimation is developed in this chapter.

The thesis ends with an outline of the scope for further research.